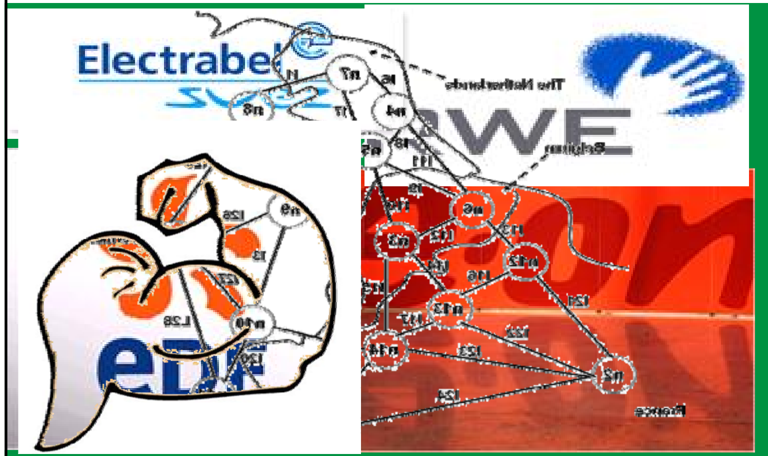
TECHNISCHE UNIVERSITÄT DRESDEN

EE²

UNIVERSITY OF MARYLAND 1856



Solving Discretely-Constrained MPEC Problems Using Disjunctive Constraints and Linearization with Applications in Electric Power Markets

Infraday, Berlin

11 October 2008

Steven A. Gabriel
Dept. of Civil & Environmental Eng./ Applied Math & Scientific Computation Program
University of Maryland
College Park, Maryland USA

Florian Leuthold
Chair of Energy Economics and Public Sector Management (EE^2)
Technische Universität Dresden, Germany

Outline of Presentation

1. Structure of Electric Power Market in Europe
2. Literature Review
3. Mathematical Formulation
4. Computational Results
5. Future Work

Motivation: Market Structures in Europe

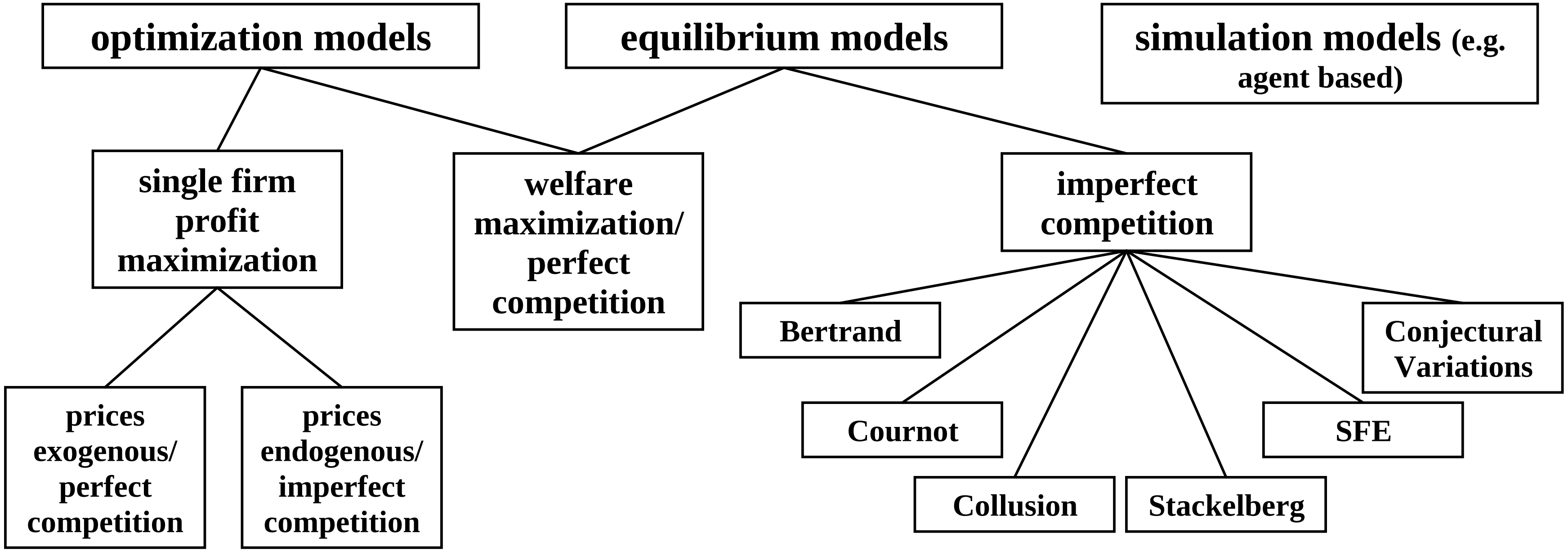
- France: EDF has a market share of 80%
- Germany: EON+RWE 55% market share; +Vattenfall+EnBW 85% market share
- Liberalization of vertical integrated companies proceeds sluggish
- Former integrated companies have information advantages in terms of geographical specifics and network knowledge
- This gives rise potentially to one (or more) dominant players in the market rest can be considered as “competitive fringe”
- Need for modeling that takes this structure into account

Source: EDF (2008), EON (2008), Google Maps (2008), RWE (2008).

