The allocation of capacity bottlenecks at a congested rail network: rail traffic maximization versus auctions

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Overview

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Introduction

Basic model without intermodal competition and environmental externalities

Linear tariffs and rail traffic maximization (regime m)

Linear tariffs and scarcity premiums/auctions (regime a)

Extended model with intermodal competition and environmental externalities

Conclusions

Introduction

"German" way of allocating railway capacity versus auctions:

	Germany	Auctions
Excess demand	Rail traffic maximization	Scarcity premium
Revenue generation	Linear tariff	Linear tariff, scarcity premium

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Excess demand	Rail traffic maximization	Scarcity premium
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What's better?

Introduction

Important aspects of rail transport markets:

- Network industry with demand complementarities
- Transport industry with delays, delay costs (e.g. missed business meetings or interruptions in production processes) and congestion externalities
- Intermodal competition (e.g. between rail and road transport service providers)
- Environmental externalities ("Mehr Verkehr auf die Schiene")
- Fixed rail network costs (marginal cost pricing will lead to heavy losses)

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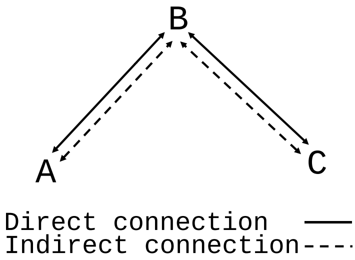
How do they affect the choice of allocation regimes?

Introduction

- We built a "simple" model that accounts for the above mentioned characteristics of rail transport markets
- This model is then used to compare the two different railway allocation regimes
  - Linear tariffs and, in the case of excess demand, rail traffic maximization
  - Linear tariffs and scarcity premiums that prevent excess demand
- We find (amongst others) that
  - auctions may reduce total revenues of the rail network provider (for a given linear tariff)
  - auctions may also reduce total surplus
  - intermodal competition *and* environmental externalities favor the use of auctions from a welfare perspective

Overview

Simple rail network



- Three cities A, B and C
- Three origin-destination (o-d) pairs AB, BC and AC
- Two direct rail links on o-d pairs AB and BC
- One indirect link on o-d pair AC

Game structure

- Static model
- Players:
  - customers (passengers, freight companies or logistics firms),
  - train operating companies (tocs) and
  - a monopolistic rail network provider
- One-stage game: players simultaneously decide upon the
  - demand for rail transport services and
  - supply of rail transport services as well as
  - allocation of rail network capacity
- The capacity allocation regime is exogenous

Benefits and delay costs

- $q_x \geq 0$ with $x \in \{AB, BC, AC\}$ is the mass of rail customers served on the different o-d pairs
- Total rail traffic on direct connections AB and BC is

$$q_{AB} + q_{AC} \quad \text{and} \quad q_{BC} + q_{AC}$$

- The representative customer's benefits from rail services are

$$B_x(q_x) := a_x\, q_x - \frac{b_x\, q_x^2}{2}$$

- Average delay costs to customers on direct connections AB and BC are

$$C_{AB}(q_{AB}, q_{AC}) := q_{AB} + q_{AC} \text{ and } C_{BC}(q_{BC}, q_{AC}) := q_{BC} + q_{AC}$$

- To control delays there is an upper limit for the rail services on each direct connection

$$q_{AB} + q_{AC} \leq \bar{q} \text{ and } q_{BC} + q_{AC} \leq \bar{q} \text{ with } \bar{q} > 0$$

Consumer surplus and welfare

- Tocs charge a unit price equal to unit costs that are normalized to zero
- Revenues of the rail network provider are $R \geq 0$
- Consumer surplus is

$$S := \underbrace{\sum_{x \in \{AB,BC,AC\}} B_x}_{\text{benefits}} - \underbrace{\sum_{y \in \{AB,BC\}} (q_y + q_{AC})\, C_y}_{\text{total delay costs}} - \underbrace{R}_{\text{payments}}$$

- Fixed costs of the network are $F > 0$
- Total surplus is

$$W := S + R - F$$

Overview

Linear tariffs and rail traffic maximization (regime m)

