Measuring the potential value of demand response using historical market data

Graziano Abrate (University of Piemonte Orientale and FEEM) and Daniele Benintendi (FEEM)

***Abstract* - One of the main barriers to the deployment of Demand Response (DR) and Distributed Generation (DG) is the ex-ante assessment of their benefits. The aim of this study is to develop a general framework to evaluate this potential using historical market data. We observe that profitability of DR-based strategies can be linked to several market characteristics. Among them are the price levels and the frequency of high prices, the price differentials between consecutive hours and their frequency, the duration of peaks, the variables affecting prices (e.g. load, seasonality, weather, temperature, etc.). Although a single indicator to take into account all the above dimensions is not readily available, we propose to compare different markets in terms of "interesting events" referred to one or more of the above listed dimensions. The empirical analysis is based on five European markets, which present interesting differences in day-ahead price patterns and are analyzed to give some insights on the different technologies and business models which may better exploit the potential of DR. This work shows a methodology which has to be refined in order to draw conclusions for specific sets of customers and technology combinations in order to understand the right price differentials and the possible DR time intervals.**

The full version of the paper, on which we are working at the moment, will include both a more detailed description of the possibilities offered by aggregation and an extended empirical analysis. It will be added a section studying the relationship of day ahead and balance market prices in order to understand the potential of Demand Response in trading activities. As a matter of fact can be often convenient for a retailer to sell in the balance market the forward positions acquired in the day ahead market if he can reduce the consumption of his customers.

***Index Terms* – electricity markets; demand response; volatility; peak prices**

I. INTRODUCTION

The paper is organised as follows: chapter II gives a general description of Demand Response (DR). Chapter III introduces an application of DR, namely the concept of aggregation of DR and Distributed Generation; Chapter IV gives an overview of the measures used to analyze volatility in Electricity Markets and develops measures to link Day Ahead price patterns and the profitability of DR. Chapter V is an empirical analysis of 5 European Markets and Chapter VI presents the conclusions and the possible future research.

II. DEMAND RESPONSE: DEFINITIONS AND ECONOMIC RATIONALE

Following the definition given by the US Department of Energy [1], demand response is any "change in electric usage by the end-use customers from their normal consumption patterns in response to change in the price of electricity over time, or to incentive payments designed to induce lower electricity use at times of high wholesale market prices or when system reliability is jeopardized."

The definition above argues that there are two sources of demand response: retail prices or programs providing incentives to reduce load in critical times. This classification can be meaningfully related to the usual alternative solutions to the problem of peak load indicated by the economic literature: pricing or rationing.

The first source of demand response directly comes from a link between wholesale cost of electricity and its retail price ("price-based" demand response). This link is at its maximum when the end-user faces real time pricing (RTP), giving a perfect signal of the real time cost of consuming. A simpler type of pricing structure is the time-of-use (TOU): in this case retail prices vary in a pre-set way within certain block of time, but clearly they cannot take into account the state contingent variability of demand and supply. A third possible retail pricing option is the critical peak pricing (CPP), which implies a sort of mix between the logic of TOU and RTP: the highest rate is pre-set but can be called on short notice by the system operator when he recognizes a critical peak hour. Time-varying retail prices provide a direct incentive to a rational use of electricity for the consumer, who can decide to modify his consumption patterns on the basis of his own economic valuations. The extent of this change will depend on the consumers' price elasticity.

The second source of demand response comes from programs specifically designed to induce load reductions in particular critical times ("incentive-based" demand response). This is somehow closer to the idea of rationing, since the amount of load reduction is at least partially beyond the control of the customer (who however voluntary enrolls into such programs). These programs are generally intended to increase system operators' confidence that demand reductions will materialize when needed. Actually, pure rationing would involve the electric load being completely under the centralized control of the utility (or system operator) during certain critical times. For example, the customer can be provided an economic compensation for enrolling in such programs, giving the right to the utility to curtail his load on short notice to address reliability contingencies [2] .

Four main aspects characterize an incentive based program and make this instrument more flexible than the traditional pricing systems: (i) the control over the real time demand reduction, which usually stays to the final consumer; (ii) the frequency of events, e.g. the program may specify the maximum number of events in a certain period; (iii) the timing of notification; (iv) the economic incentive, which is usually a combination of bill savings for enrolling to the program with the commitment to reduce load and penalties for not responding when the event is called.