Outline

1. Structure of Electric Power Market in Europe
2. Brief Literature Review
3. Mathematical Formulation of Discretely-Constrained Mathematical Program with Equilibrium Constraints (DC-MPEC) for Power Market
4. Computational Results
5. Future Work

Electricity Market Modeling Approaches



Source: Day et al. (2002), Görner et al. (2008), Kahn (1998), Smeers (1997), Ventosa et al. (2005).

- **Simulation models do not follow a single mathematical formulation**
- **For the rest: The type of competition mostly defines the resulting model**
  - Perfect vs. imperfect competition → Optimization vs. equilibrium models
  - One stage vs. two/three stages approach
- **Combining this with further characteristics of electricity markets can make models basically impossible to solve**
  - Discrete variables (e.g., investments, start-up, unit commitment)
  - Stochastic modeling (e.g., stochastic demand, stochastic wind generation)

→ **Current focus of research: Solving discretely-constrained equilibrium models**

Outline

1. Structure of Electric Power Market in Europe
2. Literature Review
3. Mathematical Formulation
4. Computational Results
5. Future Work

General Problem Formulation-DC MPEC

$$\min_{x,y} \left\{ \begin{pmatrix} d_x \\ d_y \end{pmatrix}^T \begin{pmatrix} x \\ y \end{pmatrix} + \frac{1}{2} \begin{pmatrix} x \\ y \end{pmatrix}^T \begin{pmatrix} M_{xx} & M_{xy} \\ M_{yx} & M_{yy} \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} \right\}$$

$$s.t.\ A_1 y + B_1 x = b_1 \ (\beta_1)$$

Joint x-y constraints

$$A_2 x = b_2 \ (\beta_2)$$
$$A_3 x \le b_3 \ (\beta_3)$$
$$x_i \in Z_+, i = 1, \ldots, n_1$$
$$x_i \in R, i = n_1 + 1, \ldots, n$$

x-only constraints

$$A_4 y = b_4 \ (\beta_4)$$
$$A_5 y \le b_5 \ (\beta_5)$$
$$y_j \ge 0, j = 1, \ldots, m_1$$
$$y_2 \text{ free}, j = m_1 + 1, \ldots, m$$
$$y \in S(x)$$

y-only constraints, includes lower-level problem solution set S(x) as a function of x

x: dominant firm upper- level planning variables (e.g., generation), some may be discrete/some continuous

y: lower-level market/ISO variables, all continuous (e.g., market prices, phase angles)

quadratic objective function (e.g., min costs-revenue), will involve product of price and generation, bilinear (non-convex) term

Disjunctive Constraints

$$y \in S(x) \begin{cases} 0 \leq c_1(x) + M_{11}(x)y_1 + M_{12}(x)y_2 \perp y_1 \geq 0 \\ 0 = c_2(x) + M_{21}(x)y_1 + M_{22}(x)y_2 \quad y_2 \text{ free} \end{cases}$$

Lower-level problem as mixed complementarity problem relating to a market equilibrium

Replacing perpendicular condition by disjunctive constraints

$$\begin{aligned} 0 &\leq c_1(x) + M_{11}(x)y_1 + M_{12}(x)y_2 \leq Kr \\ 0 &\leq y_1 \leq K(1-r) \\ 0 &= c_2(x) + M_{21}(x)y_1 + M_{22}(x)y_2 \quad y_2 \text{ free} \\ r &\in \{0,1\} \end{aligned}$$

Lower-level problem as Mixed Integer Problem
K is a constant
r is a vector of binary variables

Electricity Market Model I: Fundamental Idea

- **Assumption: Stackelberg competition**
  - **Leader makes output decision**
  - **Follower decides taking the leaders decision for granted**

- **Leader: Strategic production company**
  - **Maximizes individual profit under maximum generation constraints and non-negative production (upper-level problem)**
  - **Takes into account followers decisions (lower-level problem)**

- **Follower: ISO**
  - **Maximizes social welfare**
  - **Decides over the output decision of the competitive fringe**
  - **Takes into account technical constraints such as maximum fringe generation, line flow, and energy balance constraints**

Electricity Market Model II: ISO Problem

$$\min_{d_n g_{nfu} \delta_k} \left\{ \sum_n \left(-a_n d_n + \frac{b_n d_n^2}{2} \right) + \sum_n \sum_f \sum_u (g_{nfu} c_{nfu}) \right\}$$

Welfare maximization

$$s.t.\ d_n + \sum_k (B_{nk}\delta_k) - \sum_f \sum_u g_{nfu} = 0, \forall n \quad (\lambda_n)$$

