Regional network competition: using tax instruments
(DRAFT)

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Abstract

A country is divided into several regions that are connected by a network of roads. Each region strives to maximize its revenue using tax instruments such as tolls and VAT (Value Added Tax) on goods and services. Regions are non-homogenous in the sense that they have different population densities and economic activities. Based on these two elements they are either considered better off or worse off regions. For example a region with high population density and high level of economic activities is considered as a better off region. Toll and VAT are considered as local taxes. This means that each region can set its toll and VAT depending on its revenue objective. Each region has a share of the road network. Each regional network is considered to be at a non-equilibrium state, with traffic patterns that obey the LWR model. Depending on the neighboring regions, i.e. a better off region may have either better off regions or worse off regions or a mix of both as neighbors; a specific tax scenario is applied. Several tax scenarios are engaged. A model is devised to test the impact of tax scenarios on the status of a region. Each region has an initial state of either being better off or worse off. The model tests whether the application of a particular tax scenario for a period of time will change the status of the region for better or worse. A utility maximization model investigates the invariance principle. The invariance principle states that no matter which tax scenario is used, and no matter for how long (if the process is repeated indefinitely), there will still be inequality among regions. It is not possible to have uniformly better off regions. There will always be regions with different degrees of affluence. The measure of affluence is determined through region's market structure, market share, and social welfare (revenue).

Key words: regional competition, tax instruments, tolling, VAT, network, nodes, links, invariance principle, utility maximization model.
JEL: L91, R48

1. INTRODUCTION

The object of this paper is to analyze some possible implications of tax/toll competition between regions.

Many researchers studied tax competition; for example, Mintz and Tulkens (1985), Zodrow and Mieszkowski (1986), Wildasin (1988), Bucovetsky (1991), Hoyt (1993), Bayindir-Upmann and Ziad (2006) and many others. Some papers have been devoted to the study of commodity tax competition models: Kanbur and Keen (1993), Lockwood (1993), and Ohsawa (1994), Lucas (2004).

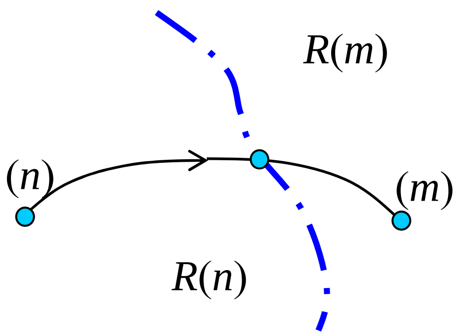
In order to carry out this analysis we build a simplified model which takes into account consumption of a single generic good, demand functions at population centres, impact on

transportation of activity, distribution, assignment on the transportation network with non constant arc costs, and node supply constraints. The model distinguishes local flows from flows induced by economic activity. Regions apply tolls on transportation and taxes (VAT) on the generic good. This model is developed in section 2. The Nash equilibrium for transit networks has been studied in Zhou et al. (2005).
A two region model is introduced in section 3. This model relies on some further simplification and can be studied semi-analytically. This model expands on Khoshyaran (2002-2004), which took into account complex transportation costs in regional tax competition in the case of two regions. Regions are assumed to compete in order to maximize their revenue (by which we measure how well-off a region is). The reaction functions are piecewise continuous and multi-valued, generalizing to the case of tax/toll competition the results obtained in Khoshyaran and Lebacque (2006) results previously obtained only for tax competition. Only reaction curves can be parameterized.
The results extend to general networks. Thus in a process of sequential optimization, in which each region maximizes in turn its revenue, regions are unlikely to reach any Nash equilibrium, and while some profit others are worse off, which expresses the invariance principle (section 4).

2. Network model

2.1. The elements of description of the network are the following.

- Population is concentrated in nodes (n), $n \in \mathbf{N}$
- The transportation system between nodes (m) (n) and is represented by an arc: $(a) = (m,n)$, $a \in \mathbf{A}$, $m,n \in \mathbf{N}$
- Regions are denoted by $(r) \in \mathbf{R}$. The region to which node (n) belongs is called $R(n)$
- Arcs may connecting two different regions (and tolled by each region): $(a) = (m,n)$ and $R(m) \neq R(n)$ will be divided into two arcs, as depicted below. Thus any arc is assumed to belong to a single region.



- Paths $(p) \in \mathbf{P}$ connect origins and destinations, which are nodes in the network.
- An OD (origin-destination) couple is denoted by $(w) = (m,n)$, $w \in \mathbf{W}$, $m,n \in \mathbf{N}$.
- The set of paths connecting a given OD $(w) \in \mathbf{W}$ is called $(\mathbf{P}_w)$.

2.2. Flows on the network

- Flows on an arc $(a) = (m,n)$, $a \in \mathbf{A}$, $m,n \in \mathbf{N}$

- Φ_{mn}: inter flow induced by economic activities (in the present case it is the activity related to the generic good)
- F_{mn}: intra flow. This is the flow on the arc such that $\Phi_{mn} + F_{mn}$ is the total flow on arc (m,n)
- T_{mn}: cost of transportation over arc (m,n). Assume that the transportation cost is a linear function of the total flow

$$T_{mn} = \alpha_{mn} + \beta_{mn}(F_{mn} + \Phi_{mn}) \quad \forall (m,n) \in \mathbf{A} \tag{1}$$

- S_{mn}: supply constraint at head node (n) for traffic coming from tail node (m)

$$F_{mn} + \Phi_{mn} \le S_{mn} \quad \forall (m,n) \in \mathbf{A} \tag{2}$$

- Inverse demand for link (m,n):

$$\Theta_{mn} = a_{mn} - b_{mn} F_{mn} \quad \forall (m,n) \in \mathbf{A} \tag{3}$$

Note that the inverse demand is a function of local flow only. This model was introduced in ref??Mcicam+tristan

- Equilibrium of link (m,n). Given the inter flow Φ_{mn}, the intra flow F_{mn} is completely determined by the equilibrium between supply and demand and the capacity constraints (2). There are three cases of equilibrium.
  - Case 1: the intra flow reaches its maximum value, for which the travel cost is less than the inverse demand Θ_{mn}:

