Estimating Economies of Scale and Scope using a Flexible Technology

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Economies of scale and scope are typically modelled and estimated using production possibilities that are shared by all firms in an industry irrespective of whether they specialize in a single output or not. We suggest a general model that does not make this assumption and show how it can be estimated using standard functional forms like the quadratic and the tranlog forms. Our application is for US public electric companies. The results suggest that assuming common production possibilities for integrated and specialized firms has no support. In particular, not allowing specialized firms separate technologies might bias estimates for economies of scope upwards.

1 Introduction

The estimation of economies of scale and scope has been of interest since Baumol et al. [1988] showed their importance for the theory of contestable markets. The theory has swayed many countries to introduce competition into industries that formerly were considered natural monopolies. In many network industries the introduction of competition was accompanied by vertical restructuring to facilitate competition. Such restructuring requires the careful weighting of gains from competition against economies of scope [Michaels, 2004]. Another example for the importance of estimating economies of integration is the current debate on the restructuring of the banking industry to ring-fence risk and prevent future financial recessions.

The estimation of economies of scale and scope requires a model of cost. Duality theory[1] allows us to estimate a cost function that is consistent with the underlying production technology. Almost the entire literature on the estimation of economies of integration (i.e. scale and scope) follows the suggestion of [Baumol et al., 1988, p. 445] to model a cost function based on "the production technology of the firms in an industry". That is all the firms in an industry share the same production possibilities. This implies that economies of integration only depend on differences in cost levels across different firms but not on differences in technologies. Such a restriction of the production possibilities for instance denies specialized firms to have different production possibilities than integrated firms potentialy leading to biassed estimates for economies of integration. Recently, Weninger [2003] and Bottasso et al. [2011] introduced models that allow flexibility across firm types. Bottasso et al. [2011] use a Generalized Composite model [Pulley and Braunstein, 1992] and allow different types of firms to have different technologies. They find that indeed integrated and specialized firms have different technologies and that estimates of economies of integration are not robust to the assumption of a common technology.

Our model allows for separate technologies across firm types by making the cost function discontinuous using a set of dummy variables. This approach can be applied to any functional form including the popular translog form [Christensen et al., 1973]. This is important because despite its many advantages the non-admission of zero values (the log of zero is not defined) does not allow the use of the translog form for the estimation of economies of scope [Caves et al., 1980]. Our approach is more general than the model used by Bottasso et al. [2011] because it allows zero values not only for outputs but also for input prices (or any other variables) . Also, our model is easier to implement for the applied researcher as it is linear and all coefficients have direct economic interpretations. It is also more general than the proposal by Weninger [2003] to estimate separate translog functions because it allows for inference to test the common technology assumption. It allows testing for differences in technology using a unified approach, thereby documenting whether the results are driven by unnecessary or unwanted restrictions on the underlying technologies.

We demonstrate the usefullness of our model by estimating economies of scale and scope for a sample of US electric companies. Our data is suitable for this task as it comprises both specialized and integrated firms. Many previous studies only had integrated firms available forcing them to estimate economies of scope based on cost complementarity only. For our application we use both a quadratic and a translog form and compare both against the results from a quadratic form assuming a common technology.

Our results provide evidence that the assumption of a common technology leads to an upward bias for estimates of economies of scope. This would suggest that at least for our industry and sample specialized firms use technologies that are not necessarily equivalent to technologies used by integrated firms. Also we find that economies of scale vary considerally between firm types and again average estimates across firm types might not contain much useful information.

This paper is organized into sections. Section 2 summarizes the relevant literature. Section 3 shortly sets out the theories of economies of scale and scope. Section 4 introduces our model and outlines our

[1]Duality theory and the implied retrictions on the cost function ensure that the latter does not violate the physics of production. That is the cost function is a dual to the underlying production possibilities. For an introduction see survey see [Fuss and McFadden, 1978].

analystical approach. Section 5 introduces the data and defines the variables. Section 6 presents the results and section 7 gives a short conclusion.

2 Literature

There is a vaste number of studies that estimate economies of scale and scope for various industries. We do not review this literature here but note that for various industries different studies have found different results [Arocena et al., 2009] underlining the importance of model choice. Instead we provide a short summary of the debate on how to model and estimate cost functions for multiproduct industries.

Almost all studies in the parametric literature use a model of an industry level production technology assuming poolability across different firm types. Even if the firms in the industry produce multiple products they share a *single* technology. But there are reasons to believe that such an assumption is unjustified [Weninger, 2003]. For instance, cost complementarity can lead to differences in the cost structure between diversified and specialized firms (the latter by definition produce no complementary goods). Bottasso et al. [2011] allow costs to depend on the firm type using a Generalized Composite function (using a Box Cox transformation). They found that it is an undue restriction to impose a common technology for two types of water companies in England and Wales, water-and-sewage and water-only companies. As far as we are aware these are the only two studies that allow for different technologies across firm types that use parametric estimators. In a related literature that uses Data Envelopment Analysis estimators Charnes et al. [1978] to measure economies of scope always use models that allow for different technologies across firm types because here it is these differences that underly economies of scope [Färe, 1986, Färe et al., 1994].