Energy balance

$$-lc_l + \sum_k (H_{lk}\delta_k) \le 0, \forall l \quad (\bar{\mu}_l)$$

$$-lc_l - \sum_k (H_{lk}\delta_k) \le 0, \forall l \quad (\underline{\mu}_l)$$

Line flow cap

$$-\overline{g}_{nfu} + g_{nfu} \le 0, \forall n, f, u \quad (\beta_{nfu})$$

Generation cap

$$-sw_k\delta_k = 0, \forall k \quad (\gamma_k)$$

Voltage angle 0 for slack

$$d_n \ge 0, \forall n$$

Non-negative demand

$$g_{nfu} \ge 0, \forall n, f, u$$

Non-negative production

→ KKT conditions for the lower-level problem are necessary and sufficient, they are S(x)

Electricity Market Model II: Overall MPEC

$$\min_{g_{nsu},\lambda_n} \left\{ \sum_n \sum_s \sum_u (c_{nsu} - \lambda_n) g_{nsu} \right\}$$

Profit maximization

$$s.t. \quad g_{nsu} - \overline{g}_{nsu} \leq 0, \ \forall n, s, u$$

Leader's generation cap (“x-only constraints”)

$$0 \leq -a_n + b_n d_n + \lambda_n \perp d_n \geq 0, \forall n$$

$$0 \leq c_{nju} - \lambda_n + \beta_{nju} \perp g_{nju} \geq 0, \forall n, j, u$$

$$0 = \sum_n (B_{nk}\lambda_n) + \sum_l (H_{lk}\overline{\mu}_l) - \sum_l \left(H_{lk}\underline{\mu}_l\right)$$

$$-\begin{cases} \gamma_k & \text{if } k = k\prime \\ 0 & \text{otherwise} \end{cases} \qquad \delta_k \ (free), \forall k$$

$$0 = d_n + \sum_k (B_{nk}\delta_k) - \sum_f \sum_u g_{nfu} = 0, \lambda_n \ (\text{free}), \forall n$$

$$0 \leq lc_l - \sum_k (H_{lk}\delta_k) \perp \overline{\mu}_l \geq 0, \forall l$$

$$0 \leq lc_l + \sum_k (H_{lk}\delta_k) \perp \underline{\mu}_l \geq 0, \forall l$$

$$0 \leq -g_{nju} + \overline{g}_{nju} \perp \beta_{nju} \geq 0, \forall n, j, u$$

$$0 = -sw_k\delta_k, \quad \gamma_k \ (\text{free}), \forall k$$

ISO KKTs including fringe firm j

→ **Problem: Objective bilinear (price*quantity) → Non-convex mixed integer problem**

Electricity Market Model III: MILP I

→ **Linearization of the objective function, bilinear term replaced by an approximation**

$$v_{nsu,i} = \begin{cases} \bar{g}_{nsu,i}\lambda_n & \text{if } q_{nsu,i} = q_n^{\lambda} = 1 \\ 0 & \text{otherwise} \end{cases}$$

$$\min_{d_n, g_{nfu}, r_n, \bar{r}_{nju}, \hat{r}_l, \tilde{r}_l, \check{r}_{nju}, \beta_{nju}, \gamma_k, \delta_k, \lambda_n, \overline{\mu}_l, \underline{\mu}_l} \left\{ \sum_n \sum_s \sum_u c_{nsu} g_{nsu} - \sum_i v_{nsu,i} \right\}$$

$$0 \leq \lambda_n \leq M q_n^{\lambda}, \forall n$$

$$g_{nsu} = \sum_i q_{nsu,i} \bar{g}_{nsu,i}, \forall n, s, u$$

$$\sum_i q_{nsu,i} = 1, \forall n, s, u$$

Discrete generation levels for leader

$$\begin{cases} q_{nsu,i}^{v} \leq q_n^{\lambda}, \forall n, s, u, i \\ q_{nsu,i}^{v} \leq q_{nsu,i}, \forall n, s, u, i \\ q_{nsu,i} + q_n^{\lambda} - 1 \leq q_{nsu,i}^{v}, \forall n, s, u \end{cases}$$

$$\begin{cases} v_{nsu,i} \leq \bar{g}_{nsu,i}\lambda_n, \forall n, s, u, i \\ 0 \leq v_{nsu,i} \leq M q_{nsu,i}^{v}, \forall n, s, u, i \end{cases}$$