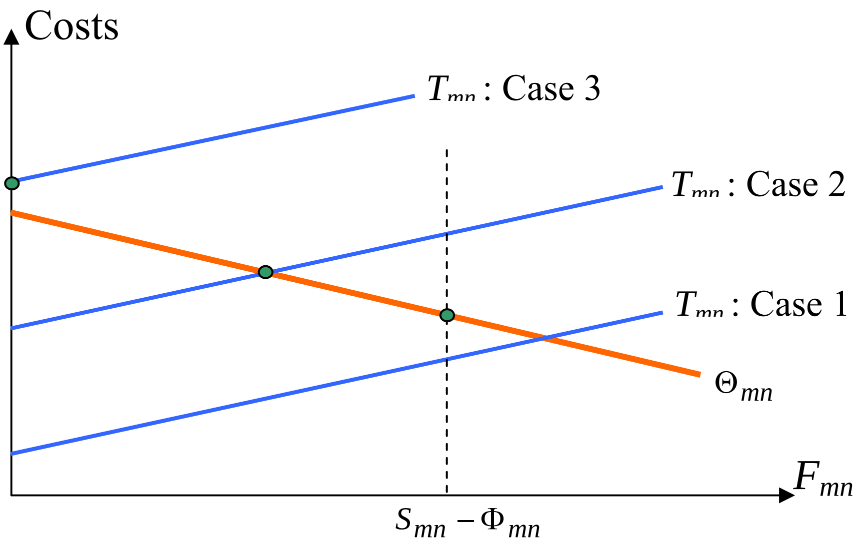
$$\begin{aligned} T_{mn} &= \alpha_{mn} + \beta_{mn} S_{mn} \\ &\le \Theta_{mn} = a_{mn} - b_{mn}(S_{mn} - \Phi_{mn}) \end{aligned}$$

then the equilibrium intra flow is given by:

$$F_{mn} = S_{mn} - \Phi_{mn} \tag{4}$$

This case if and only if

$$\Phi_{mn} \ge \frac{\alpha_{mn} - a_{mn} + (\beta_{mn} + b_{mn}) S_{mn}}{b_{mn}} \tag{5}$$



  - Case 2: Supply equals demand. $T_{mn} = \Theta_{mn}$. From (1) and (3) we deduce

$$F_{mn} = \frac{a_{mn} - \alpha_{mn} - \beta_{mn}\Phi_{mn}}{b_{mn} + \beta_{mn}} \quad (6)$$

The constraint $F_{mn} \geq 0$ implies that

$$\Phi_{mn} \leq \frac{a_{mn} - \alpha_{mn}}{\beta_{mn}}. \quad (7)$$

The constraint $F_{mn} + \Phi_{mn} \leq S_{mn}$ implies that

$$\Phi_{mn} \leq \frac{\alpha_{mn} - a_{mn} + (\beta_{mn} + b_{mn})S_{mn}}{b_{mn}} \quad (8)$$

- Case 3: the intra flow is null, and the travel cost is greater than the inverse demand: $T_{mn} = \alpha_{mn} + \beta_{mn}\Phi_{mn} \geq \Theta_{mn} = a_{mn}$.

  Thus $F_{mn} = 0$ if $\Phi_{mn} \geq \frac{a_{mn} - \alpha_{mn}}{\beta_{mn}}$ (9)

In case 2, there are two upper bounds for the inter flow Φ_{mn}.

$$\frac{a_{mn} - \alpha_{mn}}{\beta_{mn}} \leq \frac{(b_{mn} + \beta_{mn})S_{mn} - a_{mn} + \alpha_{mn}}{b_{mn}}$$

yields:

$$S_{mn} \geq \frac{a_{mn} - \alpha_{mn}}{\beta_{mn}}$$

Thus the **link equilibrium conditions** simplify as.

a. If $S_{mn} \leq \frac{a_{mn} - \alpha_{mn}}{\beta_{mn}}$, we have the following.

- Case 1.

$$\Phi_{mn} \leq \frac{a_{mn} - \alpha_{mn}}{\beta_{mn}}, \text{ then: } F_{mn} = \frac{a_{mn} - \alpha_{mn} - \beta_{mn}\Phi_{mn}}{b_{mn} + \beta_{mn}} \quad (10)$$

- Case 2.

$$S_{mn} \geq \Phi_{mn} \geq \frac{a_{mn} - \alpha_{mn}}{\beta_{mn}}, \text{ then } F_{mn} = 0 \quad (11)$$

b. If $S_{mn} \geq \frac{a_{mn} - \alpha_{mn}}{\beta_{mn}}$, we have the following.

- Case 1

$$\Phi_{mn} \leq \frac{\alpha_{mn} - a_{mn} + (\beta_{mn} + b_{mn})S_{mn}}{b_{mn}}, \text{ then } F_{mn} = \frac{a_{mn} - \alpha_{mn} - \beta_{mn}\Phi_{mn}}{b_{mn} + \beta_{mn}} \quad (12)$$

- Case 2.

$$S_{mn} \geq \Phi_{mn} \geq \frac{\alpha_{mn} - a_{mn} + (\beta_{mn} + b_{mn})S_{mn}}{b_{mn}}, \text{ then } F_{mn} = S_{mn} - \Phi_{mn} \quad (13)$$

The intra flow is given by a piecewise linear continuous expression of the inter flow and the link capacity. We summarize (10)-(13) by

$$F_{mn} \stackrel{def}{=} L(\Phi_{mn}, S_{mn}) \tag{14}$$

2.3. Network costs

Let us denote Ξ_{mn} the cost of link (m,n) at link equilibrium. If the intra flow is given by (10) or (12), it suffices to substitute the value of F_{mn} into either T_{mn} or Θ_{mn}. If the intra flow is given by (11) it suffices to substitute the value of F_{mn} into T_{mn}. If the intra flow is given by (13) it suffices to substitute the value of F_{mn} into Θ_{mn}.

The resulting expression for the costs is given by the following.

a. If $S_{mn} \le \dfrac{a_{mn} - \alpha_{mn}}{\beta_{mn}}$, we have the following.

