TECHNISCHE UNIVERSITÄT DRESDEN

Transmission Expansion Planning Applying Benders Decomposition

Infraday

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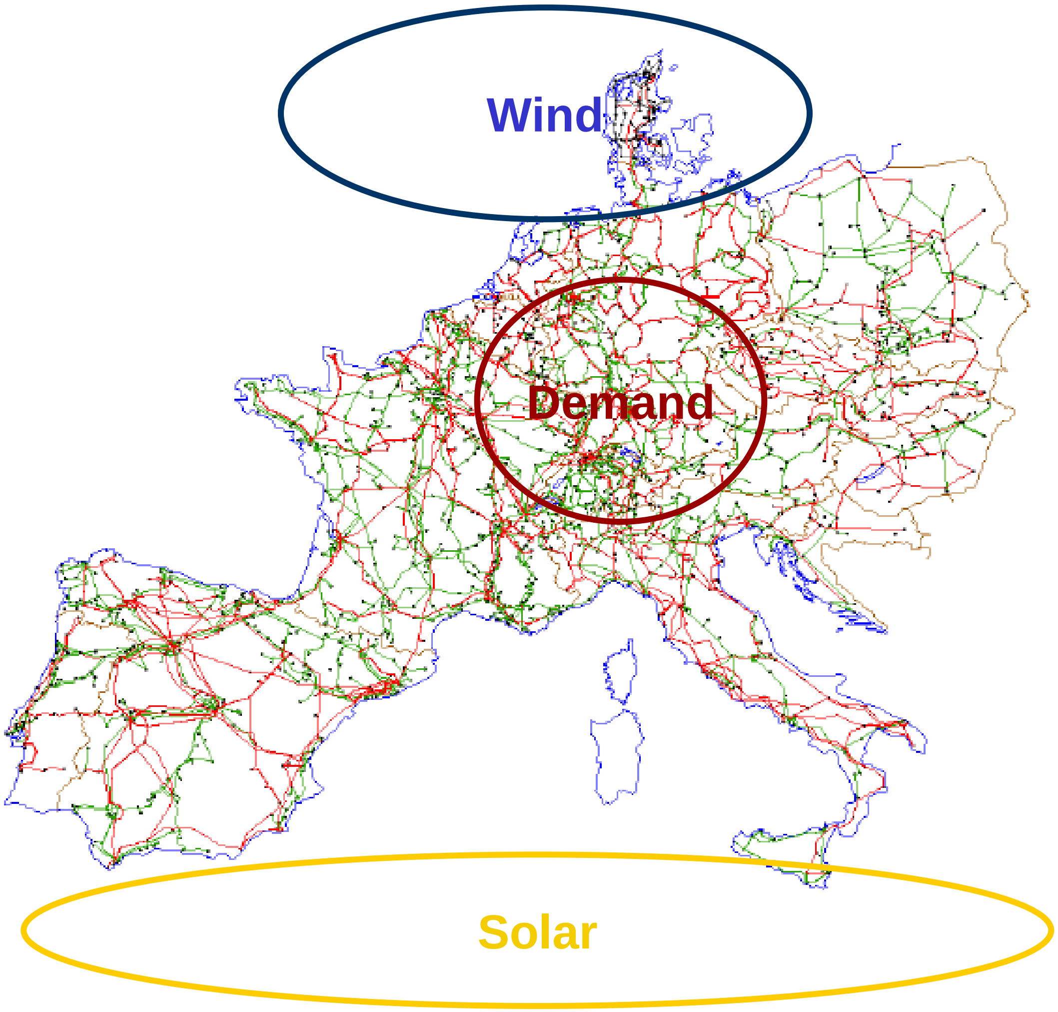
EE²

Chair of Energy Economics and Public Sector Management

Agenda

1. Motivation
2. Model Formulation
3. Decomposition
   a. Benders Decomposition
   b. GAMS Solvers
4. Results
5. Conclusion

Motivation



Source: Leuthold et al. (2008)

Benders Decomposition in TNEP

Selected Works

- **Pereira et al. (1985):**
  - First application to TNEP with linearized power flow model
- **Romero and Monticelli (1994):**
  - Hierarchical Approach to obtain global optimality despite of nonconvexity
- **Oliveira et al. (1995):**
  - Integer master problem solved only to feasibility, not optimality to make use of heuristics
- **Siddiqi and Baughman (1995):**
  - AC load flow model
- **Binato et al. (2001):**
  - Gomory Cuts and ‘Big M‘ scaling mechanism
- **Shrestha and Fonseka (2004):**
  - Competitive market considering different convergence strategies
  - Dynamic model with LP master problem and quadratic subproblem

