Road Pricing and Access Burden Avoidance:
Evidence from an Experiment with Braess's Paradox

by

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Abstract: We design a laboratory experiment that allows for testing unregulated selfish route choice behavior in a road pricing environment where the introduction of a road toll may generate network effects or excess burdens that are known from Braess's Paradox. We consider three alternative treatments: no tolls (NOT), a Braess's Paradox generating toll (BPT), a maximum toll that does *not* generate Baress's Paradox (MAT). Among other things, our results indicate that the actual excess burden generated by the BPT-treatment is significantly higher than the excess burden generated by either the NOT- or MAT-treatment. We therefore conclude that road tolls should not exceed the MAT level. To this extent, our findings have important implications for the optimal design of road pricing schemes.

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1 Introduction

Optimal provision and financing of road infrastructure has always been a policy issue in OECD countries and elsewhere, but over the last decade this debate has gained considerable drive. Essentially, financing through vehicle or fuel taxes versus direct charging for the use of road infrastructure has been the focus of the debate (e.g. see Commission 1998, Button 2005). Charges in virtually all road pricing schemes are based either on relevant road infrastructure costs, on internalizing external costs, on budgetary needs or on a mixture of these factors, while road user behavior is usually not taken into account. This is particularly true for the impact which different charges for alternative routes may have on road user behavior. Therefore, under plausible circumstances road pricing schemes may turn out to be suboptimal because they may generate an

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excess burden. In transport economics such problems could be analyzed with Braess's Paradox (e.g. see Pickhardt 2007; Meinhold and Pickhardt 2008, 2007).

In the following, we design a laboratory experiment that allows for testing endogenous road user behavior in a road pricing environment in which Braess's Paradox may emerge. We consider three alternative treatments: no tolls (NOT), a Braess's Paradox generating toll (BPT) and a maximum toll that does *not* generate Braess's Paradox (MAT). Our results indicate that the actual excess burden generated by the BPT treatment is significantly higher than the excess burdens generated by either the NOT or MAT treatment. Among other things, we therefore conclude that road tolls should not exceed the MAT level.

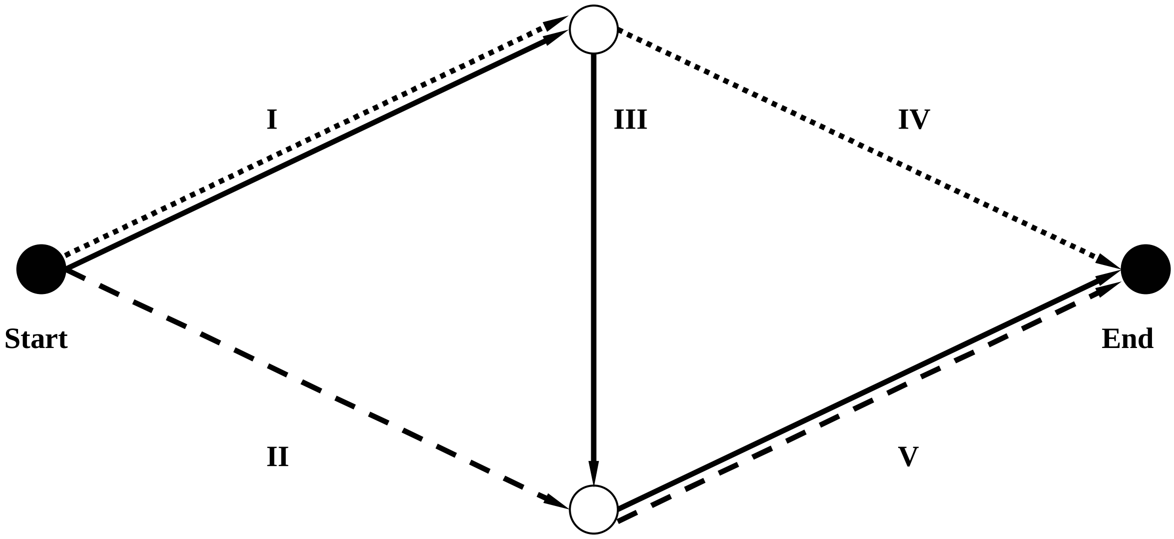
The paper is organized as follows. In the next two sections we provide some background on Braess's Paradox and the underlying model. The experimental design is introduced in section four and results are discussed thereafter. The final section concludes.