→ **Parameterizing the output decisions of the strategic player**

Electricity Market Model IV: MILP II

$$
\left.
\begin{aligned}
&0 \le -a_n + b_n d_n + \lambda_n \le K r_n, \forall n \\
&0 \le d_n \le K(1 - r_n), \forall n \\
&0 \le c_{nju} - \lambda_n + \beta_{nju} \le \bar{K}\bar{r}_{nju}, \ \forall n, j, u \\
&0 \le g_{nju} \le \bar{K}(1 - \bar{r}_{nju}), \forall n, j, u \\
&0 = \sum_n (B_{nk}\lambda_n) + \sum_l (H_{lk}\overline{\mu}_l) - \sum_l \left(H_{lk}\underline{\mu}_l\right) \\
&-\begin{cases} \gamma_k & \text{if } k = k\prime \\ 0 & \text{otherwise} \end{cases} \qquad \delta_k \ (free), \forall k \\
&0 = d_n + \sum_k (B_{nk}\delta_k) - \sum_f \sum_u g_{nfu}, \lambda_n \ (\text{free}), \forall n \\
&0 \le lc_l - \sum_k (H_{lk}\delta_k) \le \hat{K}\hat{r}_l, \forall l \\
&0 \le \overline{\mu}_l \le \hat{K}(1 - \hat{r}_l), \forall l \\
&0 \le lc_l + \sum_k (H_{lk}\delta_k) \le \tilde{K}\tilde{r}_l, \forall l \\
&0 \le \underline{\mu}_l \le \tilde{K}(1 - \tilde{r}_l), \forall l \\
&0 \le -g_{nju} + \overline{g}_{nju} \le \check{K}\check{r}_{nju}, \forall n, j, u \\
&0 \le \beta_{nju} \le \check{K}(1 - \check{r}_{nju}), \forall n, j, u \\
&0 = -sw_k\delta_k, \quad \gamma_k(free), \forall k
\end{aligned}
\right\}
$$

Replacing ISO KKT conditions by disjunctive constraints yields a mixed integer linear problem (MILP)

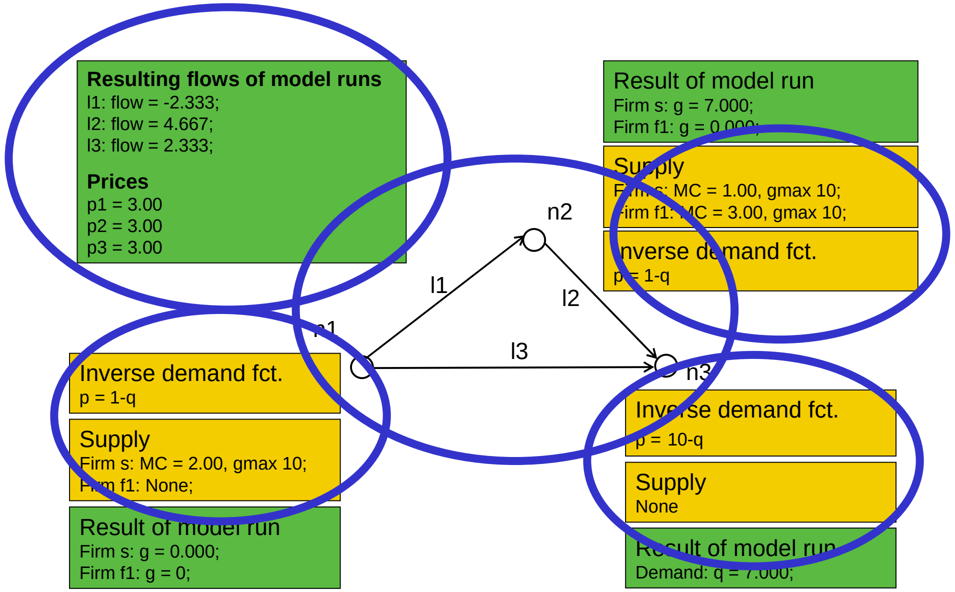
$$
\begin{aligned}
r_n, \bar{r}_{nju}, \hat{r}_l, \tilde{r}_l, \check{r}_{nju}, q_{nsu,i}, q_n^{\lambda} &\in \{0, 1\}, \forall n, s, u, i, j \\
q_{nsu,i}^{v} &\in [0, 1], \forall n, s, u, i
\end{aligned}
$$

Outline

1. Structure of Electric Power Market in Europe
2. Literature Review
3. Mathematical Formulation
4. Computational Results
5. Future Work

Three-Node Network I

- **First test for the model**
- **Simple input data**
  - **Displayed in the yellow boxes on the next slides**
- **Easy plausibility check of the results**
  - **Displayed in the green boxes on the next slides**



Three-Node Network II

Resulting flows of model runs

l1: flow = -2.333;
l2: flow = 4.667;
l3: flow = 2.333;