Irrespectively of whether the model allows separate technologies or not the researcher has to specify a functional form for the model. For multiproduct cost functions Caves et al. [1980] set out three criteria for the ex-ante choice of functional forms: satisfaction of regularity conditions, limited number of parameters, ability to admit zero values for some outputs. The latter is important because the theory of economies of scope (i.e. the costs and benefits of joint production) in a multi-output setting requires the comparison of costs between specialized and integrated firms. Specialization requires that the production of at least one of the outputs is zero [Baumol et al., 1988, Kittelsen and Magnussen, 2003]. Based on these criteria the authors suggest the Generalized translog (using a Box Cox transformation) as the most suitable form. However, in the applied literature the most popular functional form is the quadratic as it readily admits zero values and is easy to implement [Kaserman and Mayo, 1991, Greer, 2008, Jara-Diaz et al., 2004, Arocena et al., 2009, Kwoka, 2002]. But imposing homogeneity in input prices as a regularity condition on the quadratic form sacrificies flexibility [Caves et al., 1980, p. 478].[2] Other applied studies use the Generalized translog [Eftekhari, 1989], the Composite [Fraquelli et al., 2005], or Generalized Composite form [Bottasso et al., 2011]. These forms are less popular because they are non-linear and for the Composite the individual coefficients have no economic meaning. Also some studies use a translog form and replace zeros with small positive numbers before the logarithmic transformation or use a subsample of firms with strictly positive outputs only. When the sample is restricted to integrated firms cost complementarity is the only source for economies of scope. Although complementarity is sufficient it is not necessary for the existence of economies of scope as shared fixed costs are another potential source [Baumol et al., 1988]. Both the replacement of zeros and the restriction of the sample to integrated firms introduce unknown bias [Gunning and Sickles, 2009, Berger et al., 1987].

Last, the robust estimation of economies of scope requires the observation of specialized firms. Many studies do not include specialized firms at all making it impossible to estimate economies of scope without extrapolation. Mostly the reason is the non-existence of specialized firms in the industry. Although the absence of specialized firms might be taken as *prima facie* evidence for the existence of economies of scope it is not obvious that existing industry structure are only driven by costs

[2] Several authors argue that normalization can circumvent this problem but it is not clear how [Martinez-Budria et al., 2003].

considerations particularly for regulated industries. For instance Berger et al. [1987] for banking and Kwoka [2002] for electric utilities observe no specialized firms and therefore introduce unknown extrapolation error.

3 Economies of Integration with a Flexible Technology

This section sets out the theory of economies of scale and scope and modifies it to allow for different technologies across firm types. Formally, we have products $i = 1 \ldots N$ with $q^U = 1 \ldots k$ upstream products and $q^D = k+1 \ldots N$ downstream products and firm types $T = (I, U, D)$ for integrated, upstream, and downstream firms respectively. Baumol et al. [1988] define economies of scope at output bundle q as

$$S_N(q) = \frac{C(q)}{\sum_{i=1}^{n} q_i C_i(q)} \tag{1}$$

where for simplicity we ignore input prices and where C is total costs and C_i is the first derivative of cost with respect to output i. Returns to scale are said to be increasing, decreasing or constant as S is greater than, less than, or equl to unity, repectively. Economies of scope between two successive stages of production with outputs q^U for the upstream stage and q^D for the downstream stage where the two output sets are orthogonal are defined as

$$SC_{U,D}(q) = \frac{C(q^U, 0) + C(0, q^D) - C(q^U, q^D)}{C(q^U, q^D)} \tag{2}$$

The degree of economies of scope SC is measured by 2 where the separation of production is said to increase, decrease or leave unchanged the total cost as SC is greater than, less than, or equal to zero, respectively. Equation 2 shows that the estimation of economies of scope (i.e. the costs and benefits of joint production) requires the comparison of costs between specialized and non-specialized firms.

Baumol et al. [1988] and almost the entire empirical literature that estimates economies of scale and scope assume that there is a common cost function C shared by the different firm types. The assumption is that different firms share the same production possibilities though might operate in different regions of the production set. Following Weninger [2003] and Bottasso et al. [2011] we allow different firm types to have different production technologies. Essentially we allow the cost function to be discontinuous across firm types where

$$C = \begin{cases} C^I\left(q^U, q^D\right) \\ C^U\left(q^U, 0\right) \\ C^D\left(0, q^D\right) \end{cases} \tag{3}$$

Unlike the Generalized Composite model used by Bottasso et al. [2011] our model allows for any variable to take on zero values and not only output. For instance, for our application below firms that only distribute but do not generate electricity do not face a price of fuel. Using our cost function in 3 we can rewrite the textbook definition of economies of scale and scope. For scale we rewrite 1 as

$$S_N^T(q) = \frac{C^T(q)}{\sum_{i=1}^{n} q_i C_i^T(q)} \tag{4}$$

where returns to scale now depend on the firm type. Similarly, for economies of scope we rewrite 2 as

$$SC'_{U,D}(q) = \frac{C^U(q^U, 0) + C^D(0, q^D) - C^I(q^U, q^D)}{C^I(q^U, q^D)} \tag{5}$$

where we now allow for different cost function for the three firm types. Unlike in Baumol et al. [1988] both differences in cost levels and differences in technology drive economies of integration. What is important is that this model allows for a counterfactual where firms that specialize can also specialize by choosing a different technology.