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The Transmission Expansion Problem (TNEP)

$$\min C = \sum_n c_n \cdot G_n + \sum_l c_l^{inv} \cdot p_l^{max} \left(EF_l - exist_l\right)$$

subject to

$$G_n = q_n + NETIN_n \qquad \forall n \in N$$

$$G_n \leq g_n^{max} \qquad \forall n \in N$$

$$NETIN_n = \sum_{nn} B_{n,nn} \cdot \Delta_{nn} \qquad \forall n \in N$$

$$FLOW_l = \sum_n H_{l,n} \cdot \Delta_n \qquad \forall l \in L$$

$$|FLOW_l| \leq EF_l \cdot p_l^{max} \qquad \forall l \in L$$

$$H_{l,n} = \frac{inci_{l,n}}{x_l} \cdot EF_l \qquad \forall l, n \in L \times N$$

$$B_{n,nn} = \sum_l inci_{l,n} \cdot H_{l,nn} \qquad \forall n, nn \in N \times N$$

$$\Delta_1 = 0$$

$$EF_l \leq ef_l^{max} \qquad \forall l \in L$$

$$EF_{le} \geq exist_l \qquad \forall le \in E \subseteq L$$

$$G_n \geq 0 \qquad \forall n \in N$$

$$EF_l \in \mathbb{N} \qquad \forall l \in L$$

N : Set of nodes n (with nn as alternative name for set elements)

L : Set of all lines l

E : Set of already existing lines le, subset of L

C : Total cost (objective value)