Prices

p1 = 3.00
p2 = 3.00
p3 = 3.00

Result of model run

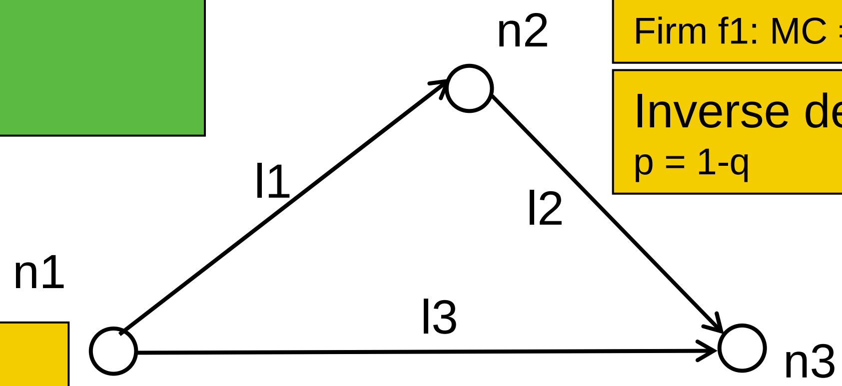
Firm s: g = 7.000;
Firm f1: g = 0.000;

Supply

Firm s: MC = 1.00, gmax 10;
Firm f1: MC = 3.00, gmax 10;

Inverse demand fct.

p = 1-q



Inverse demand fct.

p = 1-q

Supply

Firm s: MC = 2.00, gmax 10;
Firm f1: None;

Result of model run

Firm s: g = 0.000;
Firm f1: g = 0;

Inverse demand fct.

p = 10-q

Supply

None

Result of model run

Demand: q = 7.000;

Three-Node Network III

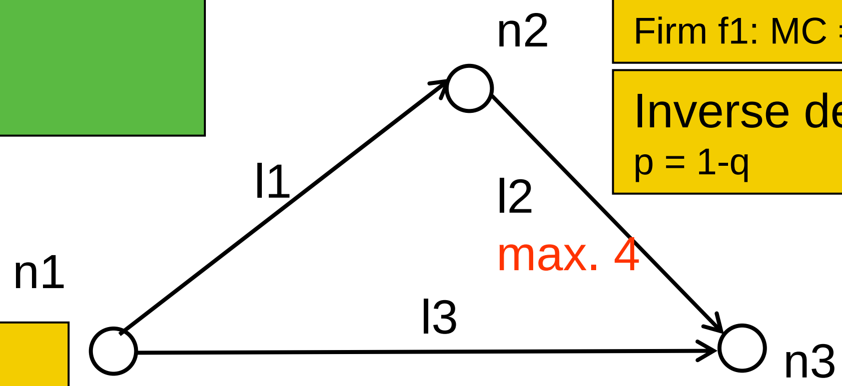
Resulting flows of model runs
l1: flow = -1.5; [-1.0]
l2: flow = 4.0; [4.0]
l3: flow = 2.5; [3.0]

Prices
p1 = 3.25 [2.0]
p2 = 3.00 [1.0]
p3 = 3.50 [3.0]

Result of model run
Firm s: g = 5.5; [5.0]
Firm f1: g = 0.0;

Supply
Firm s: MC = 1.00, gmax 10;
Firm f1: MC = 3.00, gmax 10;

Inverse demand fct.
p = 1-q



Inverse demand fct.
p = 1-q

Supply
Firm s: MC = 2.00, gmax 10;
Firm f1: None;