A second type of DR classification concerns the way consumer can respond. Three broad categories of DR strategies are available for the customer: curtailment, time-shift and shift to onsite generation. We refer to curtailment when the consumer reduces (voluntary or by load control) his total amount of consumption in response to an increase in prices (or another economic incentive). Examples of this strategies are shut down of production, reduction of end use electricity and service degradation (which implies a partial reduction of consumption). A second possibility is given by time differentiated prices, which provide an incentive to a reallocation of use of electricity over time. DR based on time shift implies a modification in the consumer load profile even if the total amount of consumption may stay at the same level. One example is given by thermal energy storage as the possibility to increase air conditioning before a price peak, so to be able to keep a sufficient temperature in the following period even with a reduced consumption. Finally, a third possibility is resorting to onsite flexible generation like CHP when the consumer can increase generation according to the signal he receives[1]. Each of these strategies can also be view as a "financial" option with a certain value for the customer [2].

We have to underline that assessing the value of DR is a multi-dimensional task, involving: (i) what type of signal is sent to the customers (the various type of price-based or incentive-based DR programs) (ii) what types of DR strategies the customer can put in place; (iii) what are the technologies available and their costs, and what will be the technologies available in the near future; (iv) what are the markets involved (the day-ahead, the real-time or the ancillary markets); (v) what is (and eventually what should be) the regulatory context.

In general, we can say that a DR strategy will have a positive value if, given the available technologies, the market is willing to pay for the demand response more than the sum of the costs perceived by the customer and those paid by the system in order to activate demand response. A cost-benefits analysis from a societal point of view is necessary, to carefully evaluate if the returns from implementing a DR program can justify the various investments that may be needed (e.g. advanced metering, energy-efficient appliances, communication equipment, automatic demand management systems, onsite generation, storage, payments to interruptible customers, etc.).

The benefits of DR have been identified in the economic literature in connection with the theoretical efficiency of time differentiated price. The classical peak load theory has emphasized market wide financial benefits, related to the efficient use and definition of the system capacity [4,5], as well as the potential benefits (by means of both pricing or rationing mechanisms) on system reliability [6]. Further benefits can be obtained in terms of congestion management and are related to the theory of transmission prices [7]. A more recent issue is the potential market performance benefits, since DR can favor a higher connection between wholesale and retail market and may reduce market power in the wholesale market [8-10].

On the customer side it is also difficult to assess the costs incurred, one way to indirectly investigate them is to assess its price elasticity by observing the actual DR induced by a certain economic incentive. What is important though to analyze is the type of consumption patterns and the technology, which determine his flexibility.

In the following section we define a general commercial application of Demand Response, which is helpful to support the logic behind our analysis.

III. Aggregation of Demand Response

Our analysis of price behavior can be widely applied to DR settings, still it is convenient to introduce the concept of Aggregator to show where it can be used in practice[2]. We refer to a situation where an entity called Aggregator is managing the energy consumption of a set of clients. The Aggregator is then selling electricity to final customers[3], but also uses the resources offered by the same customers. These can sell to the Aggregator their capability to modify their consumption and also the electricity generated at their site (Distributed Generation). The aggregator is then using the reduction in demand or the additional electricity to cover his obligations towards his clients or as a part of his trading activities. The customer themselves would not be able to participate as their size would be too small or they would lack the knowledge to participate directly to trading. The relationship between the Aggregator and the customers can include all the possibilities explained in the previous section, from an exchange of information signals to the direct control by the Aggregator of the consumption and generation at the customers site[4].

In the analysis of Demand Response clients' characteristics are extremely important in order to determine the possible profitability. As it was mentioned in chapter II there are several dimensions to take into account like the time span of availability, the number of possible calls (in a day or in a longer period), the amount of reduction in demand and the time of response after the request. These are influenced by the type of flexible load of the customers and by the financial compensation offered by the Aggregator to the customer to provide the service. These factors are important at the individual level, but they also have to be evaluated at the aggregate level as the combination of different client profiles can offer increased flexibility. Then for the Aggregator it is crucial to optimize his portfolio of clients; a simple example is given by the possibility of overriding by the customer the request of demand reduction. If the portfolio is well structured then the Aggregator could have an higher probability of having the necessary available load.