- The rail network provider charges a unit price $p \geq 0$ for the use of a direct connection (linear tariff) to, say, customers
- It can be shown that a unique set of demands $D_x(p)$ with $x \in \{AB, BC, AC\}$ exists such that

$$D_y(p) = \arg\max_{q_y} B_y(q_y) - q_y\left(p + C_y(q_y, D_{AC}(p))\right)$$

$$\text{s.t. } q_y \geq 0$$

with $y \in \{AB, BC\}$ and

$$D_{AC}(p) = \arg\max_{q_{AC}} B_{AC}(q_{AC})$$

$$-q_{AC}\left[2\,p + C_{AB}(D_{AB}(p), q_{AC}) + C_{BC}(D_{BC}(p), q_{AC})\right] \text{ s.t. } q_{AC} \geq 0$$

Linear tariffs and rail traffic maximization (regime m)

- Allocation is

$$(q_{AB}^m, q_{BC}^m, q_{AC}^m) := \arg \max_{q_{AB}, q_{BC}, q_{AC}} q_{AB} + q_{BC} + 2q_{AC}$$

$$\text{s.t. } q_y + q_{AC} \leq \bar{q} \text{ for all } y \in \{AB, BC\} \text{ and}$$

$$q_x \leq D_x(p) \ \text{ for all } \ x \in \{AB, BC, AC\}$$

- Revenues of the rail network provider are

$$R^m := p\,(q_{AB}^m + q_{BC}^m + 2q_{AC}^m)$$

Demand functions (example)

If p is low enough such that demand for the two direct services and the indirect service is positive, demand is

$$D_y(p) :=$$

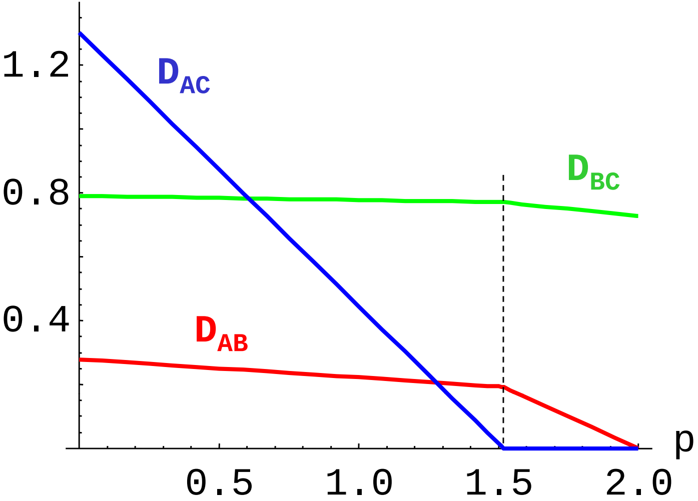
$$\frac{a_y - a_{AC}\,(1 + b_y) - (p + b_{AC}\,p)\,(1 + b_y) + a_z\,(1 + b_{AC} + b_y\,(2 + b_{AC}))}{b_y\,(1 + 2\,b_z + b_{AC} + b_z\,b_{AC})}$$

for $y, z \in \{AB, BC\}$ and $y \neq z$ and

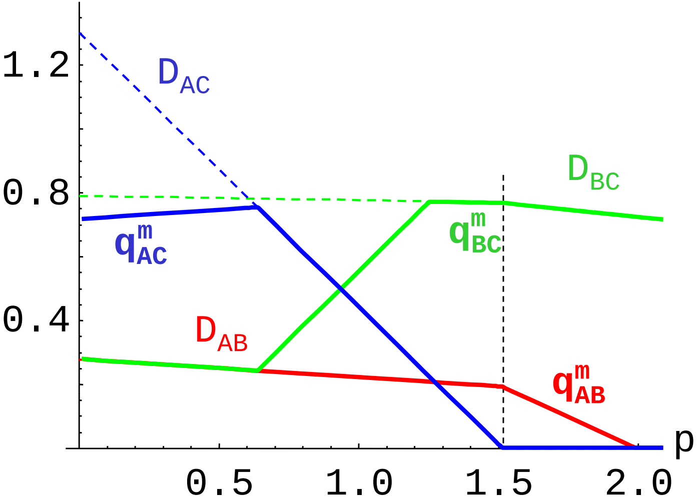
$$D_{AC}(p) :=$$

$$\frac{(a_{AC} - 2\,p)(1 + b_{AB})(1 + b_{CB}) - (a_{AB} - p)(1 + b_{CB}) - (a_{BC} - p)(1 + b}{(1 + b_{AB})\,(1 + b_{BC})\,(2 + b_{AC}) - 2 - b_{AB} - b_{BC}}$$