We do not derive any formal hypothesis on how this change in model affects the estimates for economies of scale and scope but when asserting that the motivation for specialization is efficiency our model should provide lower estimates for economies of scope than the standard model.

4 Modelling and estimation approach

To help assess our own model we compare it to the widely used quadratic model. The quadratic form [Röller, 1990] readily admits zero values and has been popular for the estimation of economies of scope. For a dual cost function $C(q, w, z)$ where C is total long-run costs and y, w, z stand for output quantity, input price, and controls respectively equation 6 gives the quadratic form

$$C = \alpha_0 + \sum_i \alpha_i q_i + \frac{1}{2}\sum_i \sum_j \alpha_{ij} q_i q_j + \sum_i \beta_i w_i + \frac{1}{2}\sum_i \sum_j \beta_{ij} w_i w_j + \quad \textstyle\sum_i \sum_i \theta_i q_i w_i \tag{6}$$

where α, β, and θ are unknown parameters. This model obviously assumes that the parameters are the same for different firm types though some authors include firm specific intercepts.

The alternative is to estimate 3. The most straightforward way to do so is to estimate separate models for each firm type [Weninger, 2003]. We propose the use of dummy variables to differentiate between the technologies. Equation 7 illustrates the separation with the help of three dummy variables I, D and U which take the value one if the firm is integrated or specializes in the downstream or upstream activity respectively. The first line of equation 7 represent integrated firms and is "activated" I takes the value one. The second and third lines represent downstream and upstream only firms respectively. We refer to this model as a flexible technology model as oppossed to joint or common technology models.

$$\begin{aligned} C &= I * C^I \left(q^I, w^I, \Gamma^I\right) \\ &+ D * C^D \left(q^D, w^D, \Gamma^D\right) \\ &+ U * C^U \left(q^U, w^U, \Gamma^U\right) \end{aligned} \tag{7}$$

where Γ are the firm specific unknown parameters. This model allows both the variables and associated parameters to vary between the three firm types. $C(\bullet)$ can take any functional form including a translog form. When using the translog form the model requires the following adjustment. Before taking logs we replace zeros with any positive number. Then we multiply all variables with the first dummy, then the seond, and so on. This has the result that previous replacement is negated for the appropritе firm as the multiplication of any value with zero equals zero. Essentialy we obtain three separate data sets that are stacked before estimation.

For our application below we will use both a quadratic and translog form[3] for 7 but in the remainder of this section we will concentrate on the translog. Applying a translog form to 7 we estimate the following model

[3]We would like to thank Luis Orea for his very helpful comments on variable selection and specification.

$$
\begin{aligned}
\ln C = I \Bigg[& \alpha_0^I + \sum_{i=1}^{N^I} \beta_i^I \ln q_i + \sum_{j=1}^{G^I} \gamma_j^I \ln w_j + \frac{1}{2} \sum_{i=1}^{N^I} \sum_{j=1}^{N^I} \rho_{ij}^I \ln q_i \ln q_j + \frac{1}{2} \sum_{j=1}^{G^I} \sum_{k=1}^{G^I} \lambda_{jk}^I \ln w_j \ln w_k \\
& + \sum_{i=1}^{N^I} \sum_{j=1}^{G^G} \theta_{ij}^I \ln q_i \ln w_j + \sum_{l=1}^{Z^I} \xi_l^I z_l + \sum_{l=1}^{Z^I} \tau_l^I z_l^2 \Bigg] \\
+U \Bigg[& \alpha_0^U + \sum_{i=1}^{N^U} \beta_i^U \ln q_i + \sum_{j=1}^{G^U} \gamma_j^U \ln w_j + \frac{1}{2} \sum_{i=1}^{N^U} \sum_{j=1}^{N^U} \rho_{ij}^U \ln q_i \ln q_j + \frac{1}{2} \sum_{j=1}^{G^U} \sum_{k=1}^{G^U} \lambda_{jk}^U \ln w_j \ln w_k \\
& + \sum_{i=1}^{N^U} \sum_{j=1}^{G^U} \theta_{ij}^U \ln q_i \ln w_j + \sum_{l=1}^{Z^U} \xi_l^U z_l + \sum_{l=1}^{Z^U} \tau_l^U z_l^2 \Bigg] \\
+D \Bigg[& \alpha_0^D + \sum_{i=1}^{N^D} \beta_i^D \ln q_i + \sum_{j=1}^{G^D} \gamma_j^D \ln w_j + \frac{1}{2} \sum_{i=1}^{N^D} \sum_{j=1}^{N^D} \rho_{ij}^D \ln q_i \ln q_j + \frac{1}{2} \sum_{j=1}^{G^D} \sum_{k=1}^{G^D} \lambda_{jk}^D \ln w_j \ln w_k \\
& + \sum_{i=1}^{N^D} \sum_{j=1}^{G^D} \theta_{ij}^D \ln q_i \ln w_j + \sum_{l=1}^{Z^D} \xi_l^D z_l + \sum_{l=1}^{Z^D} \tau_l^D z_l^2 \Bigg] \quad + v_i
\end{aligned} \tag{8}
$$