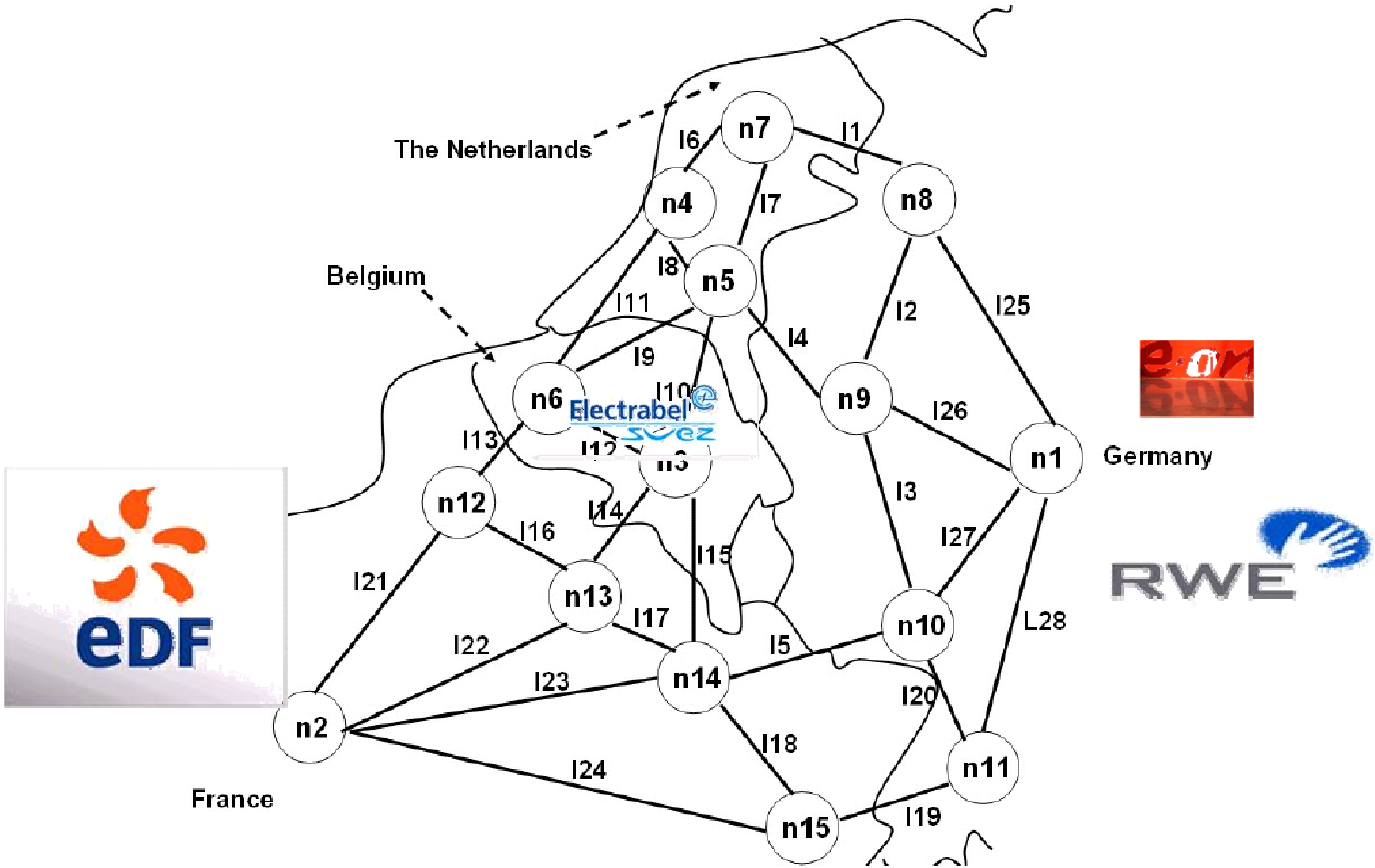
Result of model run
Firm s: g = 1.0; [2.0]
Firm f1: g = 0.0;

Inverse demand fct.
p = 10-q

Supply
None

Result of model run
Demand: q = 6.5; [7.0]

Fifteen-Node Network: Structure



Fifteen-Node Network: Results I

	Test_EDF	
	comp	strat
$g_{n2EDFu1}[MWh]$	54209	31000
$g_{n2EDFu7}[MWh]$	12000	14000
$d_{n1}[MWh]$	61867	61867
$d_{n2}[MWh]$	63000	48745
$d_{n3}[MWh]$	3500	3200
$d_{n4}[MWh]$	5250	5250
$d_{n5}[MWh]$	4534	4640
$d_{n6}[MWh]$	2133	2000
$d_{n7}[MWh]$	2716	2720

- **We compare perfect competition (comp) to an imperfect competition (strat) run**
- **It can be shown that under strategic behavior, the player produces less than in the competitive run**
- **Why? → Next slide**

Fifteen-Node Network: Results II

	Test_EDF comp	Test_EDF strat
$price_{n1}[€/MWh]$	22.0	22.0
$price_{n2}[€/MWh]$	10.0	41.7
$price_{n3}[€/MWh]$	10.0	22.0
$price_{n4}[€/MWh]$	45.0	45.0
$price_{n5}[€/MWh]$	59.3	57.2
$price_{n6}[€/MWh]$	22.0	30.0
$price_{n7}[€/MWh]$	41.3	41.2

		Test_EDF
Profit Leader [k€]	comp	140
	strat	1565
Profit Fringe [k€]	comp	483
	strat	547

- Because the player can influence the prices at nodes where it is profitable for him
- in order to maximize individual profits

Fifteen-Node Network: Results III

	Test_EDF	
Problem size	comp	582 continuous variabes 0 discrete variables
	strat	22029 continuous variables 7406 discrete variables

	Test_EDF	Tests 3nodes
Computation times	21 hours	15-33 seconds

- **Problem size increases dramatically for strategic behavior runs**
- **The size depends on the number of discrete production choice possibilities**
- **The computation times is long but varies depending on the possible discrete choices**

Outline

1. Structure of Electric Power Market in Europe
2. Literature Review
3. Mathematical Formulation
4. Computational Results
5. Future Work

Future Work

- Speeding up the solution of the DC-MPEC expressed as a mixed-integer program
  - When RWE was the leader, solution time was 4 minutes
  - When EDF was the leader, solution time was 21 hours! (presumably due to the fact that EDF had too many choices for how to generate power)
  - Need to add cuts to reduced search procedure time

- Consideration of when the lower-level problem can also have integer variables
  - For example, ISO or competitive fringe go/no decisions to make
  - May use a variant of Benders decomposition to solve this (Gabriel et al., 2007)