Demand functions (example)



Rail services as a function of p under regime m



Overview

Linear tariffs and scarcity premiums/auctions (regime a)

- The rail network provider also charges a unit price $p \geq 0$ for the use of a direct connection (linear tariff)
- Additionally she charges scarcity premiums $p_y \geq 0$ on direct connections $y \in \{AB, BC\}$
- Then, the total price customers have to pay on direct connections $y \in \{AB, BC\}$ under regime a is

$$p + p_y$$

Linear tariffs and scarcity premiums/auctions (regime a)

It can be shown that a unique set of demands $D_x(p, p_{AB}, p_{BC})$ with $x \in \{AB, BC, AC\}$ exist such that

$$D_y(p, p_{AB}, p_{BC}) = \arg\max_{q_y} B_y(q_y)$$

$$-q_y\left(p + p_y + C_y(q_y, D_{AC}(p, p_{AB}, p_{BC}))\right) \text{ s.t. } q_y \geq 0$$

with $y \in \{AB, BC\}$ and

$$D_{AC}(p, p_{AB}, p_{BC}) = \arg\max_{q_{AC}} B_{AC}(q_{AC})$$

$$-q_{AC}\left[\sum_{y \in \{AB,BC\}} (p + p_y + C_y(D_y(p, p_{AB}, p_{BC}), q_{AC}))\right] \text{ s.t. } q_{AC} \geq 0$$

Linear tariffs and scarcity premiums/auctions (regime a)

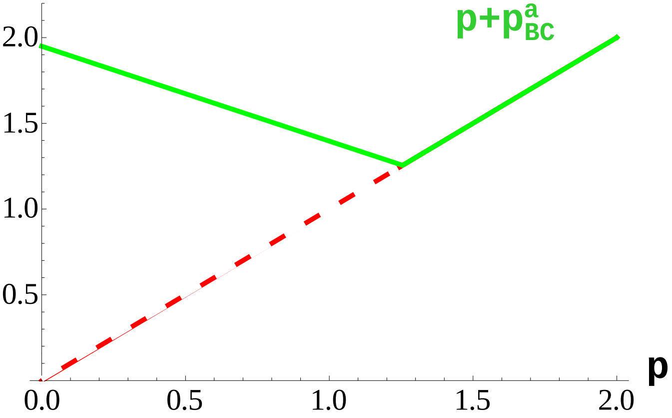
- It can also be shown that a unique pair of scarcity premiums (p^a_{AB}, p^a_{BC}) exists such that $p^a_y > 0$ implies

$$p^a_y \in \{p_y : D_y(p, p_{AB}, p_{BC}) + D_{AC}(p, p_{AB}, p_{BC}) = \bar{q}\}$$

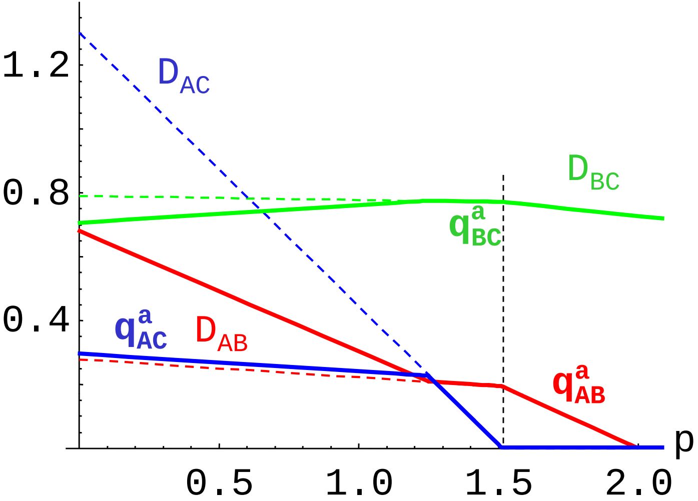
  for all $y \in \{AB, BC\}$.
- Revenues of the rail network provider are

$$R^a := \sum_{y \in \{AB,BC\}} (p+p^a_y)\,(D_y(p, p^a_{AB}, p^a_{BC})+D_{AC}(p, p^a_{AB}, p^a_{BC}))$$