G_n : Quantity of generated electricity at node n

EF_l : Number of cables on line l

$NETIN_n$: Net input into electricity grid at node n

$B_{n,nn}$: Network susceptance matrix

$H_{l,n}$: Flow sensitivity matrix

Δ_n : Phase angle at node n

$FLOW_l$: Flow on line l

c_n : Generation cost at node n

c_l^{inv} : Investment cost per MW for line l

$exist_l$: is 1 for le, 0 for all others: no cost for existing cables

q_n : Demand at node n

g_n^{max} : Generation capacity at node n

$inci_{l,n}$: Incidence Matrix

x_l : Reactance of line l

ef^{max} : maximum number of lines per corridor

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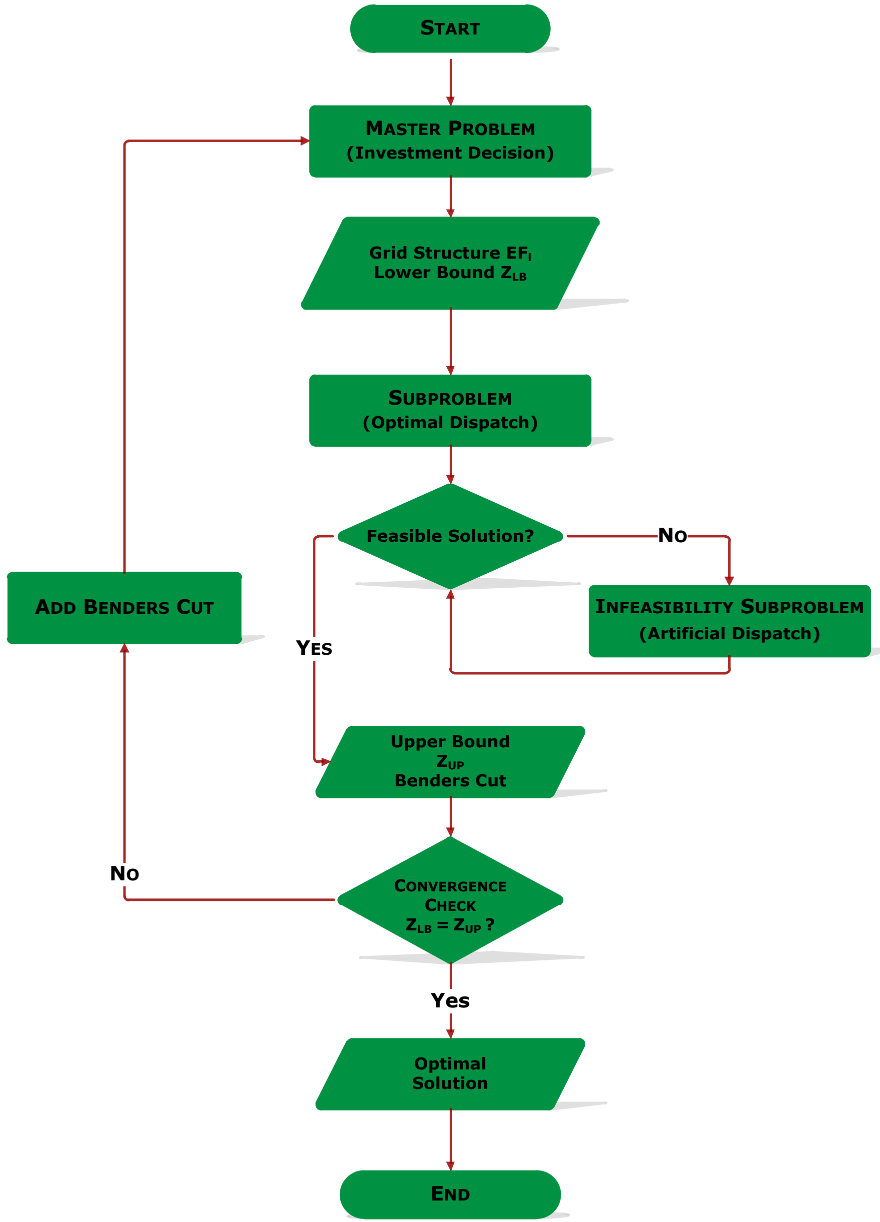
Benders Decomposition of the TNEP



The Master Problem (MP)

- **Investment Problem (MILP)**
- **Integer Expansion Factor (EF_l) as choice variable**
- **Only constraints on EF_l included → relaxed TNEP**
- **Optimal value constitutes lower bound to TNEP problem**

$$\min Z_{MP}^{(1)} = \sum_l c_l^{inv} \cdot p_l^{max} \left(EF_l - exist_l \right) + \alpha^{(1)}$$

subject to

$$EF_l \leq ef^{max} \qquad \forall l \in L$$

$$EF_{le} \geq exist_l \qquad \forall le \in E \subseteq L$$

$$\alpha^{(1)} \geq \alpha^{min}$$

$$EF_l \in \mathbb{N} \qquad \forall l \in L$$

The Subproblem (SP)

$$\min Z_{SP}^{(it)} = \sum_n c_n \cdot G_n$$

subject to

$$G_n = q_n + NETIN_n \qquad \forall n \in N$$

$$G_n \leq g_n^{max} \qquad \forall n \in N$$

$$NETIN_n = \sum_{nn} B_{n,nn} \cdot \Delta_{nn} \qquad \forall n \in N$$

$$FLOW_l = \sum_n H_{l,n} \cdot \Delta_n \qquad \forall l \in L$$

$$|FLOW_l| \leq EF_l \cdot p_l^{max} \qquad \forall l \in L$$

$$H_{l,n} = \frac{inci_{l,n}}{x_l} \cdot EF_l \qquad \forall l, n \in L \times N$$

$$B_{n,nn} = \sum_l inci_{l,n} \cdot H_{l,nn} \qquad \forall n, nn \in N \times N$$

$$\Delta_1 = 0$$

$$G_n \geq 0 \qquad \forall n \in N$$

$$EF_l = ef_l^{(it)} \quad \perp \lambda_l^{(it)} \qquad \forall l \in L$$

- **Optimal Dispatch Problem (NLP)**
- **Contains technical and economic constraints**
- **Integer variable EF_l fixed to MP solution → Upper bound to TNEP**
- **Output: λ_l and all variables of TNEP**

