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Regional network competition: using tax instruments

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Outline of the presentation

- Problem statement
- Network model
- Cost functions
- Two regions model competing using tolls and VAT (Value Added Tax)
- Network dynamics and invariance principle
- Conclusion

Problem statement 1

- Regions compete to maximize their revenue, which translates into optimizing social welfare.
  - Regions are differentiated into:
    - Better-off regions: are identified as having large population densities and high level of consumption
    - Worse-off regions: are identified as having low population densities, and low level of consumption
- Each region employs two types of tax tools to compete with the others.
- Tax tools: are Tolls, and VATs. They are considered as:
  - local taxes,
  - are regionally determined,
  - vary in time
- Regions are considered to be non homogeneous. They are characterized by variations in:
  - Network complexity
  - population density,
  - and economic activities

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Problem statement 2

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- Cost functions: function of toll application on network links and VAT on consumption.
  - It is assumed that each region sells a generic good.
- A utility maximization model is used for revenue maximization given various tax scenarios based on varying tolls and VATs
- The invariance principle:
  - The invariance principle states that no matter which tax scenario is used, and no matter for how long (if the process is repeated indefinitely), there will still be inequality among regions. It is not possible to have uniformly better off regions. There will always be regions with different degrees of affluence.

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Assumptions - 1

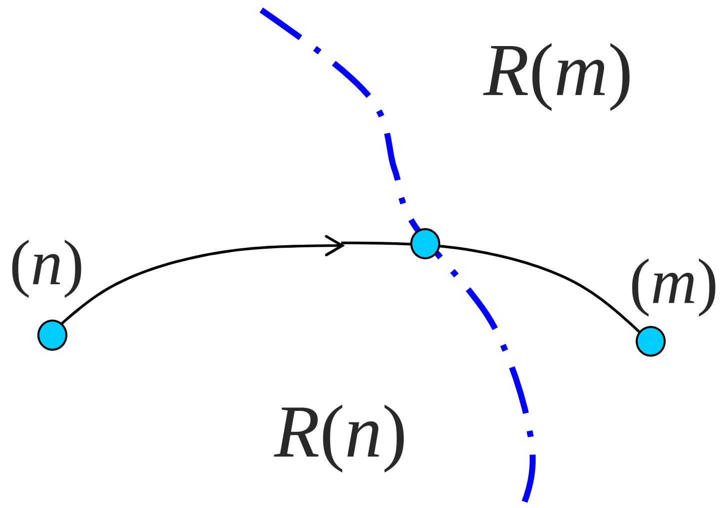
- Regions are divided into two categories: 1) better off regions, 2) worse off regions
- Non homogeneous regions are arranged in any order. Several permutations are possible: 1) Two worse off regions may be next to each other, 2) one better off region next to a worse off regions, 3) two better off regions next to each other.
- Regions use their tax instruments to raise their regional revenue.

Assumptions -2

- Regions are all disposed of the same tax instruments: tolling, and VAT on goods and services.
- VAT is based on the consumption of one generic good only
- Tax instruments: tolls and VAT
  - Toll and VAT are considered as local taxes.
  - Each regions sets its toll and VAT according to their revenue goal.

Network model 1

- Population is concentrated in nodes (n), $(n), \quad n \in \mathbf{N}$
- The transportation system between nodes (m) and (n) is represented by an arc:
$$(a) = (m, n), \quad a \in \mathbf{A}, \quad m, n \in \mathbf{N}$$
- Regions are denoted by $(r) \in \mathbf{R}$
- The region to which node (n) belongs is called $R(n)$
- Arcs may be connecting two different regions (and tolled by each region): $(a) = (m, n)$ and $R(m) \neq R(n)$ will be divided into two arcs. Thus any arc is assumed to belong to a single region.



Network model 2

- Paths $(p) \in \mathbf{P}$ connect origins and destinations, which are nodes in the network.
- An OD (origin-destination) couple is denoted by $(w) = (m,n), \quad w \in \mathbf{W}, \quad m,n \in \mathbf{N}$
- The set of paths connecting a given OD $(w) \in \mathbf{W}$ is called $(\mathbf{P}_w)$
- Flows on the network
  - Flows on an arc $(a) = (m,n), \quad a \in \mathbf{A}, \quad m,n \in \mathbf{N}$
  - Φ_{mn}: inter flow induced by economic activities (in the present case it is the activity related to the generic good)
  - F_{mn} : intra flow. This is the flow on the arc such that $\Phi_{mn} + F_{mn}$

  is the total flow on arc (m,n)
  - T_{mn} : cost of transportation over arc . Assume that the transportation cost is a linear function of the total flow

$$T_{mn} = \alpha_{mn} + \beta_{mn}\left(F_{mn} + \Phi_{mn}\right) \quad \forall (m,n) \in \mathbf{A}$$

Network model 3

- S_{mn} :supply constraint at head node (n) for traffic coming from tail node (m)

$$F_{mn} + \Phi_{mn} \leq S_{mn} \quad \forall (m,n) \in \mathbf{A} \qquad (2)$$

- Inverse demand for link : (m,n)

$$\Theta_{mn} = a_{mn} - b_{mn} F_{mn} \quad \forall (m,n) \in \mathbf{A} \qquad (3)$$

- The inverse demand is a function of local flow only.
- Equilibrium of link (m,n)Given the inter flow Φ_{mn}, the intra flow F_{mn} is completely determined by the equilibrium between supply and demand and the capacity constraints (2). There are three cases of equilibrium.

Network model 4

Case 1: the intra flow reaches its maximum value, for which the travel cost is less than the inverse demand : Θ_{mn}

$$\begin{aligned} T_{mn} &= \alpha_{mn} + \beta_{mn} S_{mn} \\ &\leq \Theta_{mn} = a_{mn} - b_{mn}\left(S_{mn} - \Phi_{mn}\right) \end{aligned}$$

then the equilibrium intra flow is given by:

$$F_{mn} = S_{mn} - \Phi_{mn} \qquad (4)$$

This case if and only if

$$\Phi_{mn} \geq \frac{\alpha_{mn} - a_{mn} + \left(\beta_{mn} + b_{mn}\right) S_{mn}}{b_{mn}} \qquad (5)$$

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Network model 5

- **Case 2**: Supply equals demand $T_{mn} = \Theta_{mn}$ From (1) and (3) we deduce

$$F_{mn} = \frac{a_{mn} - \alpha_{mn} - \beta_{mn}\Phi_{mn}}{b_{mn} + \beta_{mn}} \qquad (6)$$

- The constraint implies that $F_{mn} \geq 0$

$$\Phi_{mn} \leq \frac{a_{mn} - \alpha_{mn}}{\beta_{mn}} \qquad (7)$$

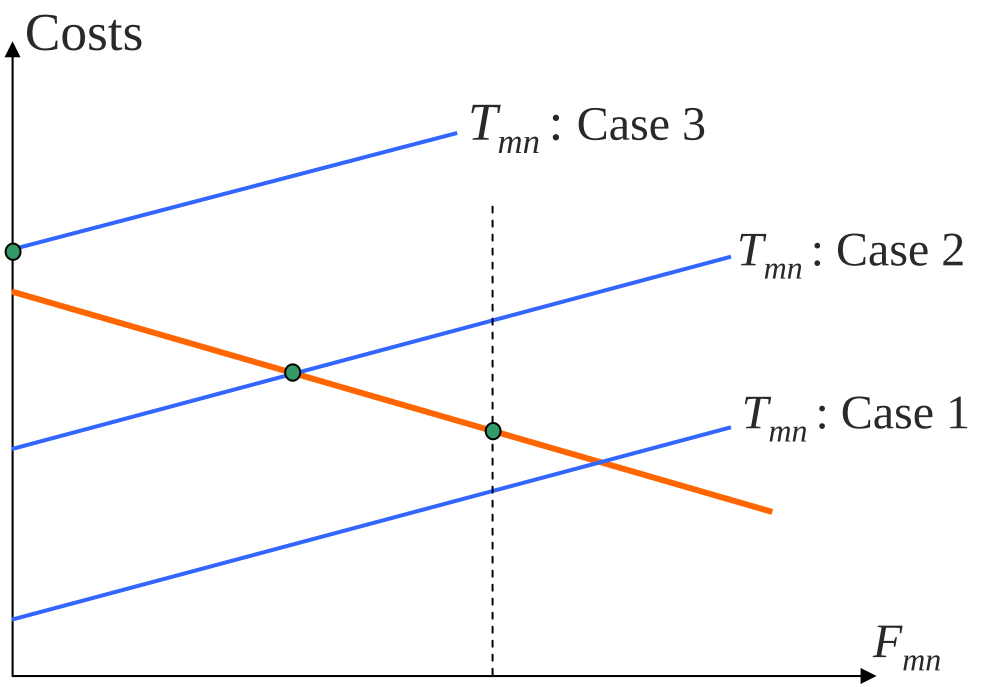
- $F_{mn} + \Phi_{mn} \leq S_{mn}$ The constraint implies that

$$\Phi_{mn} \leq \frac{\alpha_{mn} - a_{mn} + (\beta_{mn} + b_{mn})S_{mn}}{b_{mn}} \qquad (8)$$