Given that customers have different characteristics it is interesting to define them with respect to the volatility and the general pattern of prices; consequently in the context of aggregation the analysis has to be carried out also in terms of portfolios. Our analysis then can be seen from the point of view of an Aggregator, which faces certain market conditions and has to choose what kind of customers he needs in order to optimize his portfolio.

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[1] CHP electricity production normally follows the demand of heat; during these special circumstances heat is disposed.

[2] This concept is possibly the most advanced way to exploit DR and Distributed Generation potential

[3] The Aggregator being a supplier is the most effective scheme as there are strong synergies between selling electricity and aggregating DR and DG.

[4] The direct control of load is carried out giving the clients a specific notification period.

Our work then defines a series of measures, which can be used to evaluate the price patterns. It also tests these measures empirically, comparing the results of several European Markets.

IV. MEASURING DR POTENTIAL VALUE FROM DAY-AHEAD MARKET DATA

Several metrics have been proposed in the literature to describe historical electricity market data and can be helpful also for the assessment of the potential value of business and regulatory models based on DR. A first traditional way to describe historical data is given by the load duration curve (LDC) and the price duration curve (PDC) [11]. The first one indicates the percentage of cases during a time horizon T (usually one year) that the total load registered in a certain time period (l_h) is at or above any given load threshold y_l. It is fundamental for long-run capacity planning and also provides a first indication about the need of long-run DR and on the possible desirability of the application of standard peak load pricing mechanisms.

$$LDC(y_l) = \frac{1}{T}\sum_{h=1}^{T} 1, \forall l_h > y_l \tag{1}$$

$$PDC(y_p) = \frac{1}{T}\sum_{h=1}^{T} 1, \forall p_h > y_p \tag{2}$$

Similarly, PDC can be interpreted as the probability of observing price levels (pt) at or above a certain threshold y_l.[5] Again in a long-run perspective, it can be useful to evaluate investments on technologies allowing DR strategies based on curtailment or shift to own generation. Since it gives indication about expected price and their frequency, PDC can also provide a basis for risk management. However, one needs also to resort to variability and volatility measures.

The more common measure of price variability is given by the standard deviation of prices (often used in relation to average price to obtain a relative variability index). Since the standard deviation is obtained comparing each hourly price realization with the average price, it does not provide information about price sequences and about variability between subsequent hours. Instead, a way to represent this information is given by the price volatility, which is basically a measure of the variability of hourly price differences. Volatility indexes provide essential information to understand the need of short-term DR and the evaluation of DR strategies based on time-shift, when what really matters is how much price varies between two consecutive hours (or however within a limited number of hours). The interpretation of volatility indexes in terms of potential value of DR has been highlighted in [13-15].

More precisely, historical volatility ($\sigma_{h,T}$) is defined as the standard deviation of logarithmic returns ($r_{t,h}$) over a time window T.

$$\sigma_{h,T} = \sqrt{\frac{1}{(N-h)}\sum_{t=1}^{N}(r_{t,h} - \bar{r}_{h,T})^2} \tag{3}$$

N is the number of price observations in the time window (for example, for one day in electricity markets with hourly pricing it is equal to 24), while h is the temporal distance between the two price observations that are compared.[6] $r_{t,h}$ are the logarithmic returns over h, defined as

$$r_{t,h} = \ln\left(\frac{p_t}{p_{t-h}}\right) = \ln p_t - \ln p_{t-h} \tag{4}$$

Finally, $\bar{r}_{h,T}$ is the arithmetic mean of the logarithmic returns over the time window T.

The historical price volatility measures how much the actual realization of logarithmic results are expected to be close to the arithmetic mean. It provides a measure for the risk of having price spikes: a low volatility would be associated with a certain stability in price dynamics, while high volatility will be obtained in the case of severe price spikes.