- Case 1.

$$\Phi_{mn} \le \frac{a_{mn} - \alpha_{mn}}{\beta_{mn}}, \text{ then: } \Xi_{mn} = \frac{a_{mn}\beta_{mn} + \alpha_{mn}b_{mn}}{b_{mn} + \beta_{mn}} + \Phi_{mn}\frac{b_{mn}\beta_{mn}}{b_{mn} + \beta_{mn}} \tag{15}$$

- Case 2.

$$S_{mn} \ge \Phi_{mn} \ge \frac{a_{mn} - \alpha_{mn}}{\beta_{mn}}, \text{ then } \Xi_{mn} = \alpha_{mn} + \beta_{mn}\Phi_{mn} \tag{16}$$

b. If $S_{mn} \ge \dfrac{a_{mn} - \alpha_{mn}}{\beta_{mn}}$, we have the following.

- Case 1

$$\Phi_{mn} \le \frac{\alpha_{mn} - a_{mn} + (\beta_{mn} + b_{mn})S_{mn}}{b_{mn}}, \text{ then } \Xi_{mn} = \frac{a_{mn}\beta_{mn} + \alpha_{mn}b_{mn}}{b_{mn} + \beta_{mn}} + \Phi_{mn}\frac{b_{mn}\beta_{mn}}{b_{mn} + \beta_{mn}} \tag{17}$$

- Case 2.

$$S_{mn} \ge \Phi_{mn} \ge \frac{\alpha_{mn} - a_{mn} + (\beta_{mn} + b_{mn})S_{mn}}{b_{mn}}, \text{ then } \Xi_{mn} = a_{mn} - b_{mn}S_{mn} + b_{mn}\Phi_{mn} \tag{18}$$

We summarize (15)-(18) by $\Xi_{nm} = \Xi_{nm}(\Phi_{nm})$.

2.4. Toll and VAT based costs

Customers

One generic good is considered. The demand function for this good at node (n) is given by:

$$\Pi_n = e_n - f_n C_n$$

with C_n the cost.

The population at node (n) is N_n, and e_n, the maximum possible demand is a fraction of this population. The number of customers who accept a cost in the interval $[C_n, C_n + dC_n]$ is given by $f_n\, dC_n$. It is assumed that the supply at each node adjusts to the demand. The price of the generic good is uniform, only the VAT perceived depends on the region. In the formulation of the model we assume that the price of the generic good at location (m) is identical with the VAT perceived at that location, i.e. W_m.

Distribution of demand with respect to destination

We assume unlimited supply at any destination (m), i.e. supply adjusts to demand. Let C_{nm} be the OD cost $(n) \to (m)$. We assume a logit model for the choice of consumers at (n) who accept a cost in the interval $[C_n, C_n + dC_n]$. The probability of choosing the destination (m) for these consumers is given by:

$$\mathbf{P}(n \to m | C_n) = \frac{e^{-\theta C_{nm}}}{\sum_j e^{-\theta C_{nj}} + e^{-\theta C_n}}$$

This model considers the "non consuming" option as one among all options open to the consumer. This model underestimates the fraction of consumers who do not consume.
The number of consumers at (n) who choose the destination (m) to acquire the good is given by:

$$N_{nm} = \int_0^{e_n/f_n} dC_n \, \mathbf{P}(n \to m | C_n)$$

It follows:

$$N_{nm} \overset{def}{=} \Delta_{nm}(C) = \left[e_n - \frac{f_n}{\theta} \ln\left(\frac{\sum_j e^{-\theta C_{nj}} + 1}{\sum_j e^{-\theta C_{nj}} + e^{-\theta e_n / f_n}} \right) \right] \frac{e^{-\theta C_{nm}}}{\sum_j e^{-\theta C_{nj}}} \tag{19}$$

The number of potential consumers at (n) who do not consume is given by

$$N_{n0} \overset{def}{=} \frac{f_n}{\theta} \ln\left(\frac{\sum_j e^{-\theta C_{nj}} + 1}{\sum_j e^{-\theta C_{nj}} + e^{-\theta e_n / f_n}} \right)$$

2.5 Network equilibrium

- Path flows: ψ_p, $p \in \mathbf{P}$, and arc flows $\Phi_a = \sum_{p/a \in p} \psi_p$, $\quad F_a \overset{def}{=} L_a(\Phi_a, S_a), \quad a \in \mathbf{A}$
- Total flow on arcs: $\Phi_a + F_a = \Phi_a + L_a(\Phi_a, S_a), \quad a \in \mathbf{A}$
- Arc tolls: $t_a, a \in \mathbf{A}$
- Arc costs: $\Xi_a(\Phi_a), \quad a \in \mathbf{A}$
- Path costs: $\Gamma_p = \sum_{a \in p} \Xi_a(\Phi_a), \quad p \in \mathbf{P}$
- OD costs: $C_{nm} = \lambda(\mathrm{Min}_{p \in \mathbf{P_{mn}}} \Gamma_p) + W_m$
- OD demand:

$$D_{mn} = \mu \, \Delta_{mn}(C) \tag{20}$$

Here $\lambda \in]0,1[$ is the fraction of the OD cost supported by each consumer (shipping and transportation costs can be divided among several consumers). $\mu \in]0,1[$ quantifies the impact of demand for the good on travel demand.
The constraints on flows are:

$$K(C)=\left|\begin{array}{l}\psi_p \geq 0 \quad \forall p \in \mathbf{P} \\ D_{nm}=\Delta_{nm}(C)=\sum_{p\in \mathbf{P_{nm}}} \psi_p \quad \forall (m,n) \in \mathbf{W} \\ \Phi_a = \sum_{p/a\in p} \psi_p \leq S_a \quad \forall a \in \mathbf{A}\end{array}\right. \tag{21}$$

The set of constraints is convex and linear, and depends on OD costs *C*. We denote

$$\Sigma_{nm}=\lambda\left(\mathrm{Min}_{p\in \mathbf{P_{mn}}} \Gamma_p\right), \quad C_{nm}=\Sigma_{nm}+W_m \tag{22}$$

The equilibrium conditions are given by the following variational inequality:

$$\left(\lambda\Gamma_p-\Sigma_{nm}\right).\psi_p=0 \quad \forall p \in \mathbf{P_{nm}}, \quad \forall (m,n)\in \mathbf{W}, \quad \forall \psi_p \in K(C) \tag{23}$$

This variational expresses the optimality condition of an optimality problem. The preceding model expands on the kind of models introduced in Cantarella (1997) and retains some features introduced in Khoshyaran and Lebacque (2004). Solution algorithms can be based on Nagurney and Zhang (1997), and also take advantage of the piecewise linearity of the model (Murty

3. TWO REGIONS MODEL COMPETING USING TOLLS AND VAT

3.1. Notations, basics.

In the case of two competing regions, we propose a simplified model based on Khoshyaran 2002-2004, which dealt with the problem of regional tax competition (Mintz and Tulkens 1986, Lucas 2004, Bayadir-Upmann and Ziad 2005). Khoshyaran 2002-2004 expanded on the model of Kanbur and Keen 1993 by introducing the heterogeneity of population and travel times. Further the stability of the regional tax competition equilibrium was analyzed in Khoshyaran and Lebacque 2006.