And similiarly for input prices and control variables. The Greek letters stand for the unknown parameters. Duality requires the following constraints. We impose symetry of the Hessian matrix

$$
\rho_{ij} = \rho_{ji}; \lambda_{jk} = \lambda_{kj} \tag{9}
$$

and impose linera homogeneity in input prices

$$
\sum_{j=1}^{G} \gamma_j = 1; \sum_{i=1}^{N} \sum_{j=1}^{G} \theta_{ij} = 0; \sum_{j=1}^{G} \sum_{k=1}^{G} \lambda_{jk} = 0; \tag{10}
$$

which is imposed by dividing costs and input prices by one arbitrarily choosen input price and dropping the respective share equation. Using Shephard's Lemma Equation and the symmetry contraint we obtain share equation 11 for input i.

$$
\begin{aligned}
s_i = \quad & I \quad \left[\gamma_j^I + \sum \theta_{ij}^I q_i + \sum \lambda_{jk}^I w_j \right] \\
+ \quad & U \quad \left[\gamma_j^U + \theta_{ij}^U q_i + \sum \lambda_{jk}^U w_j \right] \\
+ \quad & D \quad \left[\gamma_j^D + \theta_{ij}^D q_j + \sum \lambda_{jk}^D w_j \right]
\end{aligned} \tag{11}
$$

We estimate this system of the cost function and share equations using the iterated maximum likelihood seemingly unrelated regression (SURE) technique [Zellner, 1962]. The additional structure imposed by the share equations makes the estimates more efficient. All independent variables are demeaned so that the translog expansion is around the sample mean and the first order coefficients can be interpreted as marginal effects at the sample mean.

One advantage of this approach over the estimation of separate equations is that we can readily apply standard test for inference across the types of technologies. For instance, do inference on the equality of the three different technologies. To test whether the parameters of the technology are statistically equal across firm types we test the following Null hypothesis using a Likelihood-ratio test.

$$C^I\left(q^I\right) \equiv C^U\left(q^U\right) \equiv C^D\left(q^D\right) \tag{12}$$

Using the quadratic model costs for the specialized firms are predicted by setting the respective outputs to zero. Our model is more flexible because we can predict the costs of the different firm types using 8. So the first term in the numerator of 5 is the predicted cost using the second bracketed term in 8. Below we report estimates for economies of scale and scope at the sample mean and the mean of the observation specific estimates. As ususal we can only provide observation specific estimates for integrated firms.

5 The Data and Variables

The data is for US local government owned electric utilities. The data comprises three firm types: upstream, integrated, and downstream. Whereas upstream firms generate electricity downstream firms distribute electricity and integrated firms engage in both activities. The data is an unbalanced panel for the years 2000 to 2003. Table 1 illustrates the distribution of firms across the output space (using the first distribution output only). The table gives the firm-year count by size bracket for the upstream and downstream activities. The first row and first column give the counts of fully specialized firms and the diagonal gives the count for fully integrated firms. There are 88 generation only and 149 distribution only firm-year observations. It is important to stress that our flexible technology does not add much value without observing specialized firms. The diagonal gives the count of fully integrated firms. Clearly the space between the diagonal and the two axis is less densily populated. Overall we have 444 firm-year observations.

We define the following variables. Our dependent varaibale, total cost long-run (C) is the sum of capital, fuel and operating expenses. The single upstream output is net electricity generated (yG) and the three distribution outputs are energy sales (yD1), number of customers (yD2), and distribution network length (yD3). The three distribution outputs reflect the long-run expansion of output. New settlements or industrial sites require the extension of the network. New customers in the existing supply areaq require new connections and finally existing ciutomers might demand more electricity. We include input prices for capital (wK), fuel (wF) and others (wO). Price of other imnputs is the numeraire used to impose homogeneity in input prices. We also include a variable to control for

Table 1: Firm Count in Size Bracket

	Distribution (GWh)									
Generation (GWh)	0	<250	<500	<750	<1000	<2500	<5000	<7500	<15000	Total
0	0	47	39	24	5	21	4	8	1	149
<50	3	10	9	5	0	0	0	0	0	27
<250	9	9	42	2	3	10	0	0	0	75
<500	14	0	20	10	7	17	0	0	0	68
<750	7	0	3	4	4	4	0	0	0	22
<1000	13	0	0	0	0	5	0	0	0	18
<2500	28	0	4	4	6	11	8	0	0	61
<5000	10	0	0	0	2	1	4	0	0	17
<15000	4	0	0	0	0	0	0	0	3	7
Total	88	66	117	49	27	69	16	8	4	444

Source: US electric data

generation plant capacity factor (CF). The appendix describes the contruction of the variables in more detail.