- **Case 3**: the intra flow is null, and the travel cost is greater than the inverse demand: $T_{mn} = \alpha_{mn} + \beta_{mn}\Phi_{mn} \geq \Theta_{mn} = a_{mn}$

Network model 6

Thus if $F_{mn} = 0$ $\qquad \Phi_{mn} \geq \dfrac{a_{mn} - \alpha_{mn}}{\beta_{mn}}$ (9)



Network model 7

- If $S_{mn} \le \frac{a_{mn} - \alpha_{mn}}{\beta_{mn}}$, we have the following.
- Case 1. $\Phi_{mn} \le \frac{a_{mn} - \alpha_{mn}}{\beta_{mn}}$ then: $F_{mn} = \frac{a_{mn} - \alpha_{mn} - \beta_{mn}\Phi_{mn}}{b_{mn} + \beta_{mn}}$ (10)
- Case 2. $S_{mn} \ge \Phi_{mn} \ge \frac{a_{mn} - \alpha_{mn}}{\beta_{mn}}$, then $F_{mn} = 0$ (11)
- b. $S_{mn} \ge \frac{a_{mn} - \alpha_{mn}}{\beta_{mn}}$ If , we have the following.
- Case 1 $\Phi_{mn} \le \frac{\alpha_{mn} - a_{mn} + (\beta_{mn} + b_{mn})S_{mn}}{b_{mn}}$, then $F_{mn} = \frac{a_{mn} - \alpha_{mn} - \beta_{mn}\Phi_{mn}}{b_{mn} + \beta_{mn}}$ (12)
- Case 2. $S_{mn} \ge \Phi_{mn} \ge \frac{\alpha_{mn} - a_{mn} + (\beta_{mn} + b_{mn})S_{mn}}{b_{mn}}$, then $F_{mn} = S_{mn} - \Phi_{mn}$ (13)

Network costs 1

The intra flow is given by a piecewise linear continuous expression of the inter flow and the link capacity. We summarize (10)-(13) by

(14)

$$F_{mn} \overset{def}{=} L(\Phi_{mn}, S_{mn})$$

- The intra flow is given by a piecewise linear continuous expression of the inter flow and the link capacity. We summarize (10)-(13) by

(14)

- The cost of link (m,n) at link equilibrium Ξ_{mn}

If $\Phi_{mn} \leq \frac{a_{mn} - \alpha_{mn}}{\beta_{mn}}$, we have the following.

Case 1. $S_{mn} \leq \frac{a_{mn} - \alpha_{mn}}{\beta_{mn}}$, then: $\Xi_{mn} = \frac{a_{mn}\beta_{mn} + \alpha_{mn} b_{mn}}{b_{mn} + \beta_{mn}} + \Phi_{mn} \frac{b_{mn}\beta_{mn}}{b_{mn} + \beta_{mn}}$ (15)

Case 2. $S_{mn} \geq \Phi_{mn} \geq \frac{a_{mn} - \alpha_{mn}}{\beta_{mn}}$, then $\Xi_{mn} = \alpha_{mn} + \beta_{mn}\Phi_{mn}$ (16)

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Network costs 2

b. If $S_{mn} \geq \frac{a_{mn} - \alpha_{mn}}{\beta_{mn}}$, we have the following.

Case 1 $\Phi_{mn} \leq \frac{\alpha_{mn} - a_{mn} + (\beta_{mn} + b_{mn})S_{mn}}{b_{mn}}$, then $\Xi_{mn} = \frac{a_{mn}\beta_{mn} + \alpha_{mn}b_{mn}}{b_{mn} + \beta_{mn}} + \Phi_{mn}\frac{b_{mn}\beta_{mn}}{b_{mn} + \beta_{mn}}$
(17)

Case 2. $S_{mn} \geq \Phi_{mn} \geq \frac{\alpha_{mn} - a_{mn} + (\beta_{mn} + b_{mn})S_{mn}}{b_{mn}}$, then $\Xi_{mn} = a_{mn} - b_{mn}S_{mn} + b_{mn}\Phi_{mn}$
(18)

We summarize (15)-(18) by .

$$\Xi_{nm} = \Xi_{nm}(\Phi_{nm})$$

Network costs 3

Toll and VAT based costs:

Customers

One generic good is considered.

The demand function for this good at node (n) is given by: $\Pi_n = e_n - f_n C_n$

with the cost C_n

The maximum possible demand e_n

is a fraction of population N_n

The number of customers who accept a cost in the interval $[C_n, C_n + dC_n]$

Is given by: $f_n \, dC_n$

Network costs 4

- It is assumed that the supply at each node adjusts to the demand.

- The price of the generic good is uniform, only the VAT depends on the region.

- The price of the generic good at any W_m location is identical to the VAT at the same location

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Network costs 5

Distribution of demand with respect to destination

Unlimited supply at any destination (m), i.e. supply adjusts to demand

Let C_{nm} be the OD cost $(n) \rightarrow (m)$

Logit model for the choice of consumers

$$\mathbf{P}(n \rightarrow m | C_n) = \frac{e^{-\theta C_{nm}}}{\sum_j e^{-\theta C_{nj}} + e^{-\theta C_n}}$$

The number of consumers at who choose the destination (m) to acquire the good is given by:

$$N_{nm} = \int_0^{e_n / f_n} dC_n \, \mathbf{P}(n \rightarrow m | C_n)$$

$$N_{nm} \overset{def}{=} \Delta_{nm}(C) = \left[e_n - \frac{f_n}{\theta} \ln \left(\frac{\sum_j e^{-\theta C_{nj}} + 1}{\sum_j e^{-\theta C_{nj}} + e^{-\theta e_n / f_n}} \right) \right] \frac{e^{-\theta C_{nm}}}{\sum_j e^{-\theta C_{nj}}}$$

Network costs 6

The number of potential consumers at (n) who do not consume is given by

$$N_{n0} \stackrel{def}{=} \frac{f_n}{\theta} \ln \left(\frac{\sum_j e^{-\theta C_{nj}} + 1}{\sum_j e^{-\theta C_{nj}} + e^{-\theta e_n / f_n}} \right)$$

Network equilibrium

Path flows: $\psi_p,\ p \in \mathbf{P}$, and arc flows $\Phi_a = \sum_{p/a \in p} \psi_p, \quad F_a \stackrel{def}{=} L_a(\Phi_a, S_a), \quad a \in \mathbf{A}$

Total flow on arcs: $\Phi_a + F_a = \Phi_a + L_a(\Phi_a, S_a), \quad a \in \mathbf{A}$