The Infeasible Subproblem

$$\min Z_{SP}^{(it)} = \sum_n c_n \cdot G_n + \sum_n M \cdot (\gamma_n^+ + \gamma_n^-)$$

subject to

$$G_n + \gamma_n^+ - \gamma_n^- = q_n + NETIN_n \qquad \forall n \in N$$

$$G_n \le g_n^{max} \qquad \forall n \in N$$

$$NETIN_n = \sum_{nn} B_{n,nn} \cdot \Delta_{nn} \qquad \forall n \in N$$

$$FLOW_l = \sum_n H_{l,n} \cdot \Delta_n \qquad \forall l \in L$$

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$$\Delta_1 = 0$$

$$G_n \ge 0 \qquad \forall n \in N$$

$$EF_l = ef_l^{(it)} \qquad \perp \lambda_l^{(it)} \qquad \forall l \in L$$

$$\gamma_n^+, \gamma_n^- \in \mathbb{R}^+ \qquad \forall n \in N$$

- **Introduce Slack variables into Energy Balance**
- **Obtain always-feasible solution**
- **'Punish' use of slack by adding a large value to objective function**

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New Iteration

Convergence Check: $Z_{UB}^{(it)} - Z_{LB}^{(it)} \leq \epsilon$ **?**

Optimal solution found

New Master Problem with Benders Cut:

$$\min Z_{MP}^{(it+1)} = \sum_{l} c_l^{inv} \cdot p_l^{max} \left(EF_l - exist_l\right) + \alpha^{(it+1)}$$

subject to

$$\alpha^{(it+1)} \geq \sum_{n} c_n \cdot G_n + \sum_{l} \lambda_l^{(it)} \cdot \left(EF_l - EF_l^{(it)}\right)$$

$$EF_l \leq ef^{max} \qquad \forall l \in L$$

$$EF_{le} \geq exist_l \qquad \forall le \in E \subseteq L$$

$$\alpha^{(it+1)} \geq \alpha^{min}$$

$$EF_l \in \mathbb{N} \qquad \forall l \in L$$

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GAMS Solvers

DICOPT (DIscrete and Continuous Optimizer)

- Outer Approximation (MIP and NLP problems)
- Equality Relaxation
- Augmented Penalties

SBB (Standard Branch & Bound)

- Branch & Bound algorithm (NLP subproblems)
- Pseudo Cost possible

→ **DICOPT with advantages when many discrete variables, SBB when complexer nonlinearities are faced**