Table 2 provides summary statistics by firm type. The table shows that there are important differences across the three firm types and that there are large variances within each group. Dots indicate that a variable is not applicable to the type of firm. For instance, distribution only firms do not face a price of fuel. Average total cost is higheest for generation only firms reflecting the higher capital costs of generation plant. Also generation only companies generate more than twice the amount of elecetricity as integrated firms possibly reflecting the effect that integrated firms can choose between making and buying electricity. This hypothesis is also supported by the much higher capacity factor for generation only firms. For the distribution outputs distribution only firms have about three times larger networks than integrated firms but have similiar amounts of customer connection and sales implying a much lower density. The lower density might reflect geographically and demographically different supply areas. Note that the firms in our sample are much smaller in terms of output than the firms in previous studies on US electric utilities which typically used samples of investor owned utilities[Kaserman and Mayo, 1991, Greer, 2008, Jara-Diaz et al., 2004, Arocena et al., 2009, Kwoka, 2002]. Mean prices of capital and other inputs are very similar across firm types. However, the price of fuel for integrated firms is more than twice the price for generation only firms possibly reflecting bulk discounts for the latter or better procurement policies. the three cost shares are roughly a third each for generation only firms. For the other two firm types the shares of other inputs dominate.

Table 2: Summary Statistics

	All		Generation		Integrated		Distribution	
	mean	*sd*	*mean*	*sd*	*mean*	*sd*	*mean*	*sd*
Total Cost (M. US dollars)	36.46	68.21	55.97	81.51	45.51	79.66	12.38	16.25
yG Net Generation (GWh)	1053.03	2100.08	1726.41	2697.48	766.76	1716.17	.	.
yD1 Retail Sales (GWh)	979.43	1570.33	.	.	985.78	1527.69	970.60	1632.90
yD2 Retail Customers (No.)	33450.17	49361.14	.	.	33830.66	51202.45	32921.58	46848.33
yD3 Network length (miles)	1007.76	1382.87	.	.	578.86	805.12	1603.60	1752.14
wK Price of Capital (Rate)	0.12	0.03	0.12	0.04	0.13	0.03	0.11	0.03
wF Price of Fuel (M/Mbtu)	2.23	1.71	1.27	0.65	2.64	1.86	.	.
wO Price of Other Inputs (Index)	0.91	0.12	0.92	0.12	0.92	0.13	0.91	0.11
CF capacity factor (yG/capacity)	41.65	26.37	64.59	21.52	31.90	21.84	.	.
Capital share	0.32	0.11	0.33	0.12	0.27	0.08	0.38	0.11
Fuel share	0.31	0.11	0.36	0.12	0.29	0.11	.	.
Other input share	0.47	0.16	0.31	0.13	0.44	0.11	0.62	0.11
Observations	444		88		207		149	

6 Results

In this section we present the parameter estimates and estimates for economies of scale and scope for our three models. First, Table 6 gives the coefficients for our benchmark model, the common technology quadratic form model. Table 7 gives the results for the flexible technology quadratic form model. The parameter estimates for the separate firm types are given in different columns and the first three rows give the firm type specific constants. When comparing these two models the only difference is the restrictions on the technology. The statistics for the goodness of fit at the bottom of the two tables show that the R-squared are both very high but slighly higher for the flexible technology model. When comparing individual coefficients it seems that the technologies are likely to differ. Remember that the first order coefficients are the marginal costs for the sample mean firm. Although the first order coefficients for the generation output (yG) are very similar for generation only and for integrated firms the coefficients for the distribution outputs differ substantially. In particular, some of the distribution outputs for integrated firms have the wrong signs. The common technology model does not produce such sign violations but this might be spurious as it could be the result of the undue restrictions imposed on the technology. Also it seems that the coefficients for the common technology model are closest to the coefficients for the specialized firms in the flexible technology model. As the two models are nested we can perform a log-likelihood ratio test. We reject at a 1 percent level the null hypothesis that the common technology model is nested in the flexible technology model.

Table 3: Common Technology (Quadratic)

	(1) All Total Cost
yG	37.65***
	[0.896]
yD1	9.615***
	[2.226]
yD2	123.6
	[66.13]
yD3	1002.7
	[1169.5]
wK	98383176.8***
	[2293097.8]
wF	7548749.0***
	[62777.9]
yG2	$-6.45e-08$
	[0.000000119]
yD12	0.00000240
	[0.00000357]
yD22	0.00387
	[0.00299]
yD32	0.184
	[0.423]
wK2	−364856641.5***
	[49084482.9]
wF2	−123586.5***
	[36064.7]
yD1*yG	0.00000111
	[0.00000177]
yD2*yG	−0.0000664
	[0.0000632]
yD3*yG	0.00210
	[0.00144]
yD2*yD1	−0.0000833
	[0.000105]
yD3*yD1	0.00140
	[0.00129]
yD3*yD2	−0.0969*
	[0.0402]
wK*yG	109.3***
	[1.424]
wK*yD1	23.70**
	[7.546]
wK*yD2	135.5
	[240.4]
wK*yD3	8248.9***
	[2298.4]
wF*yG	9.819***
	[0.0394]
wF*yD1	0.298
	[0.198]
wF*yD2	−0.244
	[6.278]
wF*yD3	−305.7***
	[63.99]
wF*wK	3158065.0***
	[902272.9]
CF	−26329.9
	[19129.6]
Constant	40669366.9***

	[664799.3]
Observations	444
RSS	$4.71e+16$
RMSE	10299301.86
Ll	-23011.88
R-squared	0.98

Standard errors in brackets
The estimator is iterative SUR. All variables are mean-corrected.
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 4: Flexible Technology (Quadratic)