The simplified model proposed here makes semi-analytical computations possible; it is adapted so it is compatible with the general model developed in the previous section.

The elements of the model are the following:

- There are two regions *i* = 1 and 2
- Region (i) applies a VAT W_i on consumption and a toll t_i on transportation
- The population density is $\rho_i(x_i)dx_i$, with x_i the distance to the closest centre over the border
- The density of consumers at distance x_i to the closest centre over border with travel cost τ is $\varphi_i(x_i,\tau)d\tau$
- The density of the population with respect to the travel cost τ_i is given by:

$$\mathrm{P}_i(\tau_i)d\tau_i \overset{def}{=} d\tau_i \int_{(i)} \rho(x_i)\varphi_i(x_i,\tau_i)dx_i \tag{24}$$

- The total population of region (i) is: $N_i=\int_0^{\infty} \rho_i(x_i)dx_i=\int_0^{\infty} P_i(\tau_i)d\tau_i$
- Cost of buying in region (j): $\lambda(\tau_i+t_i+t_j)+W_j+\eta_i$, with
    - η_i a random variable expressing the variability of consumers, the variability of the consumer perception of items such as travel time, the variability of travel costs,
    - λ the coefficient of impact on traffic of demand (as in the previous section ??).
- Cost of buying in region (i): $W_i+\zeta_i$, with ζ_i a random variable expressing the variability of consumers and consumer perception.
- The probability for a consumer in region (i) to buy in region (j) is given by:

$$\begin{aligned}\mathbf{P}[i \to j] &= \mathbf{P}[\eta_i - \zeta_i \le W_i - W_j - \lambda(\tau_i + t_i + t_j)] \\ &= G_i(\Delta W - \lambda t)\end{aligned}$$

- with notations:
  - $\Delta W \stackrel{def}{=} W_i - W_j$, $t = t_i + t_j$
  - $G_i(\sigma) \stackrel{def}{=} \dfrac{1}{1+\exp(-\theta_i\sigma)} \stackrel{def}{=} \int_{-\infty}^{\sigma} g_i(s)ds$ and $g_i(s) = \dfrac{\theta_i \exp(-\theta_i s)}{(1+\exp(-\theta_i s))^2}$ (a simple logistic model)

The simplified two-region model is this section is close to the model of section 2. The only differences are: the population in the simplified model is not concentrated on point-wise nodes but spread, the assignment is simplified (travel costs are independent of flows), the distribution is simplified (the demand function is simplified with $\widetilde{C}_n = 0$).

3.2. Consumers.

Now we can calculate the number of consumers in region (i) buying in region (j). Let $N_{i\to j}$ be the number of consumers of region (*i*) buying in (*j*).

$$\begin{aligned}N_{i\to j} &= \mu\int_0^{\infty} d\tau_i P(\tau_i) G(\Delta W - \lambda t - \lambda\tau_i) \\ &= \mu\int_0^{\Delta W} d\sigma\, Q_i(\sigma - \lambda t)\end{aligned} \tag{25}$$

with

$$Q_i(\varsigma) \stackrel{def}{=} \int_0^{\infty} d\tau_i\, P(\tau_i) g_i(\varsigma - \lambda\tau_i) \tag{26}$$

Q_i represents the population density of region (i) corrected by the effect of variability of consumers. If we denote by $N_{k\to\ell}$ the number of consumers in region (k) buying in region (ℓ), we deduce from the above calculation the following results:

$$\begin{aligned}N_{i\to i} &= \mu\int_{\Delta W}^{\infty} d\sigma Q_i(\sigma - \lambda t) \\ N_{i\to j} &= \mu\int_0^{\Delta W} d\sigma Q_i(\sigma - \lambda t) \\ N_{j\to j} &= \mu\int_{-\Delta W}^{\infty} d\sigma Q_j(\sigma - \lambda t) \\ N_{i\to j} &= \mu\int_0^{-\Delta W} d\sigma Q_j(\sigma - \lambda t)\end{aligned} \tag{27}$$

Also note that

$$N_{i\to j} = \mu\int_0^{\Delta W} d\sigma Q_i(\sigma - \lambda t) = \mu\int_0^{\Delta W - \lambda t} d\sigma Q_i(\sigma)$$

3.3 Revenue, reaction curves

The revenue R_i of region (i) is expressed by

$$R_i = \lambda t_i (N_{i\to j} + N_{j\to i}) + W_i (N_{i\to i} + N_{j\to i}) \tag{28}$$

Let us first calculate the reaction curves for the VATs, by expressing that $\dfrac{\partial R_i}{\partial W_i} = 0$, yielding:

$$W_i = \lambda t_i \frac{Q_i(\Delta W - \lambda t) - Q_j(-\Delta W - \lambda t)}{Q_i(\Delta W - \lambda t) + Q_j(-\Delta W - \lambda t)} + \mu^{-1}\frac{N_{i\to i} + N_{j\to i}}{Q_i(\Delta W - \lambda t) + Q_j(-\Delta W - \lambda t)}$$

By definition of $\Delta W = W_i - W_j$, the reaction curve for region (i) with respect to *W* results:

$$S_i^W(\Delta W, t) = \begin{cases} W_i = \lambda t_i \dfrac{q_i - q_j}{q_i + q_j} + \mu^{-1} \dfrac{N_{i \to i} + N_{j \to i}}{q_i + q_j} \\ W_j = -\Delta W + \lambda t_i \dfrac{q_i - q_j}{q_i + q_j} + \mu^{-1} \dfrac{N_{i \to i} + N_{j \to i}}{q_i + q_j} \end{cases} \tag{29}$$

with $\quad q_i \overset{def}{=} Q_i(\Delta W - \lambda t), q_j = Q_j(-\Delta W - \lambda t)$

This curve is parameterized by ΔW , for given values of the tolls t_i, t_j .