Arc tolls: $t_a, a \in \mathbf{A}$

Arc costs: $\Xi_a(\Phi_a), \quad a \in \mathbf{A}$

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Network costs 7

- Path costs: $\Gamma_p = \sum_{a\in p} \Xi_a(\Phi_a), \quad p \in \mathbf{P}$
- OD costs: $C_{nm} = \lambda\left(\mathrm{Min}_{p\in\mathbf{P_{mn}}} \Gamma_p\right) + W_m$
- OD demand: $D_{mn} = \mu\,\Delta_{mn}(C)$ (20)
- the fraction of the OD cost supported by each consumer (shipping and transportation costs can be divided among several consumers)

$$\lambda \in]0,1[$$

- the impact of demand for the good on travel demand.

$$\mu \in]0,1[$$

Network costs 8

- The constraints on flows are:

$$K(C)=\left|\begin{array}{l}\psi_p \geq 0 \quad \forall p \in \mathbf{P} \\ D_{nm}=\Delta_{nm}(C)=\sum_{p\in \mathbf{P_{nm}}} \psi_p \quad \forall (m,n) \in \mathbf{W} \\ \Phi_a=\sum_{p/a\in p} \psi_p \leq S_a \quad \forall a \in \mathbf{A}\end{array}\right. \qquad (21)$$

- The set of constraints is convex and linear, and depends on OD costs *C*.

$$\Sigma_{nm}=\lambda\left(\mathrm{Min}_{p\in \mathbf{P_{mn}}} \Gamma_p\right), \quad C_{nm}=\Sigma_{nm}+W_m \qquad (22)$$

- The equilibrium conditions are given by the following variational inequality:

$$(\lambda\Gamma_p-\Sigma_{nm}).\psi_p=0 \quad \forall p \in \mathbf{P_{nm}}, \quad \forall (m,n)\in \mathbf{W}, \quad \forall \psi_p \in K(C) \qquad (23)$$

- This variational inequality expresses the optimality condition

Two regions model competing using tolls and VAT 1

- The elements of the model are the following:
  - There are two regions i = 1 and 2
  - Region (i) applies a VAT W_i on consumption and a toll t_i on transportation $\rho_i(x_i)dx_i$
  - The population density is $\varphi_i(x_i,\tau)d\tau$, with x_i the distance to the closest centre over the border
  - The density of consumers at distance x_i to the closest centre over the border with travel cost τ is $\varphi_i(x_i,\tau)d\tau$
  - The density of the population with respect to the travel cost τ_i is given by:

$$\mathrm{P}_i(\tau_i)d\tau_i \overset{def}{=} d\tau_i \int_{(i)} \rho(x_i)\varphi_i(x_i,\tau_i)dx_i \qquad (24)$$

Two regions model competing using tolls and VAT 2

- The total population of region (i) is: $N_i = \int_0^\infty \rho_i(x_i)\,dx_i = \int_0^\infty P_i(\tau_i)\,d\tau_i$

- Cost of buying in region (j): $\lambda(\tau_i + t_i + t_j) + W_j + \eta_i$, with

  - η_i a random variable expressing the variability of consumers, the variability of the consumer perception of items such as travel time, the variability of travel costs,

  - λ the coefficient of impact on traffic of demand

  - Cost of buying in region (i): $W_i + \zeta_i$, with ζ_i a random variable expressing the variability of consumers and consumer perception.

Two regions model competing using tolls and VAT 3

The probability for a consumer in region (i) to buy in region (j) is given by:

$$\begin{aligned}\mathbf{P}[i \to j] &= \mathbf{P}[\eta_i - \zeta_i \leq W_i - W_j - \lambda(\tau_i + t_i + t_j)] \\ &= G_i(\Delta W - \lambda t)\end{aligned}$$

with notations:

$$\Delta W \overset{def}{=} W_i - W_j \qquad t = t_i + t_j$$

$$G_i(\sigma) \overset{def}{=} \frac{1}{1+\exp(-\theta_i\sigma)} \overset{def}{=} \int_{-\infty}^{\sigma} g_i(s)ds \qquad g_i(s) = \frac{\theta_i \exp(-\theta_i s)}{(1+\exp(-\theta_i s))^2}$$

•The population in the simplified model is not concentrated on point-wise nodes but spread,
• The assignment is simplified (travel costs are independent of flows),
• The distribution is simplified (the demand function is simplified with).

Two regions model competing using tolls and VAT 4

Consumers

The number of consumers of region (*i*) buying in (*j*).

$$\begin{aligned} N_{i\to j} &= \mu\int_0^{\infty} d\tau_i P(\tau_i) G(\Delta W - \lambda t - \lambda \tau_i) \\ &= \mu\int_0^{\Delta W} d\sigma\, Q_i(\sigma - \lambda t) \end{aligned} \qquad (25)$$

with

$$Q_i(\varsigma) \overset{def}{=} \int_0^{\infty} d\tau_i\, P(\tau_i)\, g_i(\varsigma - \lambda\tau_i) \qquad (26)$$

Q_i represents the population density of region (*i*) corrected by the effect of variability of consumers.

$N_{k\to\ell}$ the number of consumers of region (*k*) buying in (*l*).

$$\begin{aligned} N_{i\to i} &= \mu\int_{\Delta W}^{\infty} d\sigma\, Q_i(\sigma - \lambda t) \\ N_{i\to j} &= \mu\int_0^{\Delta W} d\sigma\, Q_i(\sigma - \lambda t) \\ N_{j\to j} &= \mu\int_{-\Delta W}^{\infty} d\sigma\, Q_j(\sigma - \lambda t) \\ N_{i\to j} &= \mu\int_0^{-\Delta W} d\sigma\, Q_j(\sigma - \lambda t) \end{aligned} \qquad (27)$$

Two regions model competing using tolls and VAT 5

Also

$$N_{i\to j}=\mu\int_0^{\Delta W}d\sigma\, Q_i\left(\sigma-\lambda t\right)=\mu\int_0^{\Delta W-\lambda t}d\sigma\, Q_i\left(\sigma\right)$$

Revenue, reaction curves

The revenue R_i of region (*i*) is expressed by

$$R_i=\lambda t_i\left(N_{i\to j}+N_{j\to i}\right)+W_i\left(N_{i\to i}+N_{j\to i}\right) \qquad (28)$$

The reaction curves for the VATs, $\frac{\partial R_i}{\partial W_i}=0$

$$W_i=\lambda t_i\frac{Q_i\left(\Delta W-\lambda t\right)-Q_j\left(-\Delta W-\lambda t\right)}{Q_i\left(\Delta W-\lambda t\right)+Q_j\left(-\Delta W-\lambda t\right)}+\mu^{-1}\frac{N_{i\to i}+N_{j\to i}}{Q_i\left(\Delta W-\lambda t\right)+Q_j\left(-\Delta W-\lambda t\right)}$$

Two regions model competing using tolls and VAT 6

- By definition of $\Delta W = W_i - W_j$, the reaction curve for region (*i*) with respect to *W* results:

(29)
$$S_i^W(\Delta W, t) = \begin{cases} W_i = \lambda t_i \dfrac{q_i - q_j}{q_i + q_j} + \mu^{-1} \dfrac{N_{i\to i} + N_{j\to i}}{q_i + q_j} \\ W_j = -\Delta W + \lambda t_i \dfrac{q_i - q_j}{q_i + q_j} + \mu^{-1} \dfrac{N_{i\to i} + N_{j\to i}}{q_i + q_j} \end{cases}$$

$$\text{with} \quad q_i \overset{def}{=} Q_i(\Delta W - \lambda t),\ q_j = Q_j(-\Delta W - \lambda t)$$

- This curve is parameterized by ΔW, for given values of the tolls t_i, t_j
- A similar calculation yields the reaction curve for region (j)

$$S_j^W(\Delta W, t) = \begin{cases} W_i = \Delta W + \lambda t_j \dfrac{-q_i + q_j}{q_i + q_j} + \mu^{-1} \dfrac{N_{j\to j} + N_{i\to j}}{q_i + q_j} \\ W_j = \lambda t_j \dfrac{-q_i + q_j}{q_i + q_j} + \mu^{-1} \dfrac{N_{i\to i} + N_{j\to i}}{q_i + q_j} \end{cases}$$

$$\text{with} \quad q_i \overset{def}{=} Q_i(\Delta W - \lambda t),\ q_j = Q_j(-\Delta W - \lambda t)$$