- Consideration of “n-1” problem for network resilience

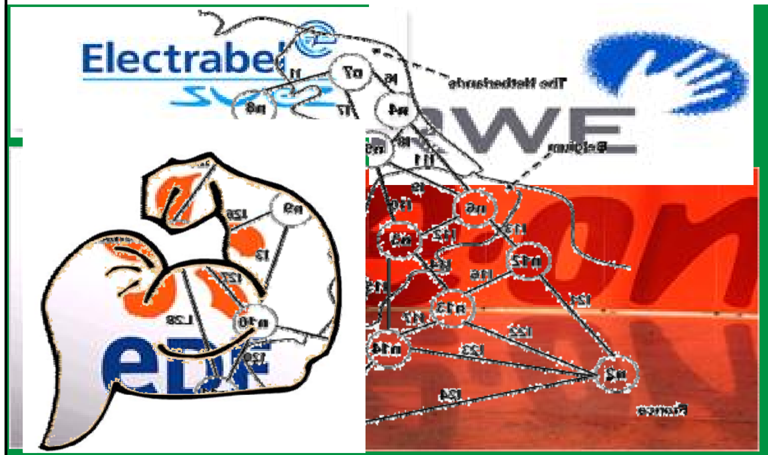
- Additional discrete variables
  - Investment decisions
  - Unit commitment decisions

References (Selected)

- M. Anitescu, On Solving Mathematical Programs with Complementarity Constraints as Nonlinear Programs. Preprint ANL/MCS-P864-1200, MCS Division, Argonne National Laboratory, Argonne, IL, USA, 2000.
- W.F. Bialas and M.H. Karwan, Two-Level Linear Programming, Management Science, 30(8), 1004-1020, 1984.
- Y. Chen, B.F. Hobbs, S. Ley¤er, T.S. Munson, Leader-Follower Equilibria for Electric Power and NOx Allowances Markets, Preprint ANL/MCS-P1191-0804, Mathematics and Computer Science Division, Argonne National Laboratory, Argonne, IL USA 60439, 2004.
- O. Daxhelet, Y. Smeers, Variational Inequality Models of Restructured Electric Systems, in: M.C. Ferris, O.L. Mangasarian, J.-S. Pang (eds.), Applications and Algorithms of Complementarity, Kluwer, Dordrech, 2001.
- C.J. Day, B.F. Hobbs, J.-S- Pang, Oligopolistic Competition in Power Networks: A Conjectured Supply Function Approach, IEEE Transactions on Power Systems, 17 (3), 597-607, 2002.
- S.P. Dirkse, M.C. Ferris, A. Meerhaus, Mathematical Programs with Equilibrium Constraints: Automatic Reformulation and Solution via Constraint Optimization, Technical Report NA-02/11, Oxford University Computing laboratory, July, 2002.
- S.A. Gabriel, Y.Shim, A.J. Conejo, S. de la Torre, R. Garcia-Bertrand, A Benders Decomposition Method for Discretely-Constrained Mathematical Programs with Equilibrium Constraints with Applications in Energy, in review, October 2007.
- M. Görner, T. Hahnemann, T. Heß, H. Weigt, Electricity Transmission Modeling – Economic Impact of Technical Characteristics,
- B.F. Hobbs, C.B. Metzler, J.-S. Pang, Strategic gaming analysis for electric power systems: an MPEC approach, IEEE Transactions on Power Systems,15 (2), 638-645, 2000.
- X. Hu, D. Ralph, Using EPECs to Model Bilevel Games in Restructured Electricity Markets with Locational Prices, Operations Research, 55 (5), pp. 809-827, 2007.
- E. P. Kahn: Numerical Techniques for Analyzing Market Power in Electricity. In: Electricity Journal Vol. 11, 6, pp. 34–43, 1998.
- K. Neuhoff, J. Barquin, M.G. Boots, A. Ehrenmann, B.F. Hobbs, F.A.M. Rijkers, M. Vázquez, Network-Constrained Cournot Models of Liberalized Electricity Markets: The Devil is in the Details, Energy Economics, 27, 495-525, 2005.
- F.C. Schweppe, M.C. Caramanis, R.E. Tabors, R.E. Bohn, Spot Pricing of Electricity, Kluwer, Norwell, Massachusetts, 1988.
- Y. Smeers, Computable Equilibrium Models and the Restructuring of the European Electricity and Gas Markets, Energy Journal Vol. 18, 4, pp. 1–31, 1997.
- M. Ventosa, B. Alvaro, R. Andres, M. Rivier, Electricity Market Modeling Trends, Energy Policy Vol. 33, pp. 897-913, 2005.