An alternative way to describe short-term price patterns is the so-called price velocity, proposed by Li and Flinn [13]. They defined the absolute variation of prices as:

$$\delta_{t,h} = \left|p_t - p_{t-h}\right| \tag{5}$$

where the subscript t and h have the usual interpretation. Then, price velocity can be obtained as the ratio between the average of $\delta_{t,h}$ and the average of prices pt. Various specifications are possible, modifying the time horizon of numerator or denominator. For example, it is possible to define the Daily Velocity with reference to Daily Average (DVDA) and the Daily Velocity with reference to Overall Average (DVOA):

$$DVDA = \frac{\left\{\left(\frac{1}{N-h}\right)\sum_{t=1}^{N}\delta_{t,h}\right\}}{\frac{1}{N}\sum_{t=1}^{N}p_t} \tag{6}$$

$$DVOA = \frac{\left\{\left(\frac{1}{N-h}\right)\sum_{t=1}^{N}\delta_{t,h}\right\}}{\frac{1}{NM}\sum_{t=1}^{NxM}p_t} \tag{7}$$

In the case of DVOA, the denominator represent the overall price average considering the whole period of analysis (for example one year). M stands for the number of days within the period.

The concept of price velocity employs the daily average of price changes to quantify price uncertainty, instead of the standard deviation of price returns used to derive historical volatility. Though the measures are different, they both try to capture the short-term price dynamic in the market.

In the empirical analysis we provide several volatility measures, changing time lags and limiting the number of hours over which to carry out the computation, still these remain general values not reflecting at what time the variation in price could trigger the intervention of DR.

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[5] Here we refer to PDC derived from historical data; however it can also be derived from price forecast simulation models [12].

[6] For example, if h=1, returns are computed comparing the price of two consecutive hours; if we refer to $r_{10,2}$, it means that we are computing the logarithmic difference between the price at 10:00 and the price at 8:00. If N is equal to 24, the number of logarithmic returns over the time window T will be (N-h), and this explain the denominator in the formula of volatility

What could give an idea of the potential of DR is as a matter of fact the number of events, which could generate profits. Very simply in our empirical analysis we attached a cost to the implementation of DR and showed the number times, when the price differential across consecutive hours would be greater than the given value. We computed three indexes: the number of price increases, price decreases and what we call true peaks or when the differential is greater then the value in the previous and subsequent hours. What can be learned is then the number of times it is possible to use the flexible load and at what time of the day these events occur. These are two fundamental characteristics for the assessment of the possibilities to have customers participating to DR programs. Even if we do not compute the magnitude of the price differentials we show the frequency of price differentials, which gives an idea of the magnitudes of the events, which can trigger DR.

V. Empirical Analysis of Demand Response Related Price Patterns in Five European Markets

The aim of Table 1 is to compare a series of measures applied to five countries in this chapter we provide comments on how the results can impact the implementation of Demand Response.

Table 1. Descriptive statistics and volatility measures for five European markets