A similar calculation yields the reaction curve for region (j):

$$S_j^W(\Delta W, t) = \begin{cases} W_i = \Delta W + \lambda t_j \dfrac{-q_i + q_j}{q_i + q_j} + \mu^{-1} \dfrac{N_{j \to j} + N_{i \to j}}{q_i + q_j} \\ W_j = \lambda t_j \dfrac{-q_i + q_j}{q_i + q_j} + \mu^{-1} \dfrac{N_{i \to i} + N_{j \to i}}{q_i + q_j} \end{cases} \tag{30}$$

with $\quad q_i \overset{def}{=} Q_i(\Delta W - \lambda t), q_j = Q_j(-\Delta W - \lambda t)$

Second we calculate the reaction functions for the tolls, assuming the VATs are given, by expressing, in the case of the region (i), that $\dfrac{\partial R_i}{\partial t_i} = 0$, yielding:

$$t_i = W_i \frac{Q_i(\Delta W - \lambda t) - Q_j(-\Delta W - \lambda t)}{Q_i(\Delta W - \lambda t) + Q_j(-\Delta W - \lambda t)} + \lambda^{-1}\mu^{-1} \frac{N_{i \to j} + N_{j \to i}}{Q_i(\Delta W - \lambda t) + Q_j(-\Delta W - \lambda t)}$$

Since by definition of *t*, $t = t_i + t_j$, the reaction curve for region (i) with respect to *t* results:

$$T_i^W(\Delta W, t) = \begin{cases} t_i = W_i \dfrac{q_i - q_j}{q_i + q_j} + \lambda^{-1}\mu^{-1} \dfrac{N_{i \to j} + N_{j \to i}}{q_i + q_j} \\ t_j = t - W_i \dfrac{q_i - q_j}{q_i + q_j} - \lambda^{-1}\mu^{-1} \dfrac{N_{i \to j} + N_{j \to i}}{q_i + q_j} \end{cases} \tag{31}$$

with $\quad q_i \overset{def}{=} Q_i(\Delta W - \lambda t), q_j = Q_j(-\Delta W - \lambda t)$

A similar calculation yields the reaction curve for region (j):

$$T_j^W(\Delta W, t) = \begin{cases} t_i = t - W_j \dfrac{-q_i + q_j}{q_i + q_j} - \lambda^{-1}\mu^{-1} \dfrac{N_{i \to j} + N_{j \to i}}{q_i + q_j} \\ t_j = W_j \dfrac{-q_i + q_j}{q_i + q_j} + \lambda^{-1}\mu^{-1} \dfrac{N_{i \to j} + N_{j \to i}}{q_i + q_j} \end{cases} \tag{32}$$

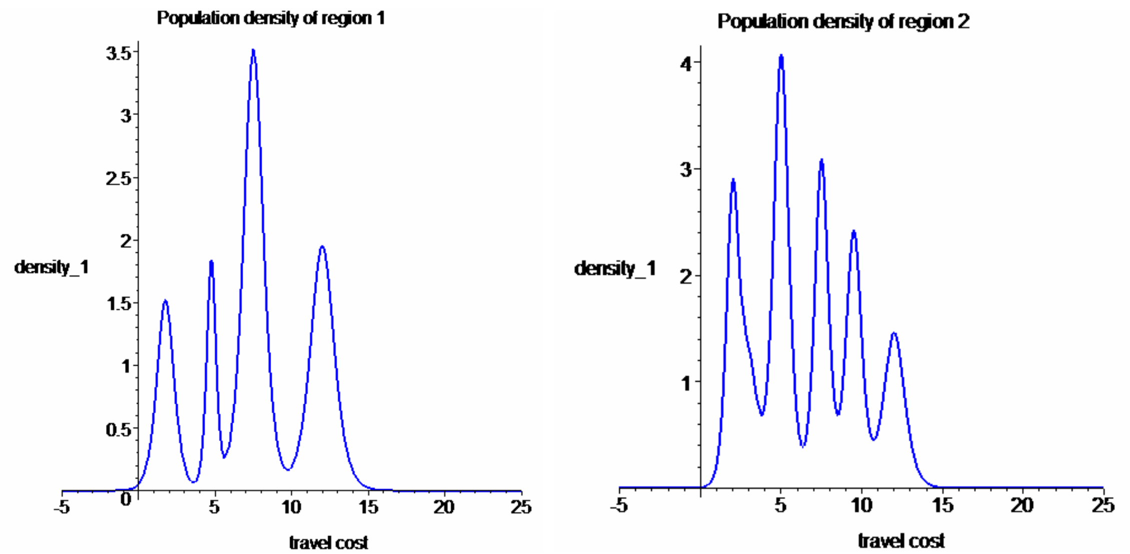
with $\quad q_i \overset{def}{=} Q_i(\Delta W - \lambda t), q_j = Q_j(-\Delta W - \lambda t)$

Thus we can study, given the tolls, the equilibrium of regional competition for the VATs, or given the VATs, study the equilibrium of regional competition for the tolls.

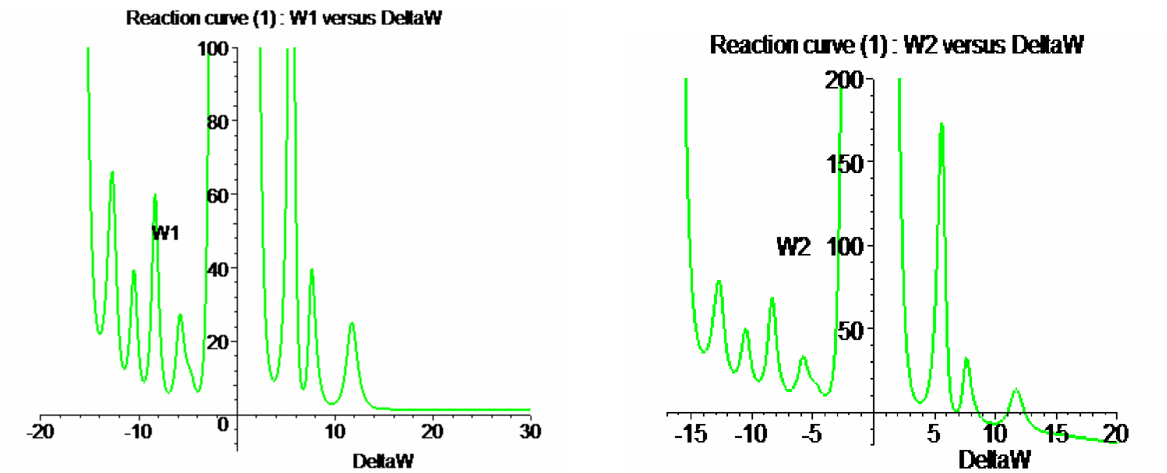
4. NETWORK DYNAMICS AND INVARIANCE PRINCIPLE