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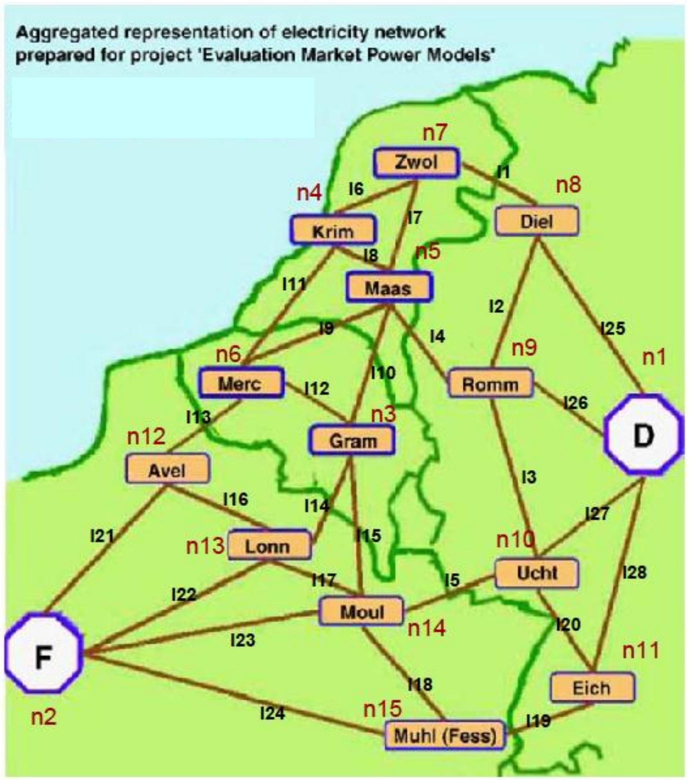
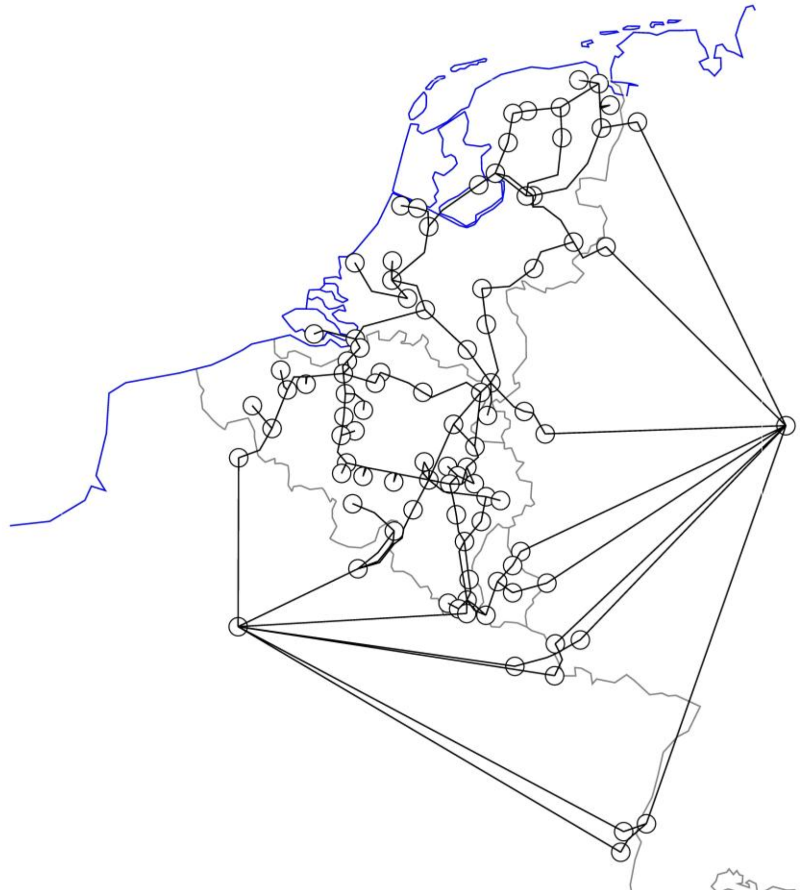
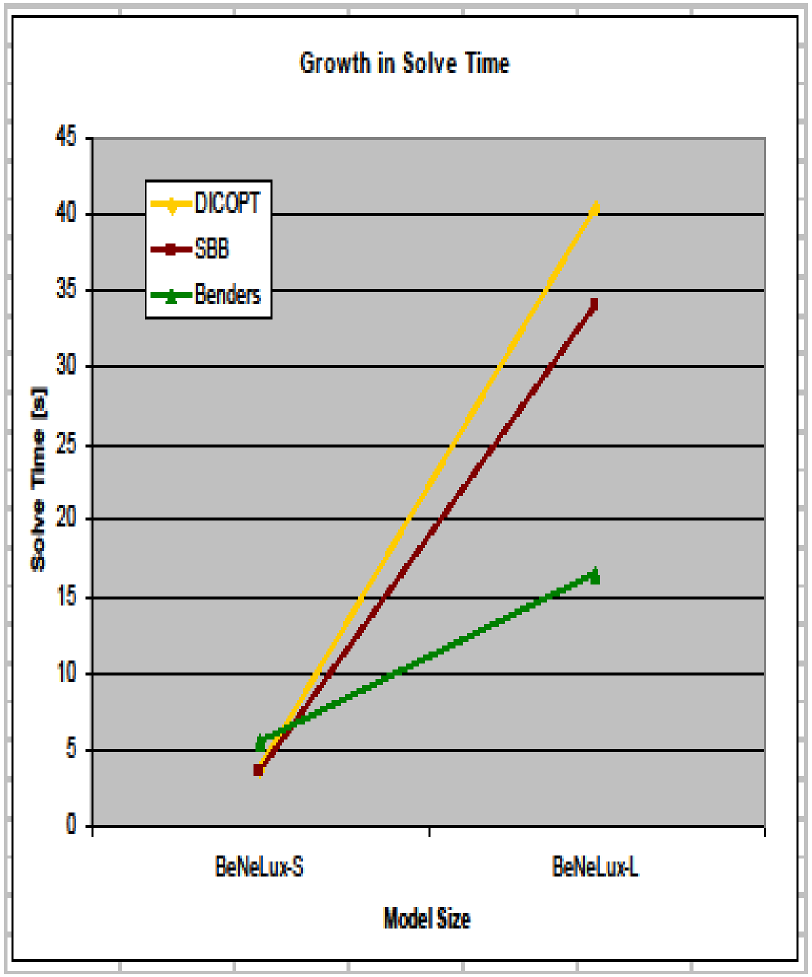
Grid Structures