Statistic	Description	Italy	Belgium	Uk (£)	France	Spain
	Standard descriptive statistics					
$\overline{p}_{YEAR}$	Average price over year 2007	70,99	41,78	27,85	40,88	39,35
σ_{YEAR}	Standard deviation	37,02	54,47	24,23	49,45	13,19
VI_{YEAR}	Variability index (σ_{YEAR}/ VI_{YEAR})	0,52	1,30	0,87	1,21	0,34
Max	Maximum value of price	242,42	2.500	639,54	2.500	130
$90\%q$	90% quantile (10% observation above this price)	119,61	71,00	42,43	69,61	56,50
$95\%q$	95% quantile	149,78	95,51	58,55	92,50	66,11
$\overline{\sigma}_{DAY}$	Average value of the daily standard deviation	31,86	19,92	14,30	18,15	9,42
$\overline{VI}_{DAY}$	Average value of the variability index computed day by day	0,44	0,38	0,44	0,36	0,24
$\rho_{LOAD,PRICE}$	Correlation between load and price	0,81	0,28	-	-	0,76
	Volatility measures					
$\overline{\sigma}_{1/2,DAY}$ (UK)	Average value of the daily volatility, computed considering consecutive half-hours			0,176		
$\overline{\sigma}_{1,DAY}$	Average value of the daily volatility, computed considering consecutive hours (lag 1 hour)	0,240	0,262	0,271	0,248	0,128
$\overline{\sigma}_{2,DAY}$	Average value of the daily volatility, computed considering a lag of 2 hours	0,391	0,398	0,331	0,373	0,208
$\overline{\sigma}_{3,DAY}$	Average value of the daily volatility, computed considering a lag of 3 hours	0,502	0,510	0,369	0,470	0,262
$\overline{\sigma}_{4,DAY}$	Average value of the daily volatility, computed considering a lag of 4 hours	0,578	0,580	0,404	0,530	0,297
$\overline{\sigma}_{1/2,WORK}$ (UK)	Average value of the daily volatility, computed considering consecutive half-hours, 8am-7pm (or 7am-8pm)			0,221 – 0,224		
$\overline{\sigma}_{1,WORK}$	Average value of the daily volatility during work days, computed considering consecutive hours, 8am-7pm (or 7am-8pm)	0,263 – 0,271	0,150 – 0,198	0,348 – 0,346	0,129 – 0,159	0,093 – 0,109
$\overline{\sigma}_{2,WORK}$	Average value of the daily volatility during work days, computed considering a lag of 2 hours, 8am-7pm (or 7am – 8pm)	0,413 – 0,429	0,235 – 0,291	0,417 - 0,414	0,201 – 0,237	0,147 – 0,167
$D\overline{VDA}$	Average value of the Daily Velocity Daily Average	0,171	0,15	0,148	0,139	0,091
$D\overline{VOA}$	Average value of the Daily Velocity Overall Average (average value of price differences between consecutive hours divided by $\overline{p}_{YEAR}$)	0,176	0,19	0,164	0,178	0,091

- On the top part of the table there are a series of measures on the level of prices. High level of prices can induce higher volatility, they are also an indicator for the cases of curtailment and use of CHP. The 90% and 95% quantiles indicate the price above which we find the 10% and 5% of the observations. It has to be remarked that the 5% represents in terms of DER more than one call per day, which would be an extremely high level of participation for a DR program. It could make sense then to analyze also higher quantiles as they would still bring useful information considering a threshold of 100 calls per year a relevant activity. The highest level of prices also gives an indication of market behavior, we can see in the table significant differences across countries. Extreme peaks are clearly very interesting as they could represent huge profits. Belgium and France have the highest extreme value, while these events are very rare. Instead in Italy we register a relatively low maximum price but data on upper quantiles indicate that prices remain at high levels more frequently.

- In terms of standard deviation we can notice the difference between yearly and the daily value. The first measure is affected by variation of prices among seasons or also by weekly variability of prices. The second one depends only on daily price patterns and is more useful to evaluate short term DR. Prices in Italy and Uk seem the least variable from the first indicator, but day-by-day the variability indexes are the highest.
- Seasonality is a factor we did not compute, but it could be of a great importance as some technologies can work only in certain parts of the year. As we do not refer to any specific case, it would have been difficult to provide a selection of relevant days. This should be clearly carried out taking into account temperatures.
- A high correlation between price and load indicates that price spikes are typical during peak hours. This is the case in Italy and as well in Spain, while in Belgium the correlation is quite low. As noted by Li and Flinn [14], this means that price patterns in Italy and Spain are more predictable (because total load is driven by weather extremes and weather is predictable with a reasonable degree of accuracy one or more days in advance). They notice that this might favor DR as it would b easier to manage customers if they could have a better idea of the periods when they should be available.
- Volatility measures have been computed considering consecutive hours and also with lags up to 4 hours. In all markets volatility increases with the lag, indicating that prices tend to follow a trend rather than having repeated peaks. Nevertheless, this dynamic is quite different across markets. In Uk we observe the highest volatility with the lag of 1 hour, while increasing the lag Italy and Belgium become the most volatile. This can indicate that technologies allowing a higher duration for DR can be more profitable, in particular in Italy and Belgium.
- An example of interesting filtration is considering only working hours in the attempt to capture the potential of the Commercial and Industrial sectors. In order to show some sensitivity analysis, two large set of hours have been tested, 8am to 7pm and 7am to 8pm. The results show two opposite types of markets: in Italy and Uk the volatility is notably higher during this smaller sets of hours, while Belgium, France and Spain show lower volatility indexes. The sensitivity analysis proposed shows that volatility is usually slightly higher considering also 7am and 8pm in the sample (apart from Uk). Especially for the larger set of hours the demand has to be decomposed according to the type of customers and their ability to offer DR. The reason is that residential and commercial industrial consumption could have strong overlaps at these particular periods. This means that similar price patterns could have radically different properties.