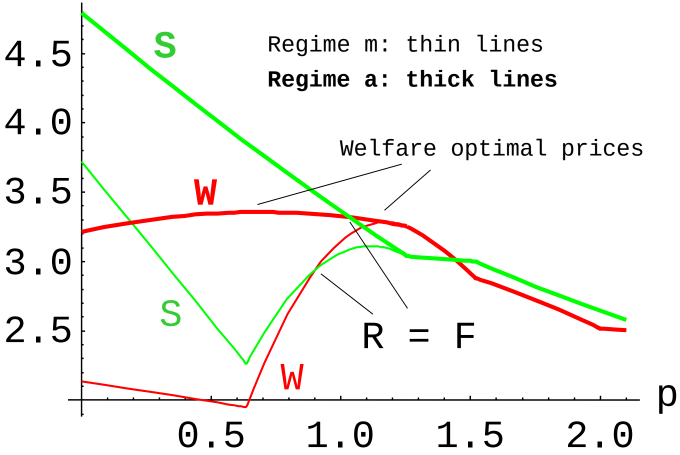
Total prices under regime *a* (example)



Rail services as a function of *p* under regime *a*



Welfare and consumer surplus as a function of p: regime m versus regime a



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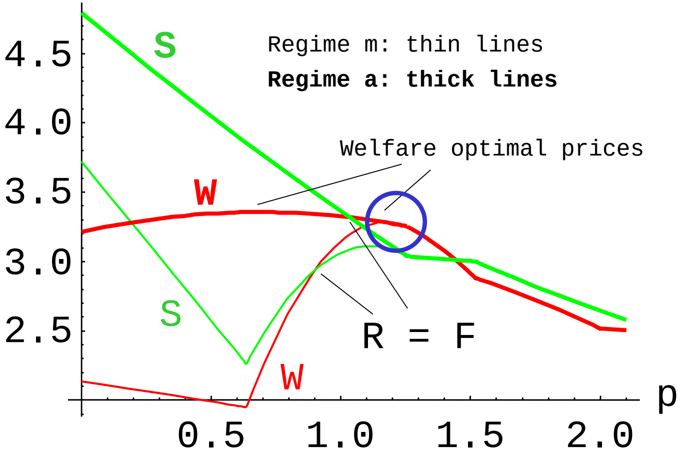

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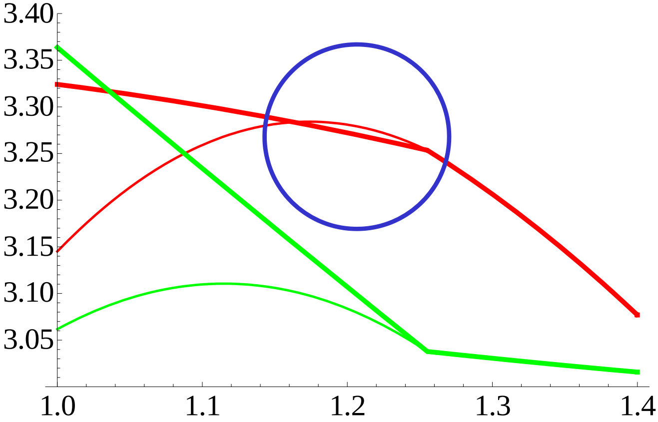

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Extended model with intermodal competition and environmental externalities

- Marginal cost of road transport is constant
- Perfect competition in road transport markets
- Road transport charges are determined by marginal costs (which may include congestion costs) and equal to $\bar{p} > 0$ for $x \in \{AB, BC\}$ and $2\,\bar{p}$ for $x = AC$
- Demand for rail services (index r) on o-d pairs $y \in \{AB, BC\}$ is

$$D^r_y(p, p_{AC}, p_{BC}) :=$$

$$\begin{cases} D_y(p, p_{AC}, p_{BC}) & \text{for} \quad p + p_y + C_y < \bar{p} \\ \max\{0, \bar{p} - p - D_{AC}(p, p_{AC}, p_{BC})\} & \text{for} \quad p + p_y + C_y = \bar{p} \\ 0 & \text{for} \quad p + p_y \geq \bar{p} \end{cases}$$