Two regions model competing using tolls and VAT 7

- Calculating the reaction functions for the tolls, assuming the VATs are given, in the case of the region (i)

$$\frac{\partial R_i}{\partial t_i} = 0$$

yielding:

$$t_i = W_i \frac{Q_i(\Delta W - \lambda t) - Q_j(-\Delta W - \lambda t)}{Q_i(\Delta W - \lambda t) + Q_j(-\Delta W - \lambda t)} + \lambda^{-1}\mu^{-1} \frac{N_{i\to j} + N_{j\to i}}{Q_i(\Delta W - \lambda t) + Q_j(-\Delta W - \lambda t)}$$

- Since by definition of t, $t = t_i + t_j$, the reaction curve for region (i) , with respect to t:

(31)

$$T_i^W(\Delta W, t) = \begin{cases} t_i = W_i \dfrac{q_i - q_j}{q_i + q_j} + \lambda^{-1}\mu^{-1} \dfrac{N_{i\to j} + N_{j\to i}}{q_i + q_j} \\ t_j = t - W_i \dfrac{q_i - q_j}{q_i + q_j} - \lambda^{-1}\mu^{-1} \dfrac{N_{i\to j} + N_{j\to i}}{q_i + q_j} \end{cases}$$

$$\text{with} \quad q_i \stackrel{def}{=} Q_i(\Delta W - \lambda t),\ q_j = Q_j(-\Delta W - \lambda t)$$

Two regions model competing using tolls and VAT 8

A similar calculation yields the reaction curve for region (j) :

$$T_j^W(\Delta W, t) = \begin{cases} t_i = t - W_j \dfrac{-q_i + q_j}{q_i + q_j} - \lambda^{-1}\mu^{-1} \dfrac{N_{i\to j} + N_{j\to i}}{q_i + q_j} \\ t_j = W_j \dfrac{-q_i + q_j}{q_i + q_j} + \lambda^{-1}\mu^{-1} \dfrac{N_{i\to j} + N_{j\to i}}{q_i + q_j} \end{cases} \quad (32)$$

$$\text{with} \quad q_i \overset{def}{=} Q_i(\Delta W - \lambda t), q_j = Q_j(-\Delta W - \lambda t)$$

•Thus we can study, given the tolls, the equilibrium of regional competition for the VATs, or given the VATs, study the equilibrium of regional competition for the tolls.

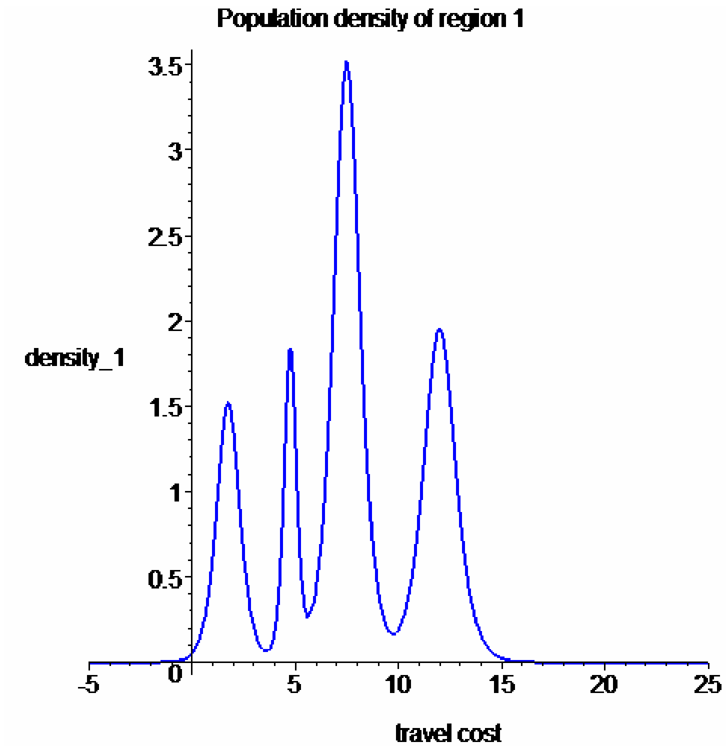
network dynamics and invariance principle 1

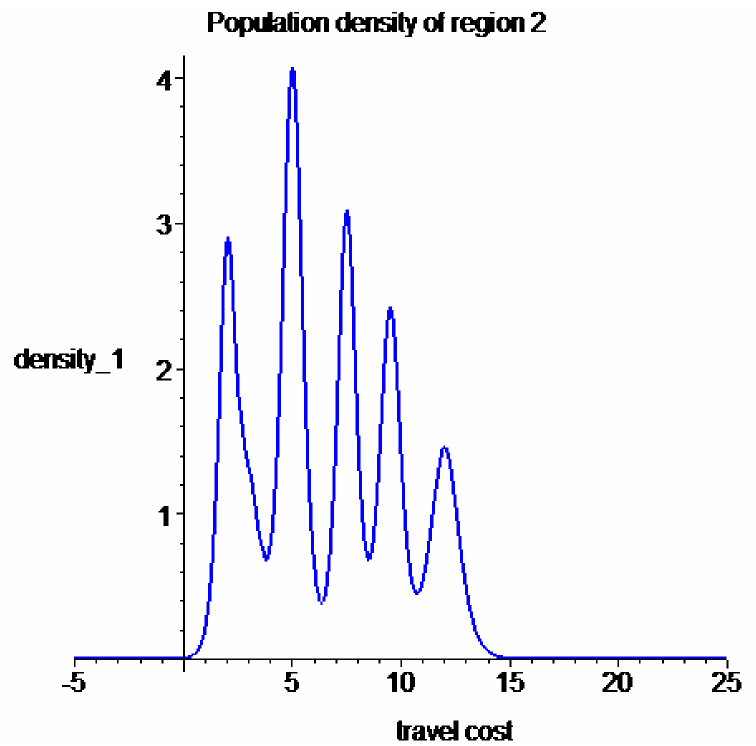
The case of two regions:

- Population densities

Q_1 Q_2

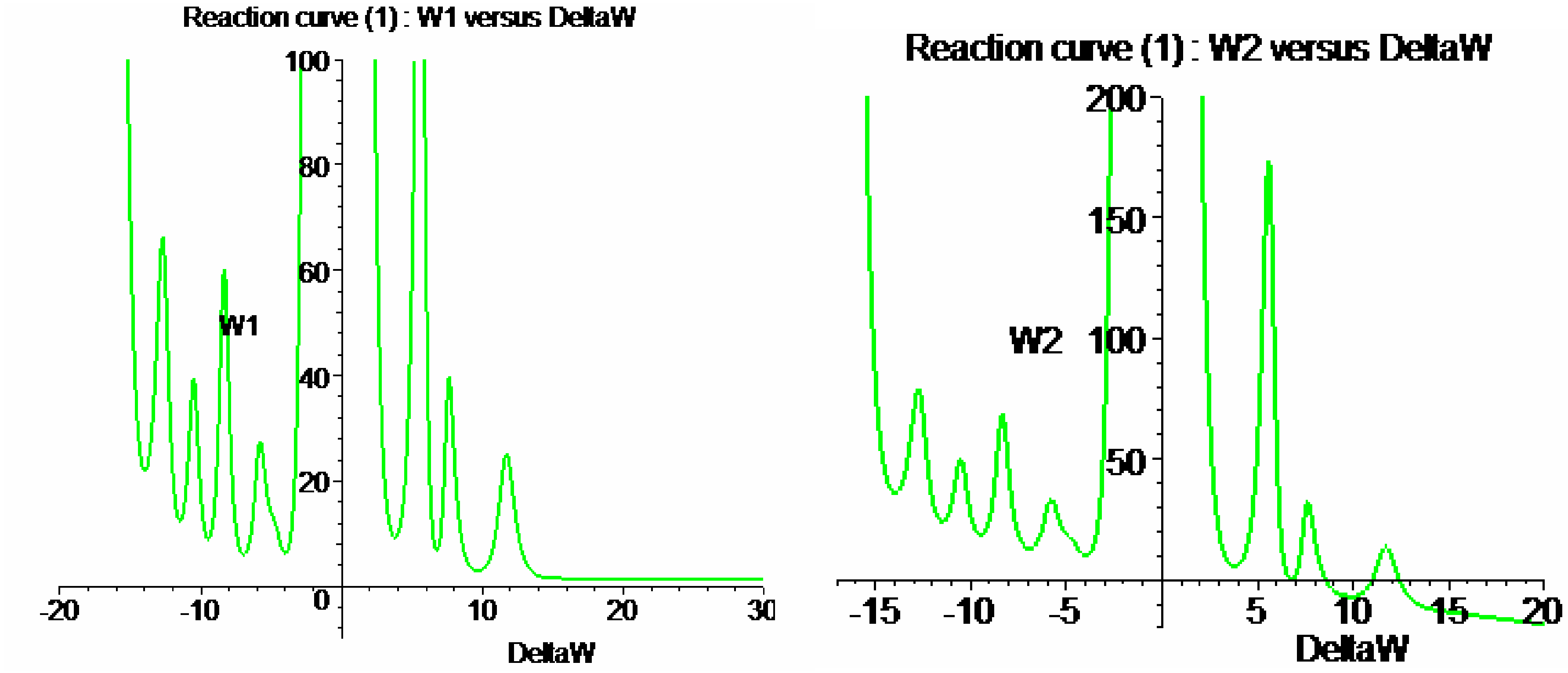
The density is the distribution of population with respect to travel costs as modified by the variability of consumers