Shifting to the graphs in Figure 1, we compare the markets counting the number of times that price changes exceeded a threshold of 30 Euros counting only the working hours. These are what we can call interesting events, which should express the possibility to implement DR actions.

The value of 30 is arbitrary, but indicates a reasonable difference, clearly it is possible to make sensitivity analysis on this parameter, but Figures 2a and 2b present a quick indicator of other price differentials. We can see that Italy and Uk register the highest number of events, in the case of Uk these are more frequent in evening. In Italy, there is a quite typical price change associated to the end of the morning, which is registered in more than 50% of working days. Belgium and France have lower case of high price differentials, even if we need to remind that in some cases these price differentials are very extreme.

Figure 1. Frequency of “interesting” price differentials

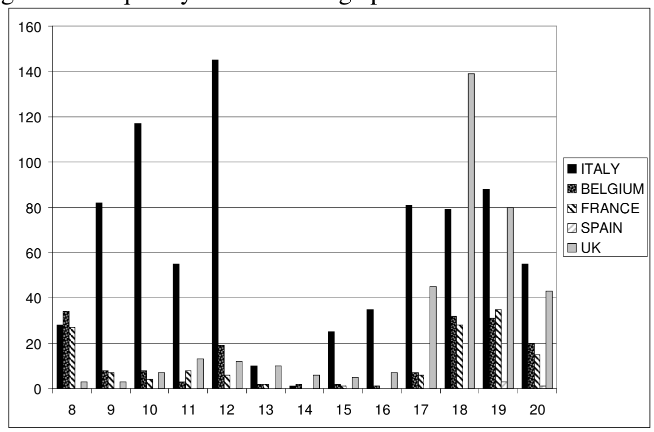


Figure 2a. Summary statistics of price differentials (Italy)

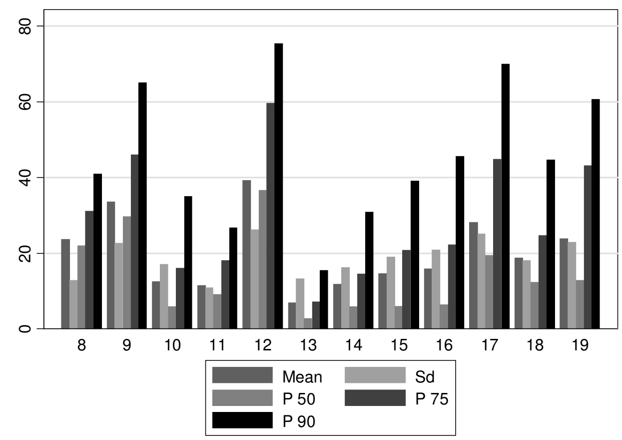


Figure 2b. Summary statistics of price differentials (Belgium)

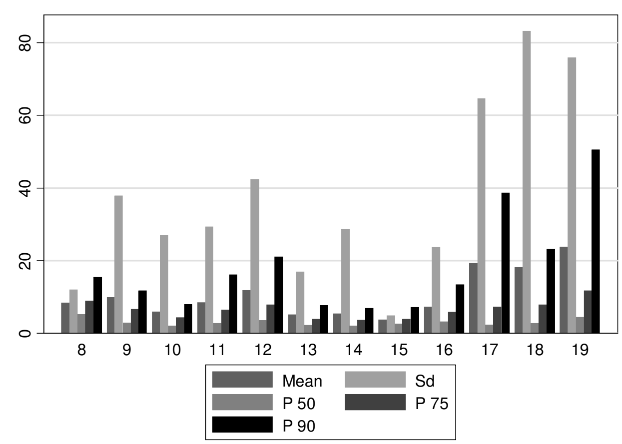


Figure 2a and 2b presents descriptive statistics of price differentials computed for each hour during working days only, in particular mean, standard deviation (Sd), median (P 50), the 3rd quartile (P 75) and the 9th decile (P 90). In Italy the average price differential registered during each hour is higher than in Belgium, which instead shows a sensibly higher standard deviation. Data on quantiles indicate the probability of having price changes above a certain threshold during a certain hour. For example, at 12 in Italy, the median value is close to 40 Euros, and on one day in 10 we can expect having price changes higher than 70 Euros.