	(1) Total Cost *Integrated*	*Generation*	*Distribution*
I	39215194.1***		
	[1090475.4]		
U		30397126.0***	
		[1309040.0]	
D			10781551.6***
			[865742.9]
yG	34.02***	35.94***	
	[2.147]	[0.817]	
yD1	30.67***		8.708*
	[5.132]		[3.632]
yD2	−32.16		85.37
	[89.36]		[110.0]
yD3	−8449.2***		727.9
	[2391.8]		[920.9]
yG2	−0.0000153***	$-1.42e-08$	
	[0.00000204]	[0.000000127]	
yD12	−0.00000618		0.0000219
	[0.0000118]		[0.0000115]
yD22	0.00312		0.0288*
	[0.00294]		[0.0139]
yD32	26.34***		0.0306
	[5.103]		[0.379]
wK	87603768.4***	64344849.4***	32703075.2***
	[4545094.7]	[5032003.4]	[4390261.0]
wF	7851745.9***	7944234.0***	
	[113231.5]	[125670.1]	
wK2	−408884616.9***	−561890794.1***	9412653.9
	[63966060.2]	[90434556.3]	[102812527.4]
wF2	−217082.7***	−586032.5***	
	[45485.0]	[151137.4]	
yD1XyG	0.0000212***		
	[0.00000411]		
yD2XyG	−0.000105		
	[0.0000845]		
yD3XyG	−0.0147***		
	[0.00376]		
yD2XyD1	−0.000130		−0.000771
	[0.000167]		[0.000404]
yD3XyD1	0.0134		−0.00161
	[0.00838]		[0.00172]
yD3XyD2	−0.528***		0.00380
	[0.135]		[0.0451]
wKXyG	70.87***	116.4***	
	[5.145]	[1.828]	
wKXyD1	94.00***		11.54
	[13.38]		[14.45]
wKXyD2	−543.1		631.5
	[287.8]		[512.5]
wKXyD3	−12965.2		5601.3*
	[9171.7]		[2487.9]
wFXyG	10.35***	9.462***	
	[0.132]	[0.0428]	
wFXyD1	0.159		
	[0.318]		
wFXyD2	−1.765		
	[6.596]		
wFXyD3	−752.9***		

	[212.6]	
wFXwK	−1809893.3	15827478.0***
	[1187340.9]	[2212593.9]
CF	−63135.1*	−24544.5
	[32108.5]	[29572.3]
Observations	444	
RSS	$2.76e+16$	
RMSE	7878843.99	
Ll	−22784.43	
R-squared	0.99	

Standard errors in brackets

The estimator is iterative SUR. All variables are mean-corrected.

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Next Table 8 gives the parameter estimates for a flexible technology model using a translog form instead of the quadratic form. Besides its theoretical advantages discussed above we find similarly to Caves et al. [1980] that the translog form produces less curvature violations for outputs and prices compared to the quadratic model. The fit based on the R-squared statistic is slightly worse than for the quadratic form.

Table 5: Flexible Technology (Translog)

	(1) Total Cost		
	Integrated	*Generation*	*Distribution*
I	0.025		
	[0.04]		
U		−0.198***	
		[0.04]	
D			−1.205***
			[0.03]
yG	0.506***	0.896***	
	[0.03]	[0.02]	
yD1	0.489***		0.288***
	[0.09]		[0.07]
yD2	−0.015		0.568***
	[0.06]		[0.07]
yD3	−0.062		0.039*
	[0.05]		[0.02]
yG2	0.111***	0.008	
	[0.02]	[0.01]	
yD12	0.067		0.211
	[0.15]		[0.15]
yD22	0.153***		0.221*
	[0.04]		[0.10]
yD32	0.001		0.048**
	[0.06]		[0.02]
wK	0.250***	0.310***	0.393***
	[0.01]	[0.01]	[0.01]
wF	0.317***	0.409***	
	[0.01]	[0.01]	
wK2	−0.007	−0.097**	0.133***
	[0.02]	[0.03]	[0.03]
wF2	0.144***	0.207***	
	[0.01]	[0.01]	
yD1XyG	0.024		
	[0.04]		
yD2XyG	−0.083**		
	[0.03]		
yD3XyG	−0.050		
	[0.03]		
yD2XyD1	−0.078		−0.099
	[0.05]		[0.10]
yD3XyD1	0.108		−0.240***
	[0.08]		[0.05]
yD3XyD2	−0.080**		0.068
	[0.03]		[0.04]
wKXyG	−0.027***	0.023**	
	[0.01]	[0.01]	
wKXyD1	0.095***		0.040
	[0.02]		[0.02]
wKXyD2	−0.033***		−0.061**
	[0.01]		[0.02]
wKXyD3	−0.029**		0.038***
	[0.01]		[0.01]
wFXyG	0.105***	0.028***	
	[0.00]	[0.01]	
wFXyD1	−0.106***		
	[0.01]		
wFXyD2	0.017**		
	[0.01]		
wFXyD3	−0.002		

	[0.01]	
wFXwK	−0.048***	−0.050***
	[0.01]	[0.01]
CF	−0.085**	−0.073
	[0.03]	[0.04]
Observations	444	
RSS	18.09	
RMSE	0.20	
Ll	1325.56	
R-squared	0.98	

Standard errors in brackets

The estimator is iterative SUR. All variables are mean-corrected.