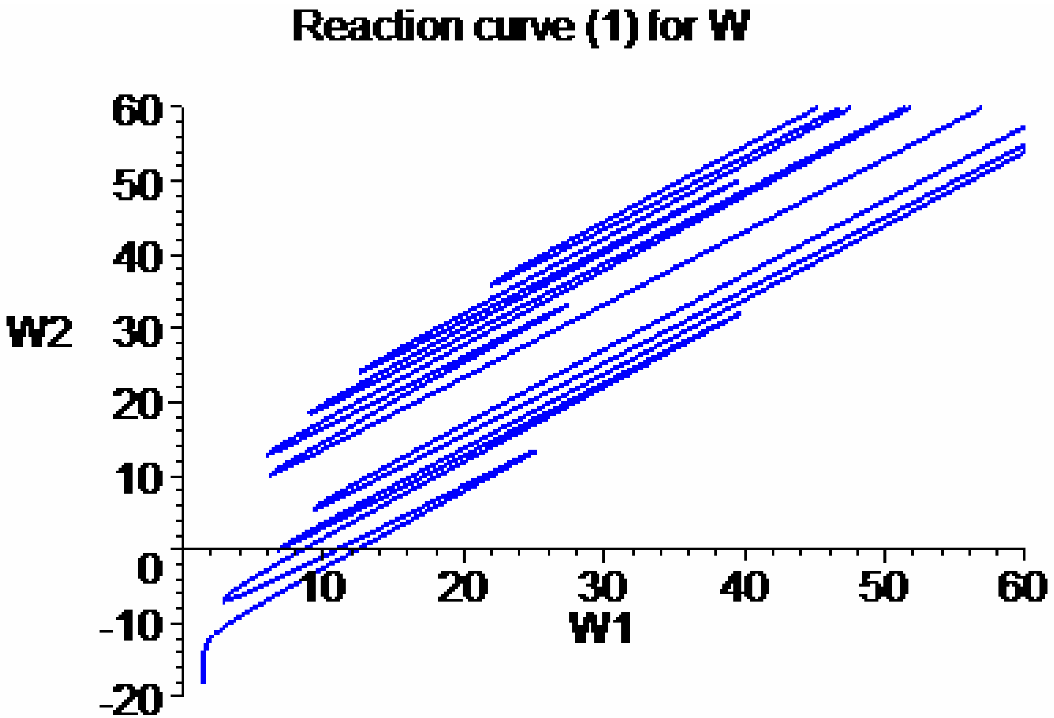
network dynamics and invariance principle 2

The shapes of the components of the reaction function (W1), and (W2) of the two regions are depicted by the following figure:



Network dynamics and invariance principle 3

The reaction curve of region (1) is described by the figure below. This curve does not define a function. Given *W1*, the reaction curve yields all possible values *W2* for which, i.e. local extrema (half of which actually correspond to a local maximum) of the revenue of region (1).



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Network dynamics and invariance principle 4

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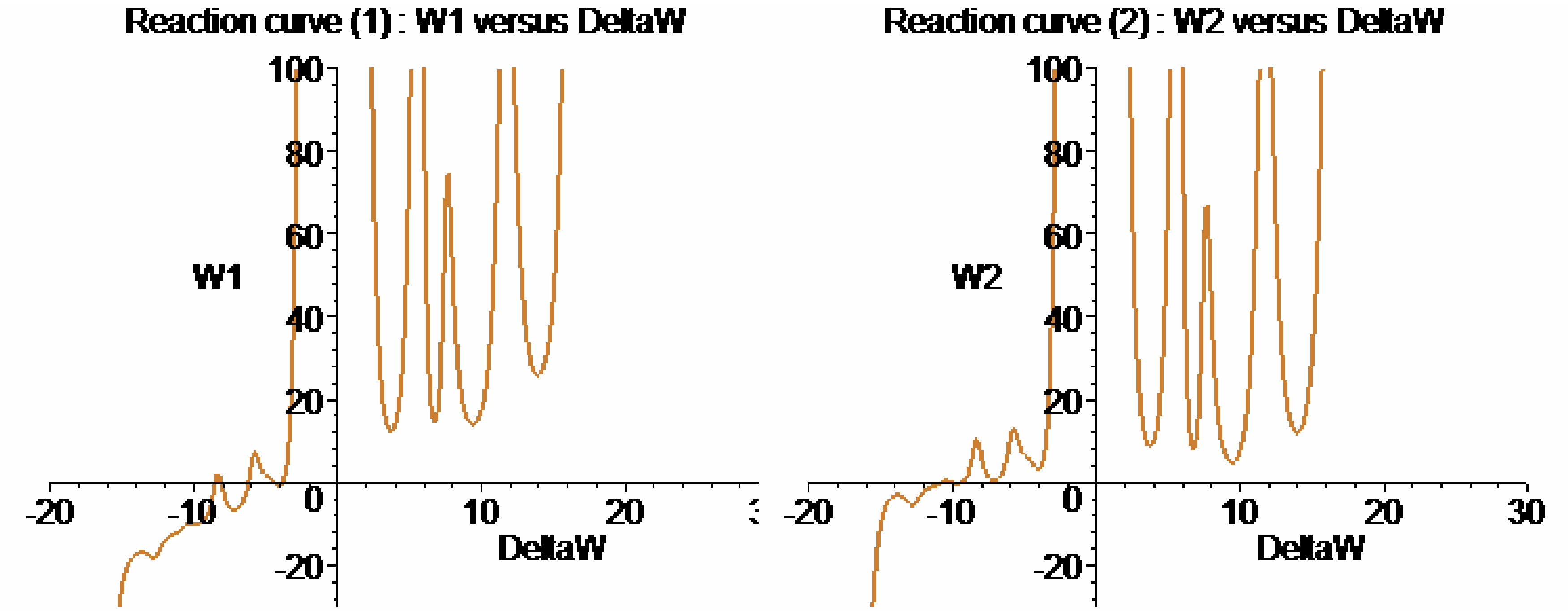
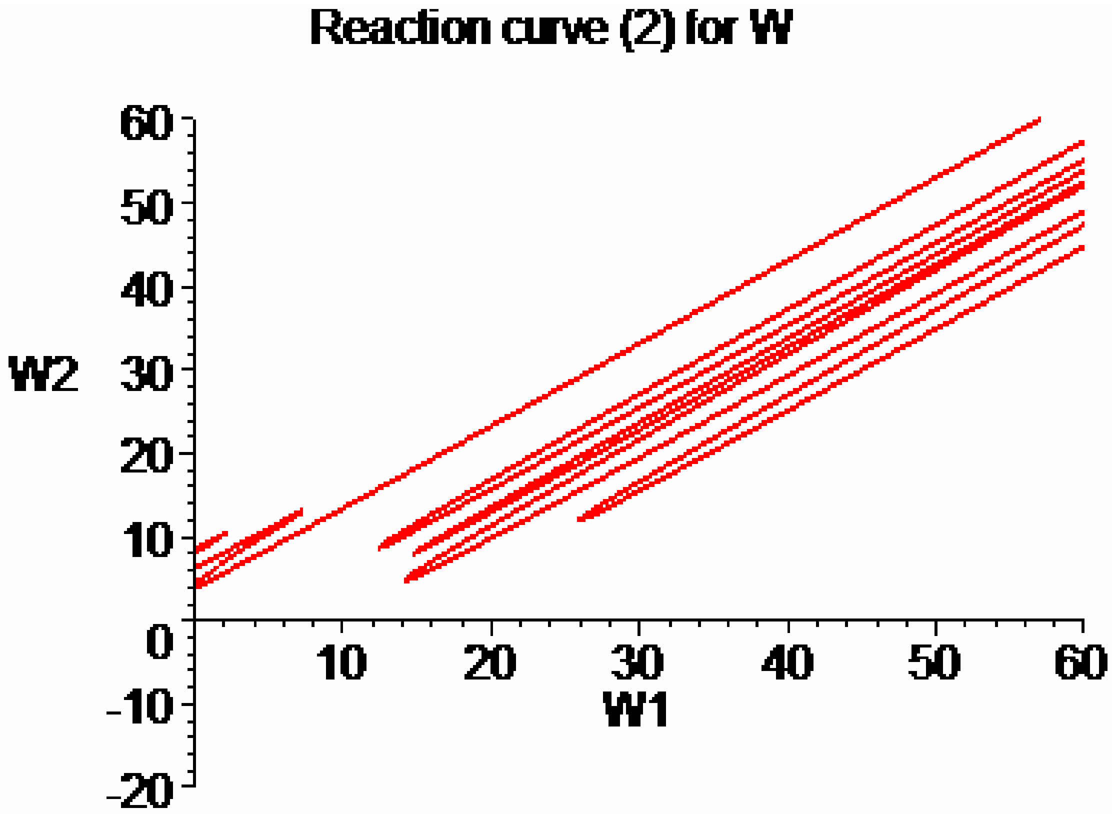
The components of the reaction function for region (2)