Let us first consider the case of two regions, as analyzed in section 3, and let us consider a numerical example. The densities Q_1 and Q_2 are described by the following figures.

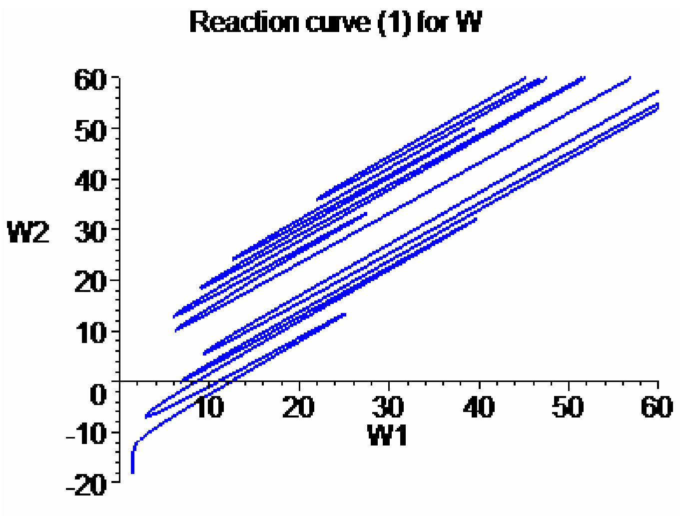
(The density is the distribution of population with respect to travel costs as modified by the variability of consumers).



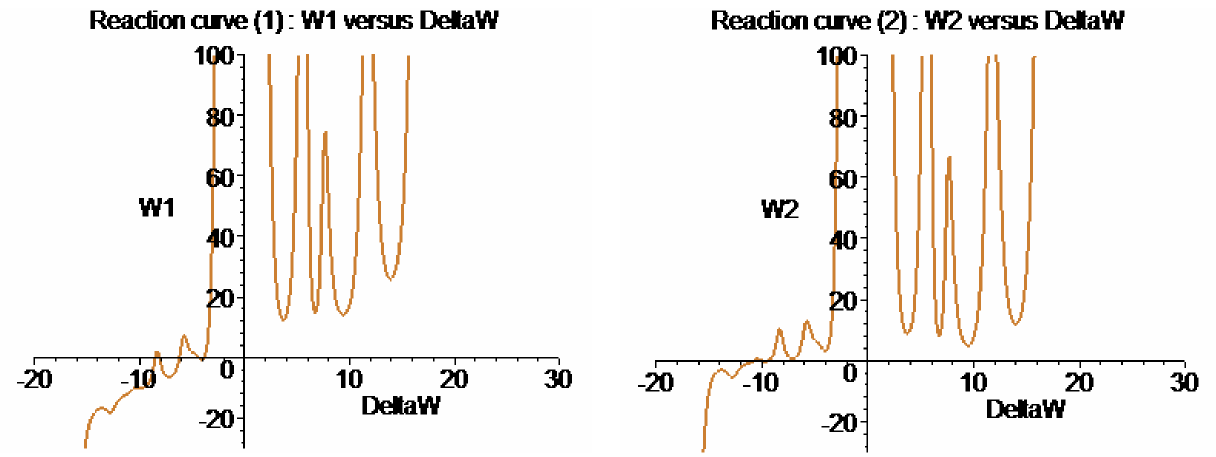
The shapes of the components of the reaction function (29) are depicted by the following figure:



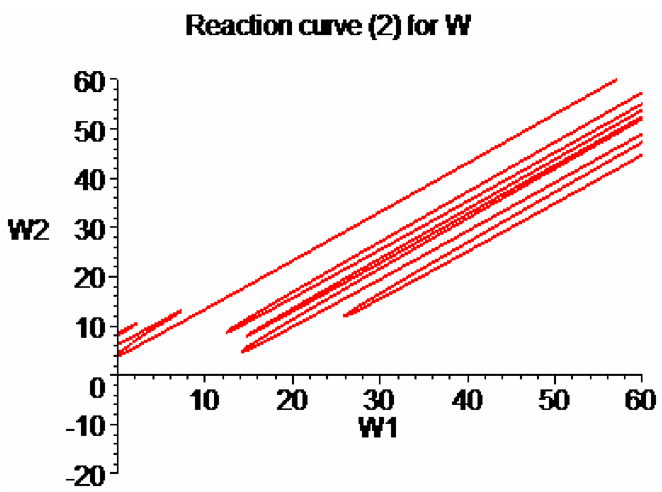
The reaction curve of region (1) is described by the figure below. This curve does not define a function.