TECHNISCHE UNIVERSITÄT DRESDEN

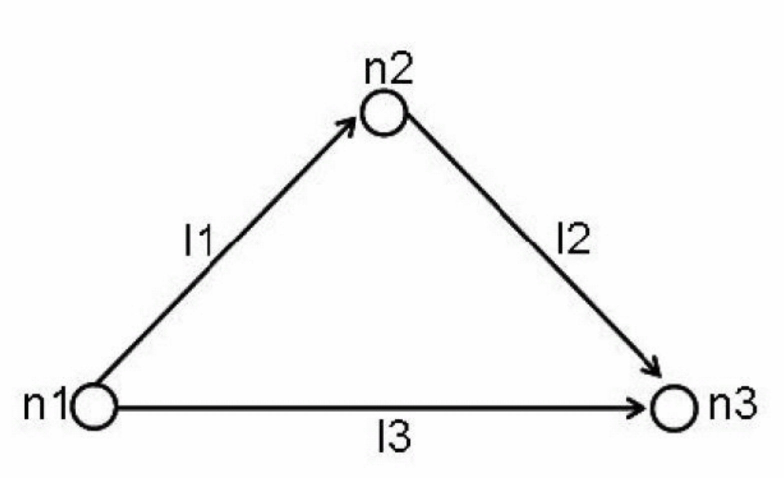
EE²



Thank you very much
for your attention!
Any questions or comments?

sgabriel@umd.edu
florian.leuthold@tu-dresden.de

Three-Node Network: Structure



	n1	n2	n3
a_n	1	1	10
b_n	1	1	1

	Test1	Test2	Test3	Test4	Test5
$c_{n1su1}[€/MWh]$	2	2	2	2	2
$c_{n2su2}[€/MWh]$	1	1	1	1	1
$c_{n2j1u3}[€/MWh]$	3	7	8	9	3
$\bar{g}_{n1su1}[MWh]$	10	10	10	10	10
$\bar{g}_{n2su2}[MWh]$	10	10	10	10	10
$\bar{g}_{n2j1u3}[MWh]$	10	10	10	10	10
$lc_{l1}[MW]$	10	10	10	10	10
$lc_{l2}[MW]$	10	10	10	10	4
$lc_{l3}[MW]$	10	10	10	10	10

Fifteen-Node Network: Results I

	Test EDF	
	comp	strat
$g_{n2EDFu1}[MWh]$	54209	31000
$g_{n2EDFu7}[MWh]$	12000	14000
$d_{n1}[MWh]$	61867	61867
$d_{n2}[MWh]$	63000	48745
$d_{n3}[MWh]$	3500	3200
$d_{n4}[MWh]$	5250	5250
$d_{n5}[MWh]$	4534	4640
$d_{n6}[MWh]$	2133	2000
$d_{n7}[MWh]$	2716	2720

	Test_RWE	
	comp	strat
$g_{n2RWEu1}[MWh]$	6000	6000
$g_{n2RWEu2}[MWh]$	11000	1000
$g_{n2RWEu3}[MWh]$	2985	0
$d_{n1}[MWh]$	61867	58270
$d_{n2}[MWh]$	63000	63000
$d_{n3}[MWh]$	3500	3500
$d_{n4}[MWh]$	5250	5250
$d_{n5}[MWh]$	4534	4554
$d_{n6}[MWh]$	2133	2133
$d_{n7}[MWh]$	2716	2630

- **We compare perfect competition (comp) to an imperfect competition (strat) run**
- **It can be shown that under strategic behavior, the player produces less than in the competitive run**
- **Why? → Next slide**

Fifteen-Node Network: Results II

	Test EDF comp	Test EDF strat	Test RWE comp	Test RWE strat
$price_{n1} [€/MWh]$	22.0	22.0	22.0	29.4
$price_{n2} [€/MWh]$	10.0	41.7	10.0	10.0
$price_{n3} [€/MWh]$	10.0	22.0	10.0	10.0
$price_{n4} [€/MWh]$	45.0	45.0	45.0	45.0
$price_{n5} [€/MWh]$	59.3	57.2	59.3	58.9
$price_{n6} [€/MWh]$	22.0	30.0	22.0	22.0
$price_{n7} [€/MWh]$	41.3	41.2	41.3	44.8

		Test EDF	Test RWE
Profit Leader [k€]	comp	140	94
	strat	1565	126
Profit Fringe [k€]	comp	483	529
	strat	547	944

- **Because the player can influence the prices at nodes where it is profitable for him**
- **in order to maximize individual profits**

Fifteen-Node Network: Results III

	Test_EDF	Test_RWE
Problem size	comp { 582 continuous variabes, 0 discrete variables }	comp { 582 continuous variabes, 0 discrete variables }
	strat { 22029 continuous variables, 7406 discrete variables }	strat { 5109 continuous variables, 1766 discrete variables }

	Test_EDF	Test_RWE	Tests 3nodes
Computation times	**21 hours**	**4 minutes**	**15-33 seconds**

- **Problem size increases dramatically for strategic behavior runs**
- **The size depends on the number of discrete production choice possibilities**
- **The computation times vary dramatically, too**