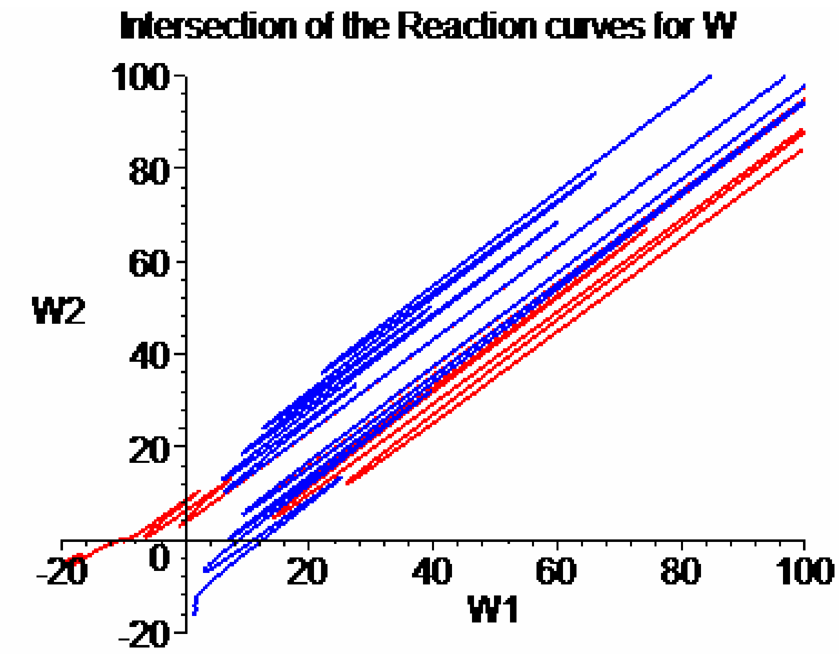
Network dynamics and invariance principle 5

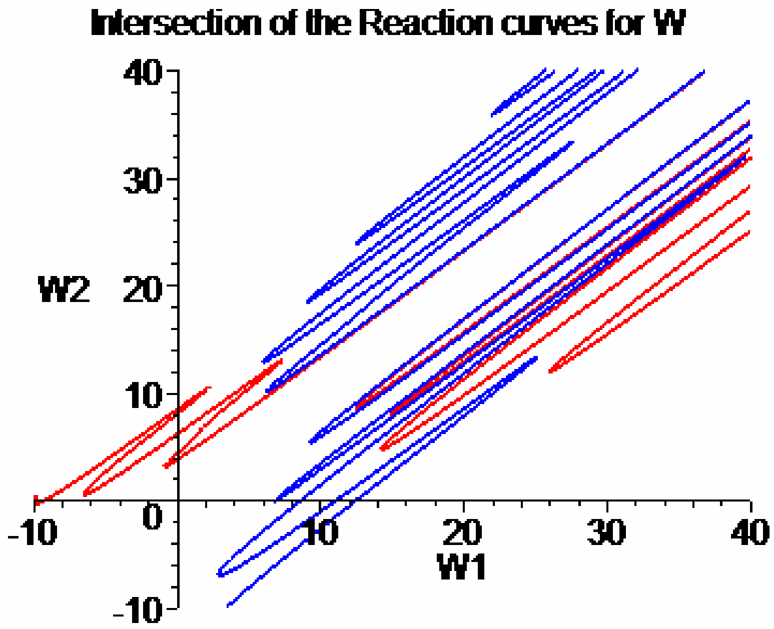
- The corresponding reaction curve for VATs (for region (2)) is depicted below.



Network dynamics and invariance principle 6

- The figure below illustrates the intersection of reaction curves for VATs for the 2 regions figure (blue for region (1), red for region (2)).





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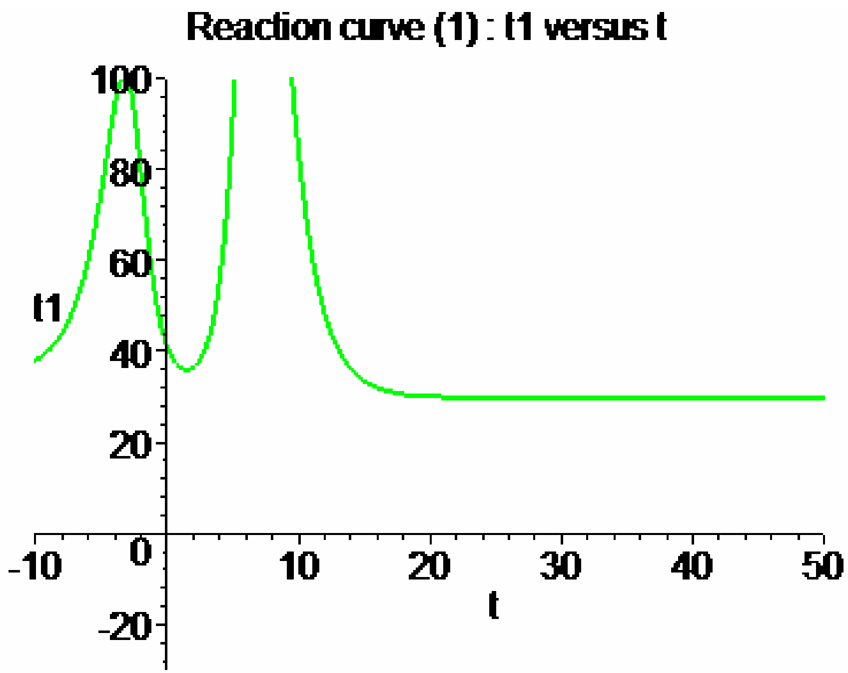
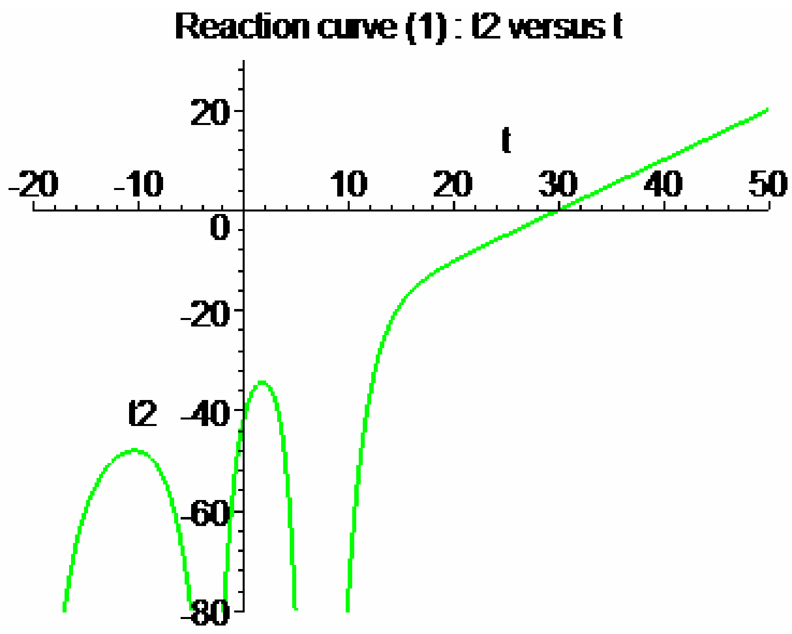
Network dynamics and invariance principle 7