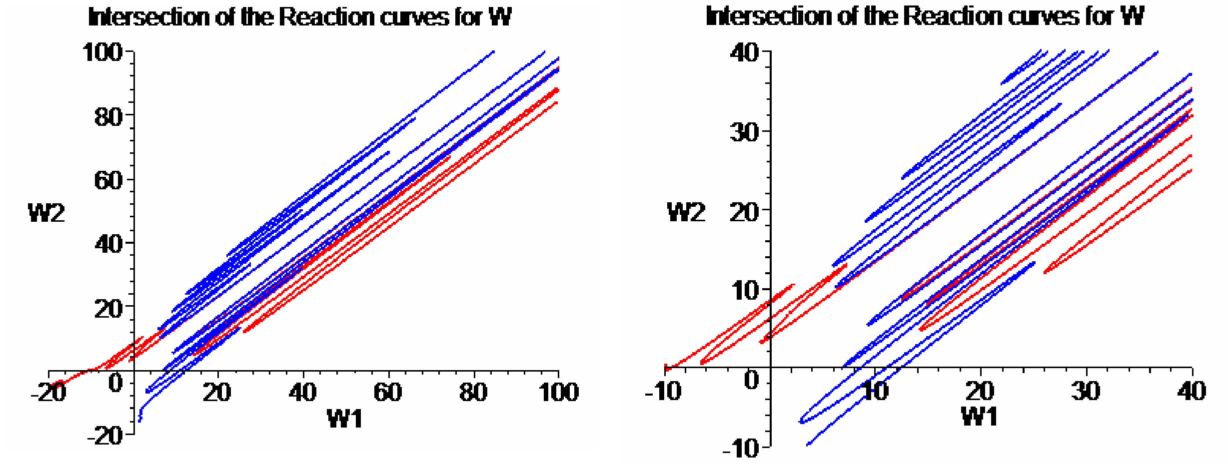
The meaning of the reaction function is the following. Given W_1, the reaction curve yields all possible values W_2 for which, i.e. local extrema (half of which actually correspond to a local maximum) of the revenue of region (1).



The above figure depicts the components of the reaction function (30). The corresponding reaction curve for VATs (for region (2)) is depicted below.



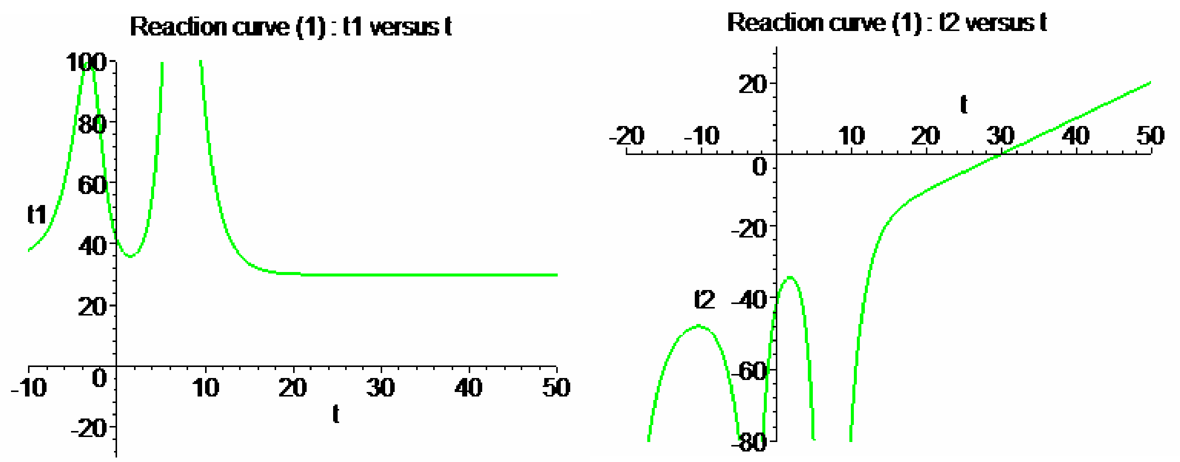
The figure below illustrates the intersection of reaction curves for VATs for the 2 regions figure (blue for region (1), red for region (2)).



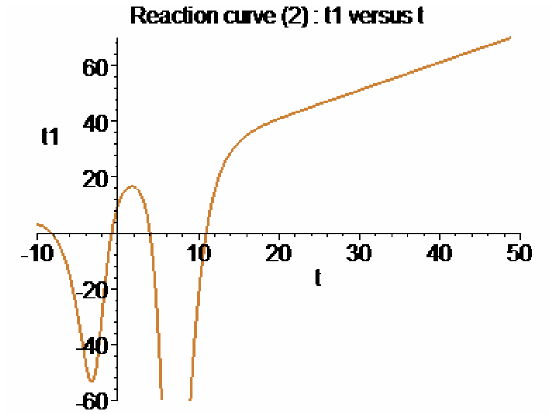
The many loops in the reaction functions for the VATs express the impact of the transportation system costs and the heterogeneity of population density. When considering the procedure of successive optimal choices, i.e. each region optimizing in turn its revenue, there is obviously no reason to reach a Nash equilibrium. Each region has a choice between many alternative values of VAT.

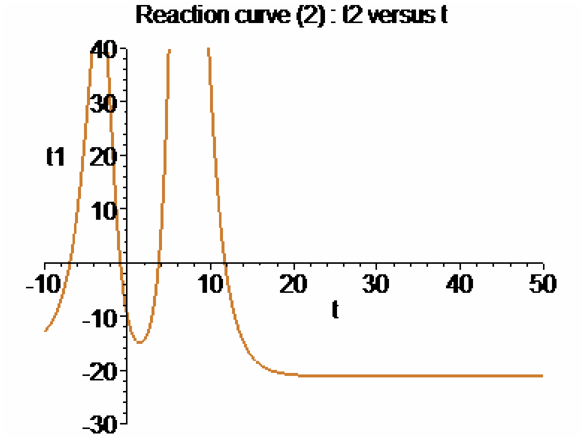
We can carry out similar calculations for tolls.

The figure below illustrates coordinate functions of the reaction curves for tolls (31).



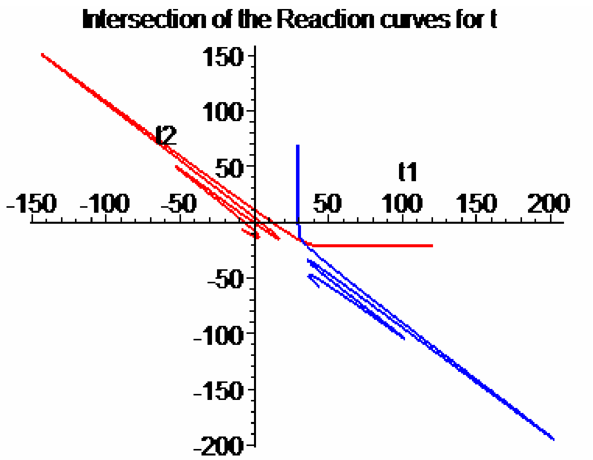
and the coordinate functions of the reaction curves for tolls (32) are depicted by the following figure:





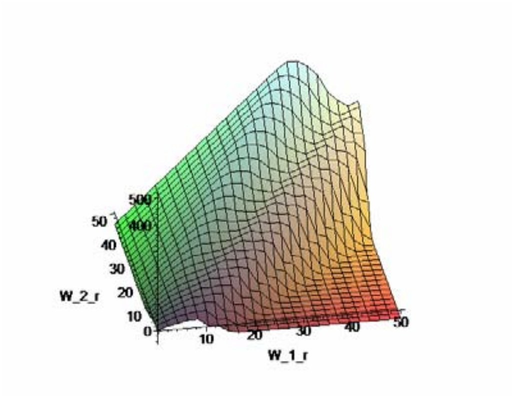
The difference between these and the toll reaction functions is du to the parameterization; the toll reaction functions are parameterized by the sum of tolls, whereas the VAT reaction curves are parameterized by the difference between VATs

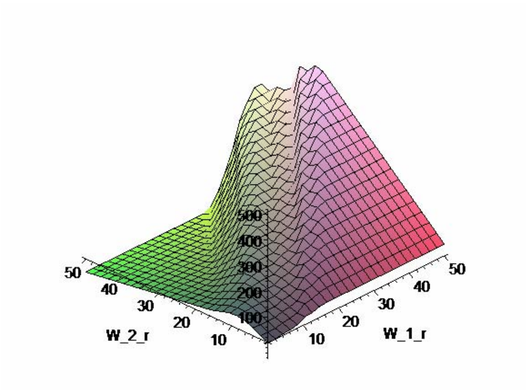
The intersection of toll reaction curves is illustrated by the below figure (blue for region (1), red for region (2)). Negative tolls are interpreted as subventions.



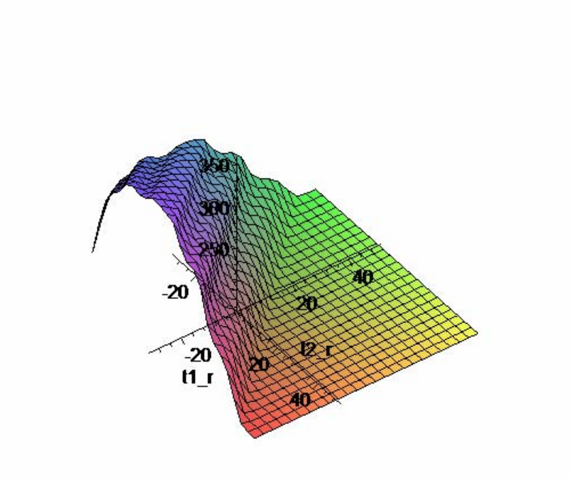
Although in this case the Nash equilibrium is unique, this is dependent on the VAT values choosen for the example. Obviously the unicity of the Nash equilibrium is not the general case: the toll reaction curves do not define reaction functions. The process of iterative choice of their optimal toll by the two regions each in turn need not converge. Indeed for each region, there exist also several alternative values of toll possible for each value of the toll of the other region (or no value at all).

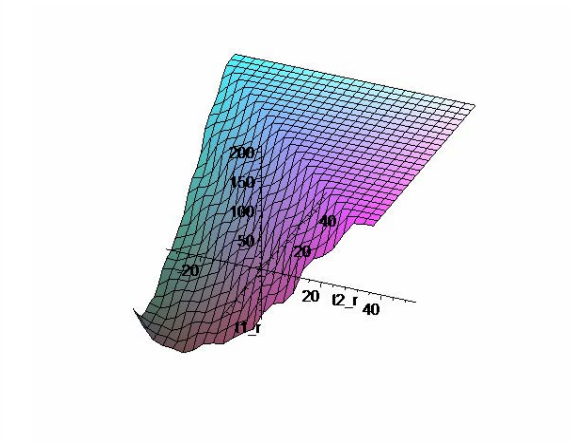
The figure below depicts the regional revenues of region (1) (left) and region (2) (right) in the VAT equilibrium (tolls are fixed). The revenues are unbounded because the simplified two-region model does not include node demand functions.





The figure below depicts the regional revenues of region (1) (left) and region (2) (right) in the toll equilibrium (VATs are fixed).





In view of the reaction curves and their intersections, a competition process whereby regions try to optimize in turn their VAT / toll will not achieve any equilibrium but will always result in one region losing out (which has been postulated as the invariance principle).

Let us now consider the general model. The differences between the general and the two-region model have been described, they are of limited impact. In the general model, following subsection 2.5, the total revenue of region (r) is given by:

$$\mathbf{R_r} = \sum_{m\in(r)} W_m \, \Delta_{mn}(C) + \sum_{a\in(r)} t_a \left(\Phi_a + L(\Phi_a, S_a)\right)$$

Let us define:

$$x^r \overset{def}{=} \left((W_m)_{m\in(r)}, (t_a)_{a\in(r)}\right), \quad \bar{x}^r \overset{def}{=} \left((W_m)_{m\notin(r)}, (t_a)_{a\notin(r)}\right)$$

Partial reaction functions can be defined for region (r):

$$H_r(\bar{x}^r) = \operatorname{Arg}_{x^r} \operatorname{Max} \mathbf{R_r}(x^r, \bar{x}^r)$$

A Nash equilibrium is reached if $x = H(x)$. The reaction functions are called partial because they are only piecewise continuous, as in the case of two regions. Indeed, the number of selections of possible optimal paths solving (23) is finite, even though usually very large. For each choice of tolls and VATs, a selection of paths is optimal and the revenue admits a closed expression yielding an expression of the reaction function dependent on the selection of paths. Thus as in the case of two regions, the iterative process of regions optimizing their revenues in turn normally does not converge and the process results in some regions losing out, again in conformity with the invariance principle postulate.

5. CONCLUSION

In this paper we have analyzed two models of regional tax/toll competition with transportation costs. The general model includes local demand for the generic good, with distribution and assignment mechanisms, whereas the simpler two-region model simplifies the distribution and assignment processes while retaining the essentials of the complex behavior of the system.
In this complex system the reaction functions are necessarily multivalued and only piecewise continuous. Since the affluence of regions is being measured by their revenue, the invariance principle results.

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