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

We now turn to the estimates for scale and scope. Table 6 gives the economies of scale S and scope SC for the three models. For the common technology model we report estimates at the sample mean. For the flexible technology models we report estimates at the sample mean and the mean estimates. We report both because there is no empirical support at the mean of the sample. When taking the sample mean for distribution and generation outputs and comparing it to the mapping in Table 1 the sample mean does not correspond to any actual observation. Also Table 2 shows that the sample mean costs overestimate the costs for a distribution only firm and underestimate costs for an integrated or generation only firm. However, the observation specific estimates are only for integrated firms as can can predict the costs for all three firm types only for these firms. The first block of estimates is for the common technology quadratic form model. The next two blocks are for the flexible technology quadratic form model, and the last two blocks are for the flexible technology translog form model.

First, we comment on the estimates for economies of scale. The first row in Table 6 shows that the common technology quadratic form model finds economies of scale for all firms of 7 percent. They are lower for specialized firms and almost neutral for generation only firms. For the two specialized firm types the estimates from the flexible technology models for economies of scale are substantially higher. The estimates for specialized firms from the common technology model rely on average incremental costs which might be biassed because the model does not include firm type specific intercepts. Importantly policy advice based on the common technology model would suggest that entry is feasible (based on economies of scale not subadditivity) into the generation only market whereas the less restrictive models suggest it is not. Also the estimates for specialized firms tend to be lower for the translog form than for the quadratic form. However, the estimates for integrated firms are similar across the models (with the exeption of the mean estimate for the flexible technology quadratic form model).

Second, we turn to the estimates for economies of scope which seem much less robust across the models. The two sources of economies of scope are shared fixed costs and cost complementarity. The interaction terms provide evidence for the latter. There are some important qualitative differences between the models. Whereas the common technology quadratic model finds no statistically significant positive or negative complementarities, the flexible technology quadratic form model finds both significant positive and negative complementarities (depending on the distribution output). The flexible technology quadratic model only finds significant negative complementarities. The estimate for economies of scope for our benchmark model is 4.6 percent at the sample mean. That is the firm would increase its cost by 4.6 percent if it was to break up into two specialized firms. When we keep the quadratic form but allow for a flexible technology the estimate is very similar at 5 percent suggesting that the estimate for the sample mean is robust to the assumption of poolability. However when we compare Tables 8 and 9 we see that away from the sample mean this is no longer true. Tables 8, 9 and 10 provide estimates for economies of scope at the sample mean (the entry for the row and column labeled 1) and deviations from the mean. For instance, the last row of the first column indicates that a firm that has distribution outputs of one quarter of the sample mean and generation output of twice the sample mean would increase its total cost by 3.7 percent if it was to break up fully.[4] Returning to our argument if we look at the columns only in Table 9 we see that to the left of the column for the distribution mean the flexible technology model produces higher estimates whereas to the right the estimates are lower than for the common technology model. Integrated firms with relatively large distribution businesses would do better to fully divest their generation businesses. When looking at the rows of Table 9 we see that for any size of the distribution business the cost advantage of having more generation first decreases and then increases again. By comparison in Table 8 all the estimates decrease monotonically in all directions from the first entry in the top left corner. This is an artefact of the model as we do not allow for firm type specific fixed costs (i.e. constants). Our ad hoc hypothesis was that easing the restriction of a common technology would lower the estimates of economies of scope. Comparing Table 8 and 9 we find some evidence that this is the case. Also when constraining all the coefficients Tables 4 and 5 to be equal estimates for economies of scope increase substantially. For both constraint models the estimate at the sample mean is 100 percent.

[4]All distribution outputs are scaled equally.

Table 6: Economies of Scale and Scope

	mean	*sd*	*p10*	*p50*	*p90*
		Quadratic form			
Common Technology - at sample mean:					
S(All)	1.070	.	.	.	.
S(Upstream)	1.001	.	.	.	.
S(Downstream)	1.066	.	.	.	.
SC	0.046	.	.	.	.
Flexible Technology - at sample mean:					
S(Upstream)	1.209	.	.	.	.
S(Integrated)	0.976	.	.	.	.
S(Downstream)	1.110	.	.	.	.
SC	0.050	.	.	.	.
Flexible Technology - all observations:					
S(Upstream)	1.259	0.554	1.010	1.105	1.418
S(Integrated)	1.045	0.235	0.784	1.016	1.364
S(Downstream)	1.209	0.222	0.962	1.212	1.463
SC	0.197	0.423	−0.114	0.083	0.635
		Translog form			
Flexible Technology - at sample mean:					
S(Upstream)	1.116	.	.	.	.
S(Integrated)	1.090	.	.	.	.
S(Downstream)	1.117	.	.	.	.
SC	0.092	.	.	.	.
Flexible Technology - all observations:					
S(Upstream)	1.124	0.022	1.093	1.122	1.153
S(Integrated)	1.065	0.058	1.016	1.078	1.122
S(Downstream)	1.147	0.215	0.894	1.172	1.444
SC	−0.012	0.136	−0.140	−0.017	0.144

Source: own calculations.