- The many loops in the reaction functions for the VATs express the impact of the transportation system costs and the heterogeneity of population density.
- When considering the procedure of successive optimal choices, i.e. each region optimizing in turn its revenue, there is obviously no reason to reach a Nash equilibrium. Each region has a choice between many alternative values of VAT.

Network dynamics and invariance principle 8

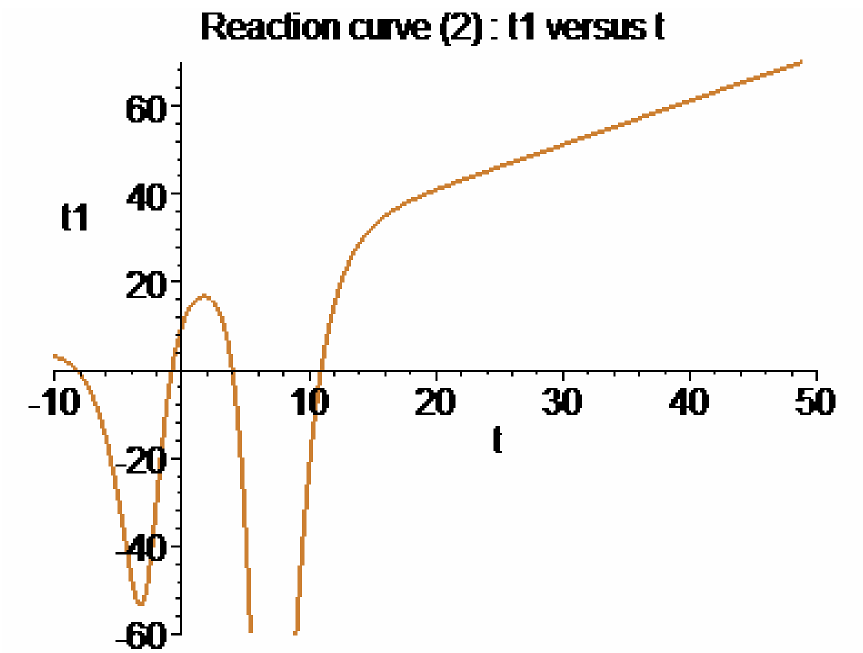
Tolls

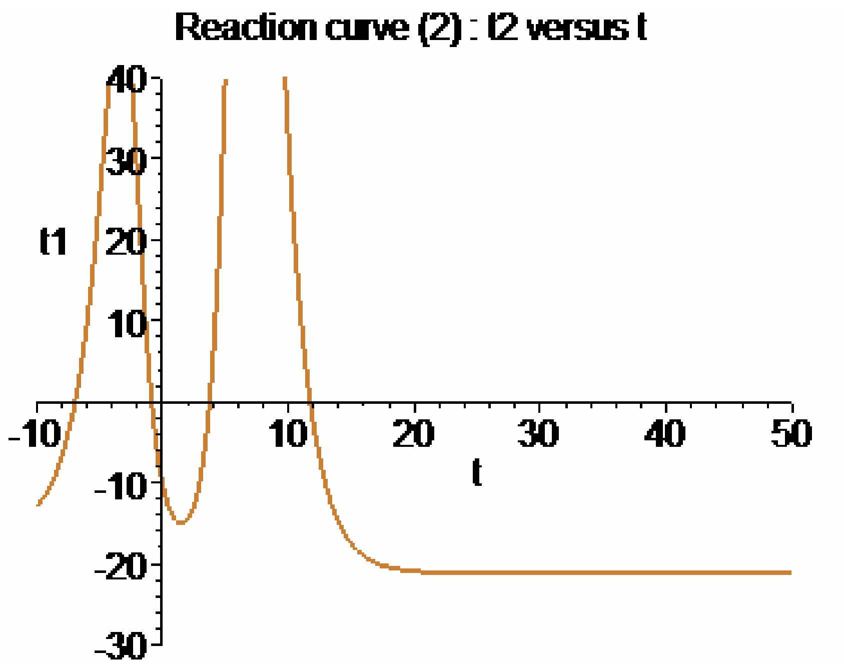




Network dynamics and invariance principle 9

- The coordinate functions of the reaction curves for tolls are depicted by the following figure
- The difference between these and the toll reaction functions is due to the parameterization; the toll reaction functions are parameterized by the sum of tolls, whereas the VAT reaction curves are parameterized by the difference between VATs