- **BeNeLux-S**
  - **15 nodes, 28 (+5) lines**
- **BeNeLux-L**
  - **94 nodes, 120 lines**

Results - Comparison to GAMS Solvers



		DICOPT	SBB	Benders
BeNeLux-S	Lines Expanded	l(4) = 4	l(4) = 4	l(4) = 4
		l(10) = 2	l(10) = 2	l(10) = 2
	Cost [€]	337.340.000	337.340.000	337.339.800
	Time [s]	3,594	3,703	5,530
		3,656	3,641	5,125
		3,719	3,515	5,905
	Average Time [s]	**3,656**	**3,620**	**5,520**
BeNeLux-L	Cost [€]	7.409.900	7.409.900	7.347.008
	Lines Expanded	l(55) = 5	l(55) = 5	l(55) = 5
	Time [s]	40,356	32,545	16,327
		41,278	37,107	18,124
		39,544	32,450	14,937
	Average Time [s]	**40,393**	**34,034**	**16,463**
ime Growth	Rate S - L	**11,05**	**9,40**	**2,98**

- **For small problems GAMS solvers outdo Benders Algorithm**
- **Already medium-sized BeNeLux-L with large savings**
- **For TNEP SBB slightly faster than DICOPT**

Grid Expansion Scenarios

Base Case

- Based on data sets from 2007, no additional features

Security

- Base Case plus reliability margin of 20% on thermal limits of all lines

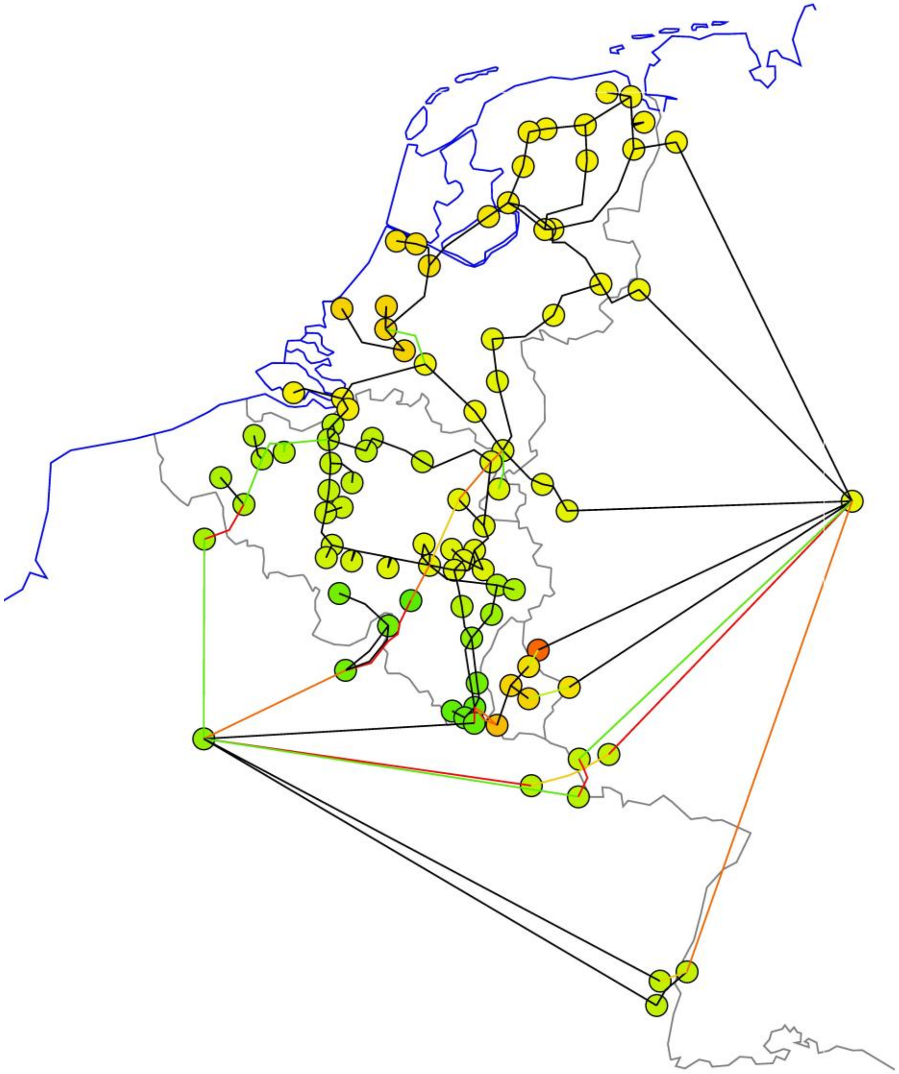
Wind 2020

- Additional 15 GW wind installed along coastal line of Belgium and The Netherlands