Extended model with intermodal competition

- Demand for road services (index s) on o-d pair $y \in \{AB, BC\}$ is

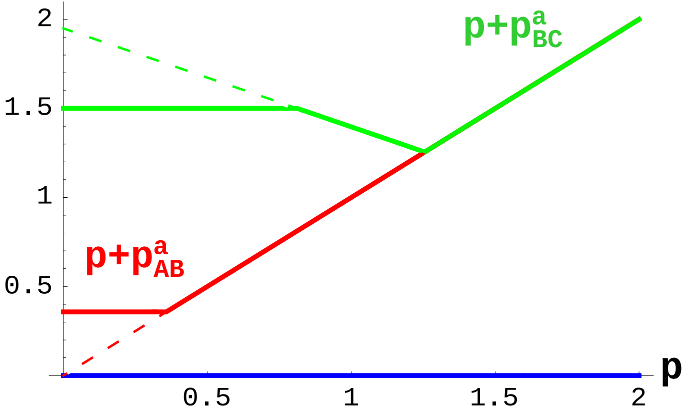
$$D^s_y(p) := \max\{0, D_y(\bar{p}, p_{AB}, p_{BC}) - D^r_y(p, p_{AB}, p_{BC})\} \quad (1)$$

- Demand for rail and road services on o-d pair $x = AC$ analog
- Environmental costs (i.e. the difference between environmental costs generated by road and rail transport services)

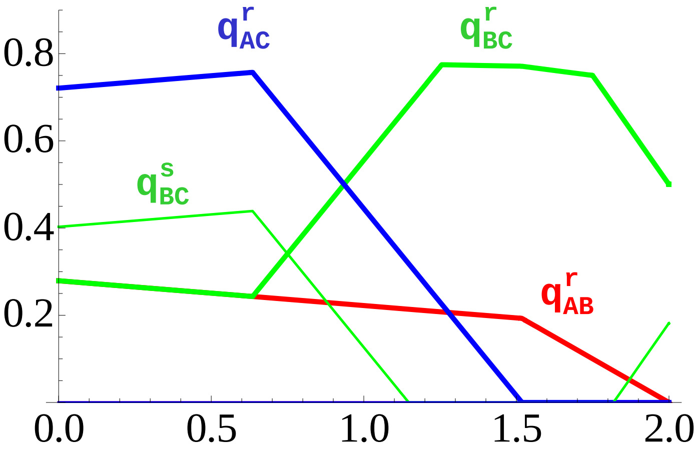
$$\phi\,(q^s_{AB} + q^s_{BC} + 2\,q^s_{AC}) \quad (2)$$

with $\phi \geq 0$

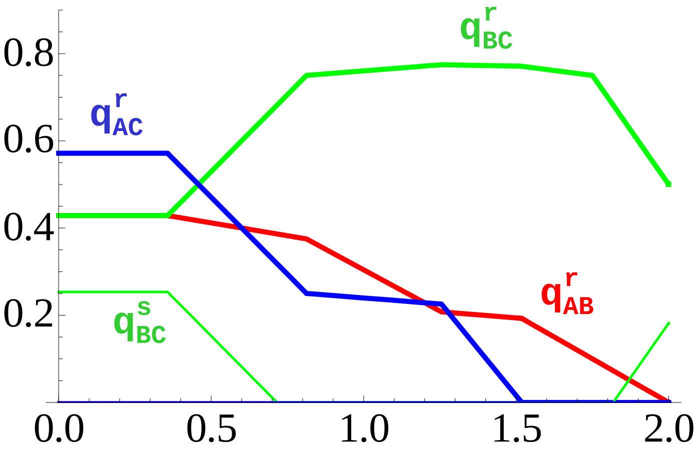
Total prices under regime a with intermodal competition and environmental externalities ($\bar{p} = 5/2$)



Road and rail services as a function of p under regime m ($\bar{p} = 5/2$)



Road and rail services as a function of p under regime a ($\bar{p} = 5/2$)



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Thank you!