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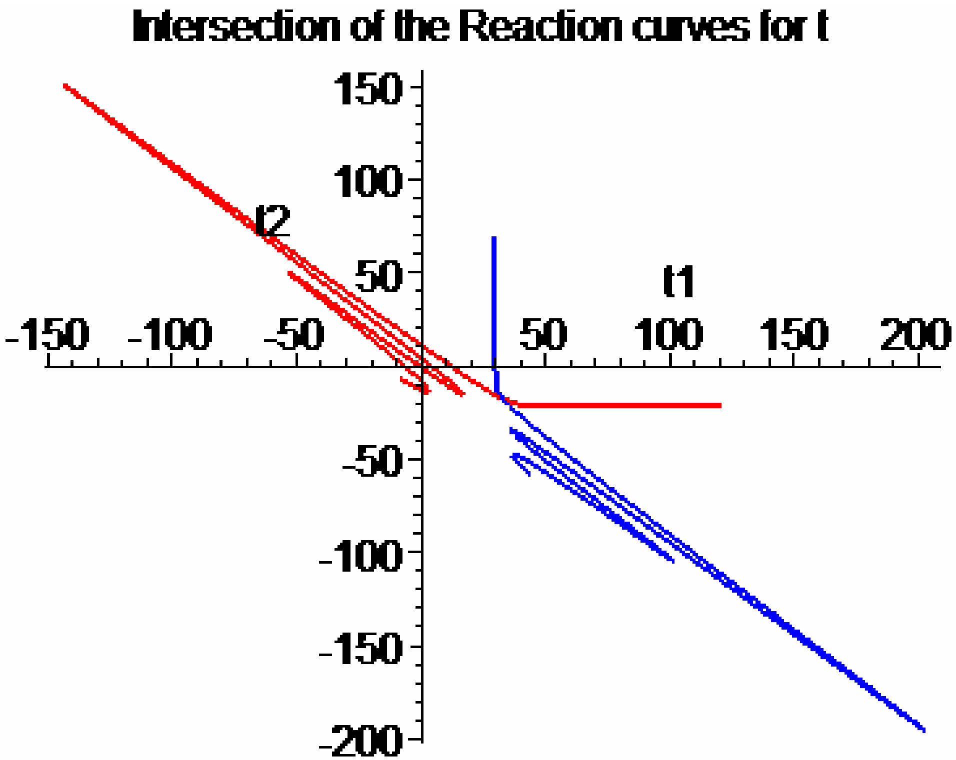
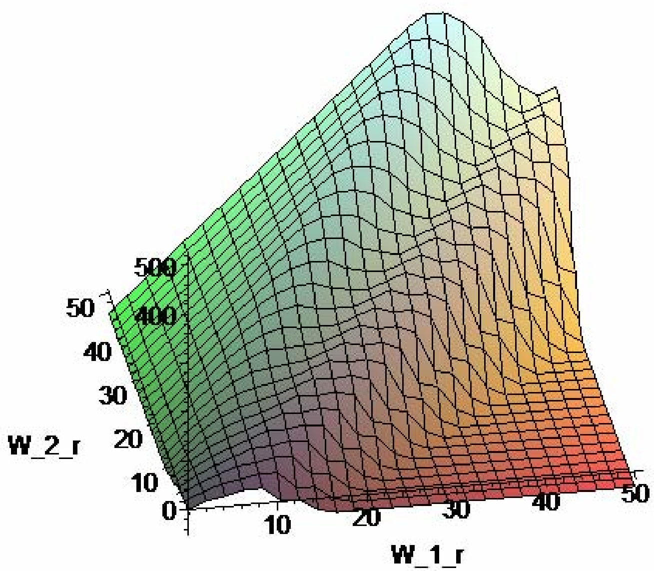
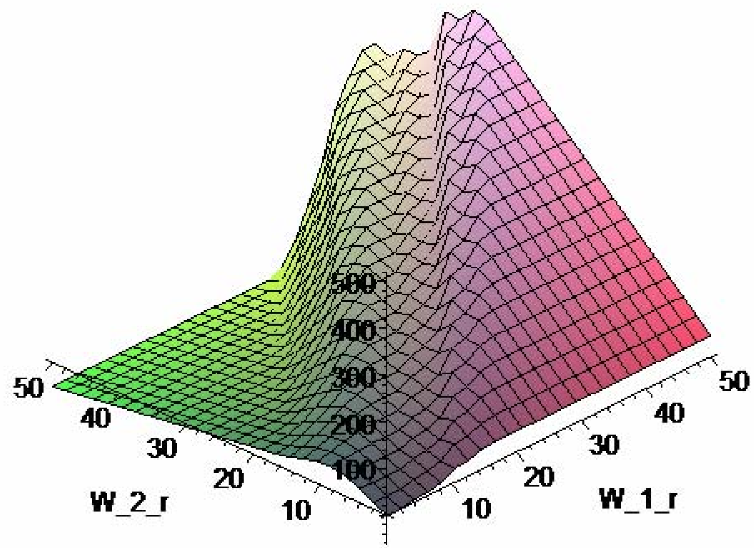
- The intersection of toll reaction curves is illustrated by the below figure (blue for region (1), red for region (2)). Negative tolls are interpreted as subventions.



Network dynamics and invariance principle 11

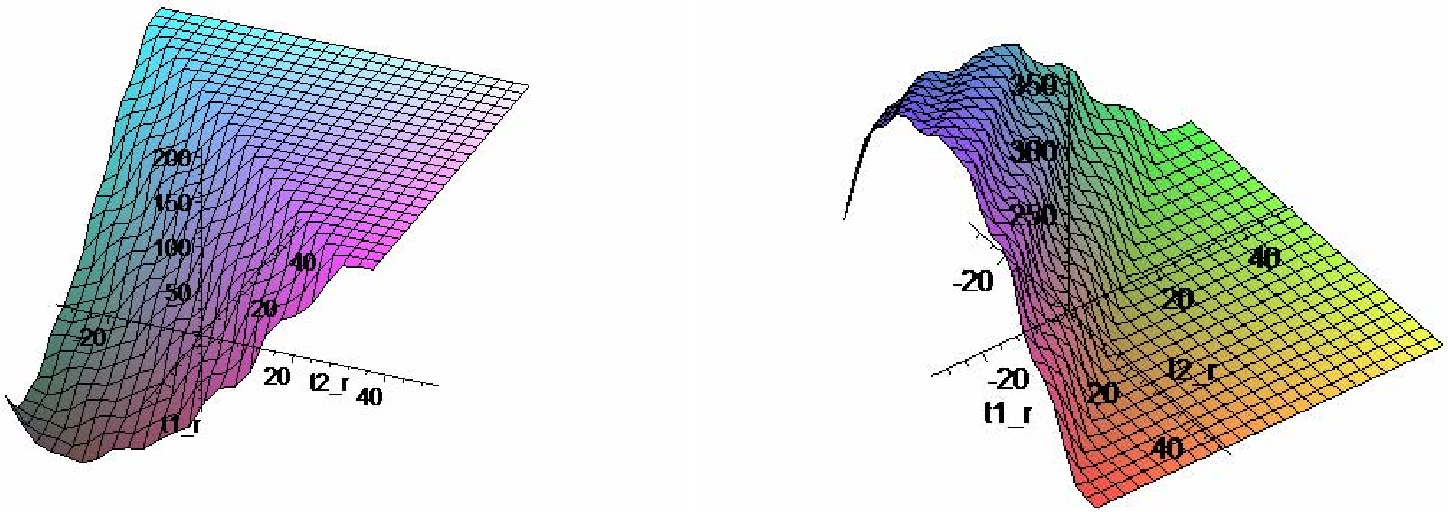
- The figure below depicts the regional revenues of region (1) (left) and region (2) (right) in the VAT equilibrium (tolls are fixed). The revenues are unbounded because the simplified two-region model does not include node demand functions.





Network dynamics and invariance principle 12

- The figure below depicts the regional revenues of region (1) (left) and region (2) (right) in the toll equilibrium (VATs are fixed).



Network dynamics and invariance principle 13

- In view of the reaction curves and their intersections, a competition process whereby regions try to optimize in turn their VAT / toll will not achieve any equilibrium but will always result in one region losing out (which has been postulated as the invariance principle).

- The general model

The total revenue of region (r) is given by:

$$\mathbf{R_r} = \sum_{m\in(r)} W_m \Delta_{mn}(C) + \sum_{a\in(r)} t_a \left(\Phi_a + L(\Phi_a, S_a)\right)$$

Let us define:

$$x^r \overset{def}{=} \left((W_m)_{m\in(r)}, (t_a)_{a\in(r)}\right) \qquad \overline{x}^r \overset{def}{=} \left((W_m)_{m\notin(r)}, (t_a)_{a\notin(r)}\right)$$

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- Partial reaction functions can be defined for region (r) :

$$H_r\left(\overline{x}^r\right) = \mathrm{Arg}_{x^r}\mathrm{Max}\,\mathbf{R_r}\left(x^r, \overline{x}^r\right)$$

- A Nash equilibrium is reached if $x = H(x)$
- The reaction functions are called partial because they are only piecewise continuous, as in the case of two regions.
- For each choice of tolls and VATs, a selection of paths is optimal and the revenue admits a closed expression yielding an expression of the reaction function dependent on the selection of paths.
- Thus as in the case of two regions, the iterative process of regions optimizing their revenues in turn normally does not converge and the process results in some regions losing out, again in conformity with the invariance principle postulate.

Conclusion

- A general regional competition model is introduced which includes local demand for a generic good, with distribution and assignment mechanisms
- A simpler version of the model is tried out with two models of regional tax/toll competition given transportation costs.
- The reaction functions are necessarily multi-valued and only piecewise continuous.
- Simulation of a two region model shows that though regional competition leads to some regions becoming more affluent (increase in regional revenue), at least one region stays worse off. This demonstrates the invariance principle.