Base Case

- **Prices in North higher than in South**
- **Expansion of cross-border line capacities**
- **Massive line expansion in Southern part to satisfy demand center in Southern NL**

- **In today's grid line expansion brings overall cost savings**



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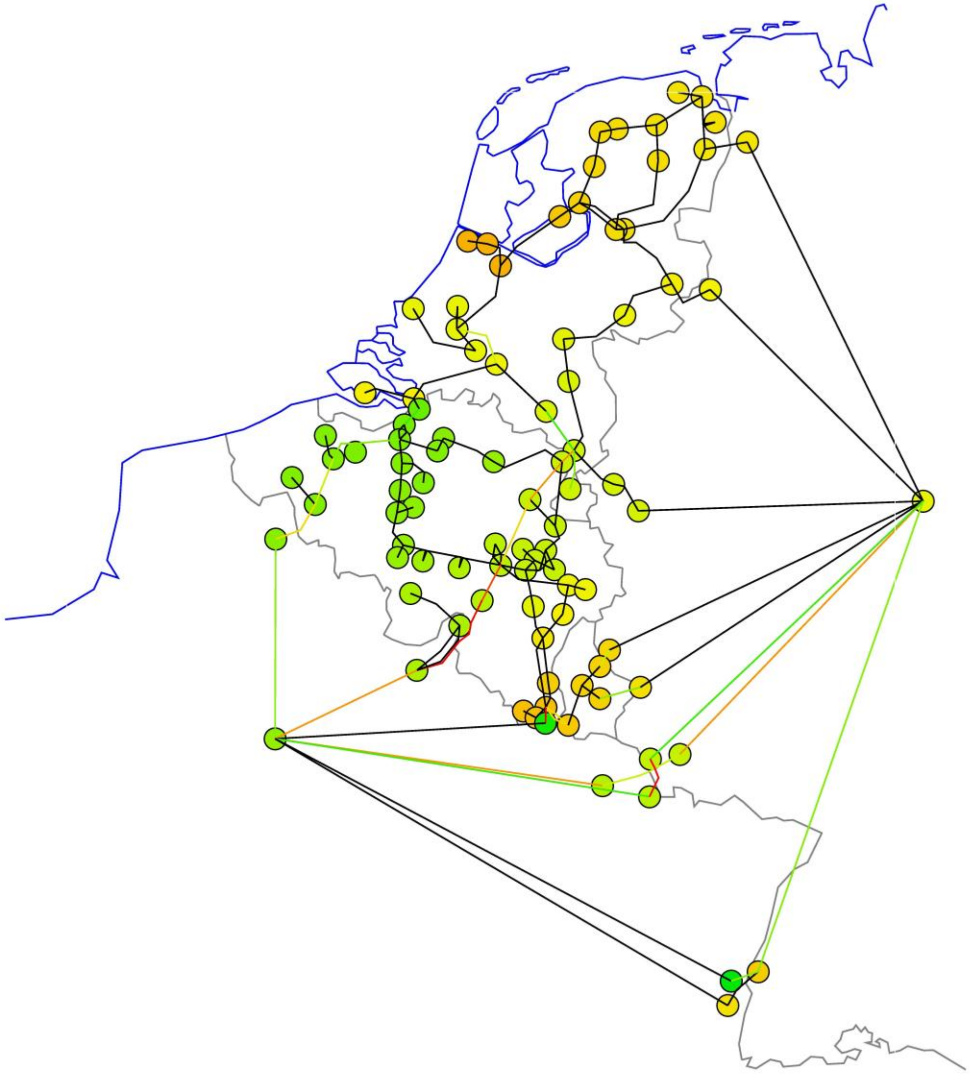
Security Scenario

- **Only small differences to Base Case:**
  - **Overall higher price level**
  - **More lines expanded**
  - **Same corridors expanded as in Base Case**