Implementing DR in Italy in certain hours (9, 12, 17, 19) is expected to give relevant returns very frequently, while in Belgium interesting events are far less frequent and, given the high value of standard deviation, less predictable.

These two figure also give a general assessment of the magnitude of possible gains induced by DR activities.

Figure 3. Characterization of the peak direction in the Italian market

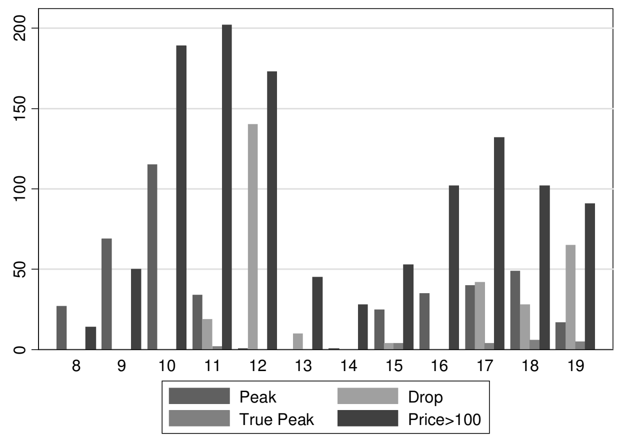
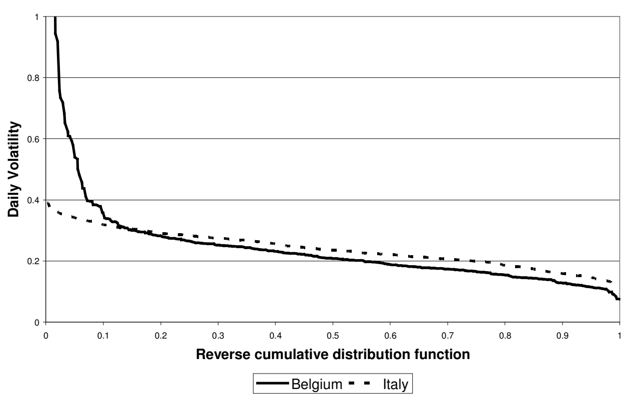


Figure 3 integrates the information from Figure 1 showing the peaks' direction. We have a series of price increases in the morning, and then we have a frequent drop from 12 am to 1pm. In the evening we have a similar pattern, but increases and drops are less concentrated in a single hour. We see that peaks typically last more than 1 hour (we have very limited cases of "true peak").

Figure 4. Daily volatility distribution



In Figure 4 we have a comparison of daily volatilities for the whole year for Belgium and Italy. On the vertical axis is the level of volatility and on the horizontal axis the percentage of the days attained by a certain level of volatility. As we can see in Belgium there are for around a 5% of the observations very high values of daily volatility, instead in Italy values are more constant. If we link this with the information showed in the previous tables and graphs we can remark how in Belgium there seems to be some extreme events, which clearly could an interesting characteristic for DR. If we have to trace a comparison then Italy presents characteristics of a country with a certain regularity, instead Belgium has sporadic events. In terms of customers then Italy would favor customers with frequent availability, a characteristic not necessary in Belgium, which instead could be an interesting case for customers, which could accept few curtailments per year with very low probability of overriding.

VI. Conclusions

The paper shows how simple analysis of volatility cannot fully retain DR commercial potential. It is necessary to match the customers profiles with the price patterns to verify their capability to offer flexible load. We then provide an analysis of volatility measures and show how to define and quantify events, which can be potentially used for DR programs. Future research should extend the analysis to other applications like Balance and Ancillary Services Markets, which are also fundamental sources of revenues for DR activities; still Day Ahead price patterns drive the whole market so we expect some of the trends transfer to other settings.

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