Table 8: Economies of Scope for Common Technology (Quadratic)

Generation	Distribution 1/4	1/2	3/4	1	5/4	3/2	7/4	2
1/4	0.198	0.155	0.128	0.109	0.095	0.084	0.075	0.068
1/2	0.127	0.105	0.090	0.078	0.068	0.061	0.054	0.049
3/4	0.092	0.078	0.067	0.059	0.051	0.045	0.040	0.035
1	0.072	0.062	0.053	0.046	0.040	0.034	0.030	0.025
5/4	0.059	0.050	0.043	0.036	0.031	0.026	0.022	0.018
3/2	0.050	0.042	0.035	0.029	0.024	0.020	0.015	0.012
7/4	0.043	0.036	0.029	0.024	0.019	0.015	0.010	0.007
2	0.037	0.031	0.025	0.020	0.015	0.010	0.006	0.003

At deviations from mean output. All other variables at means.

Table 9: Economies of Scope for Flexible Technology (Quadratic)

Generation	Distribution 1/4	1/2	3/4	1	5/4	3/2	7/4	2
1/4	0.684	0.443	0.279	0.159	0.064	-0.012	-0.076	-0.131
1/2	0.349	0.246	0.159	0.084	0.019	-0.038	-0.089	-0.135
3/4	0.243	0.176	0.114	0.056	0.004	-0.045	-0.089	-0.130
1	0.204	0.151	0.099	0.050	0.004	-0.040	-0.081	-0.120
5/4	0.195	0.148	0.101	0.056	0.013	-0.028	-0.067	-0.105
3/2	0.203	0.159	0.114	0.071	0.029	-0.011	-0.049	-0.086
7/4	0.223	0.179	0.135	0.092	0.051	0.011	-0.028	-0.064
2	0.250	0.206	0.162	0.119	0.077	0.036	-0.003	-0.040

At deviations from mean output. All other variables at means.

Now we turn to the estimates for economies of scope for the translog form model and compare these with the flexible technology quadratic model. Like for economies of scale the estimates for the translog model are lower. The mean of the estimates is actually negative indicating diseconomies of scope. Also the standard deviations are much lower for the translog model. When comparing Tables 9 and 10 there are also some qualitative differences. We still find the lowest estimates for companies with relatively high distribution to generation output ratios. However, when looking at the rows economies of scope are now increasing in generation output. For instance when looking at the first column in Table 9 increasing the amount of generation output lowers the cost disadvantage of specialization whereas in Table 10 it is the opposite.

Above we showed that the common technology quadratic form is not nested in flexible technology quadratic form technology. For both the quadratic and translog form models with flexible technology we can also reject the null hypothesis 12 that the technologies are the same across the different firm types at a 1 percent level. Also when we constrain the coefficients to be equal across firm types for the separate technology translog model the R-squared drops to 0.83.

Table 10: Economies of Scope for Flexible Technology (Translog)

Generation	Distribution 1/4	1/2	3/4	1	5/4	3/2	7/4	2
1/4	0.248	0.052	-0.021	-0.049	-0.055	-0.049	-0.034	-0.013
1/2	0.337	0.135	0.044	-0.001	-0.023	-0.030	-0.027	-0.017
3/4	0.404	0.202	0.103	0.048	0.017	0.002	-0.004	-0.002
1	0.454	0.257	0.153	0.092	0.055	0.033	0.022	0.019
5/4	0.494	0.302	0.196	0.131	0.089	0.063	0.048	0.040
3/2	0.525	0.339	0.233	0.165	0.120	0.091	0.072	0.061
7/4	0.551	0.372	0.265	0.196	0.149	0.117	0.095	0.082
2	0.573	0.400	0.294	0.223	0.175	0.140	0.117	0.101

At deviations from mean output. All other variables at means.

7 Conclusion

This paper contributes to the literature on how to estimate economies of scale and scope. We modify the standard tranlog model to allow for zero outputs and different technologies across firm types. We find that these changes impact on the quantitative and qualitative assessment of economies of scale and scope. Most importantly our evidence suggests that economies of scope are driver both by differences in cost level and differences in technology. Allowing for different technology often drastically lowers the estimates for economies of scope. Therefore policy makers have to be careful not to rely on estimates with restricted counterfactuals.

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Variable definitions

Total cost: the sum of capital, fuel and operating expenses. Operating expenses is the sum generation O&M, distribution O&M, Customer Accounts Expenses, Customer Service & Informational Expenses, Sales Expenses and a pro-rata Admin & General O&M. We do not include any transmission expenses. Total cost is measured US dollars.

Capital expense is the capital stock multiplied with a rate-of-return plus depreciation expenses. The capital stock is the written down accounting value of fixed assets. The rate-of-return is a measure of interestd paid on long-term debt.

Outputs: upstream output (yG) is the annual amount of net generation. Downstream outputs are the annual sales of electricity excluding wholesale sales (yD1) and the number of customer connections (yD2). Both are measured in GWhs.

Input prices: the capital price (wK) is capital expense divided by the capital stock. The fuel price (wF) is the price of fuel for generators and the price of other (wO) is a wage index.

Controls: we include control variables for density as the number of customers divided by distribution network length (DEN), and the capacity factor for the genartion activity (CF).