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Wind 2020

- **Lower Price Levels**
- **In North: Highly negative prices**

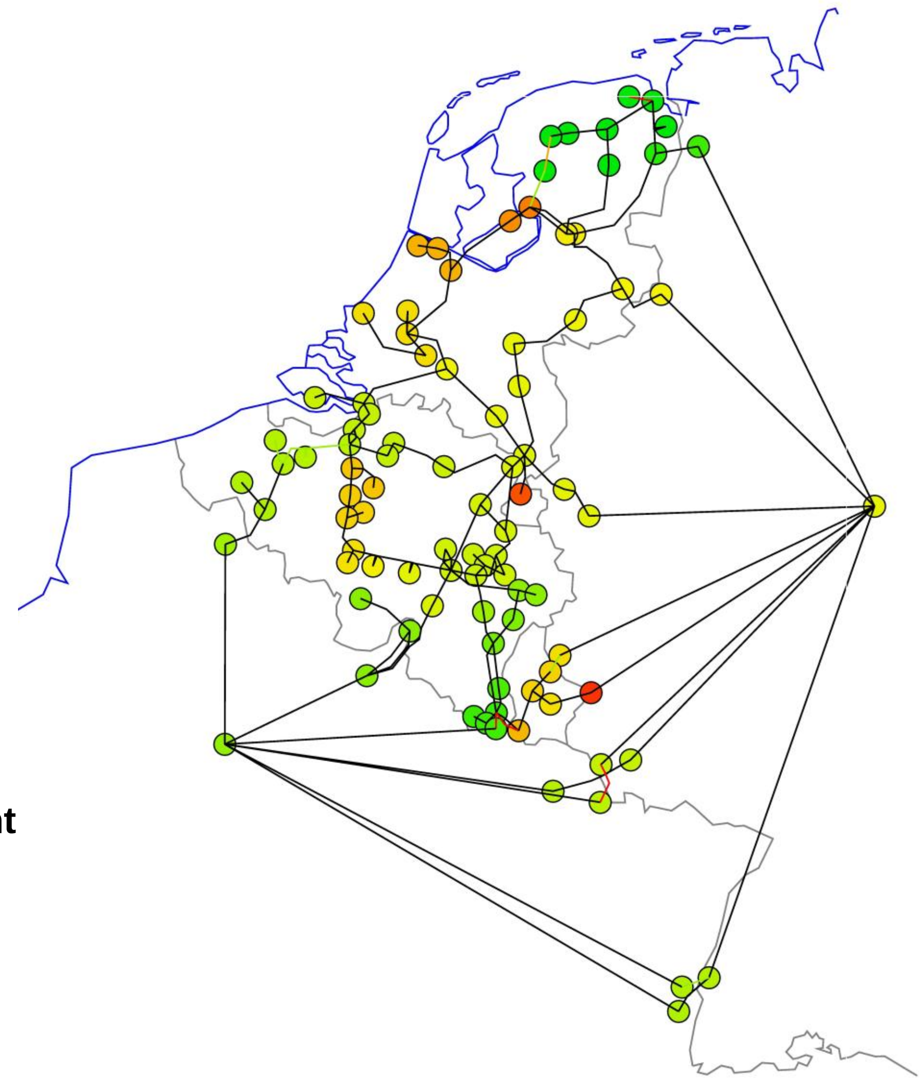
  Interpretation

  - **Large amounts of wind fed in as must-run condition**
  - **Economic gains from selling at negative prices to lower line congestion rents**

- **Hardly any line expansion**

  Interpretation

  - **No need to import due to high amount of cheap electricity along the coast**



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Conclusion

- **Already in the existing grid network expansion pays off**
- **Higher security in grid structure comes with higher expansion cost and price levels**
- **Wind can (in BeNeLux) have positive impact on network**

- **Benders Decomposition is applied successfully to TNEP**
- **For large model sizes more time-efficient than intern GAMS solvers**

Future Work:

- **Enhance Algorithm via**
  - **'Big M' scaling**
  - **Hierarchical Approach**
  - **Introduction of Gomory Cuts**
- **Apply to larger datasets**
- **Include HVDC expansion choice**
- **Differentiate generation technologies and line investment cost**

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Thank you very much for your attention!
Any questions or comments?

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