2 Braess's Paradox

Braess's Paradox is an illustration of the counterintuitive fact that removing (adding) links from (to) a network in which unregulated selfish route choice prevails may actually decrease (increase) costs incurred by agents in an equilibrium flow distribution (e.g. see Valiant and Roughgarden 2006, Korilis et al. 1999). From a game theoretical perspective Braess's Paradox is a network game that may coincide in some cases with a minority game (see Coolen 2005), for example, in the experiment we discuss here. The paradox was introduced in a seminal paper by the German mathematician Dietrich Braess in the late 1960s and has ever since received attention in transport economics, mathematical and computer sciences (Braess 1968, 2005; Murchland 1970). Since the late 1990s, however, new insights led to a fresh wave of interest in Braess's Paradox (see Pas and Principio 1997; Penchina 1997; among others). For example, Rapoport et al. (2005, 2006) conduct the first laboratory experiments with Braess's Paradox and Valiant and Roughgarden (2006) show that Braess's Paradox occurs with high probability in a natural random network and that the 'global' behavior of an equilibrium flow in a large random network is similar to Braess's original four-node example. This latter result indicates that Braess's Paradox is a relevant policy issue for large transportation networks and that it is sufficient to analyze the associated problems within small four-node examples. Therefore, in the following, we introduce Braess`s original example and the version we use in this paper.

Figure 1 shows a simple transportation network with four nodes and five links labeled *I* to *V*.

Figure 1: Braess's Paradox



Permissible directions of traffic flows are indicated by arrows. Hence, all traffic from the start node (origin) to the end node (destination) may take three alternative routes: route 1, which involves links *I* and *IV* (dotted arrows); route 2, which involves links *II* and *V* (dashed arrows); or route 3, which involves links *I*, *III* and *V* (straight arrows). Figure 1 is a slightly amended version of Braess's (1968, p. 263; 2005, p. 448) original example. Now assume that the following costs are associated with links *I* to *V*:

$$I = 2x_I, \quad (1)$$

$$II = 10 + x_{II}, \quad (2)$$

$$III = 2x_{III}, \quad (3)$$

$$IV = 10 + x_{IV}, \quad (4)$$

$$V = 2x_V, \quad (5)$$

where $x_{I\text{-}V}$ denotes the number of homogenous agents or vehicles traveling on links *I* to *V* and travel costs are measured in units of time (see also Braess (1968, p. 263) or Rapoport et al. (2005, 2006), who use somewhat different cost structures). Suppose that the total number of agents *X* traveling from start to end is given, with $\overline{X} = 6$. In this case, the equilibrium flow distribution over the three possible routes, $R_{1\text{-}3}$, is: $R_1 = R_2 = R_3 = 2$. Then, because of $x_{\mathrm{II}} = x_{\mathrm{III}} = x_{\mathrm{IV}} = 2$, and $x_{\mathrm{I}} = x_{\mathrm{V}} = 4$, each agent has travel costs of 20 units of time and total travel costs amount to (6 · 20 =) 120 units of time.

Now assume that link *III* is blocked (or removed) by an appropriate road toll or some other ruling, so that route 3 cannot be used anymore. The equilibrium flow distribution over the two remaining routes, $R_{1\text{-}2}$, is now: $R_1 = R_2 = 3$, which leads to $x_{\mathrm{I}} = x_{\mathrm{II}} = x_{\mathrm{IV}} = x_{\mathrm{V}} = 3$, and yields travel costs of just 19 units of time for each agent and total travel costs of (6 · 19 =) 114 units of time. Hence, blocking certain parts of a transportation network (link *III* in Figure 1), for example, by introducing a congestion fee, may improve both individual and overall welfare, or in other words, may represent a Pareto-improvement. In modern transport economics Braess's Paradox is used to illustrate this point (e.g. see Hagstrom and Abrams 2001; Johansson and Mattsson 1995, p. 24–25).

Now look at Braess's Paradox the other way round, that is, by assuming that link *III* does not exist and that the equilibrium flow distribution $R_1 = R_2 = 3$ prevails. Adding new infrastructure to the existing road network, that is, link *III*, leads to the seemingly paradoxical situation that rational, payoff maximizing road users will adjust their route choice in a way that leads to the new equilibrium flow distribution $R_1 = R_2 = R_3 = 2$, which is characterized by higher individual and overall travel costs in terms of time to get from start to end. This is why Braess called it a paradox, but effectively it is simply a situation in which the resulting non-cooperative Nash user equilibrium is not a Pareto-optimum.

In this paper, however, we interpret Braess's Paradox in a somewhat different way. That is, we specify the transportation network shown in Figure 1 as a two-tier road network, where links *II* and *IV* represent motorways *M* and links *I*, *III* and *V* represent highways *H*. Next we assume that cost functions (1), (3) and (5) remain unchanged, whereas the cost functions of the motorways, (2) and (4), are changed to:

$$II = x_{II}, \tag{2'}$$

$$IV = x_{IV}. \tag{4'}$$

Under these circumstances the equilibrium flow distribution will be $R_1 = R_2 = 3$ again, and because of $x_I = x_{II} = x_{IV} = x_V = 3$, this distribution yields travel costs of just 9 units of time for each agent and total travel costs of (6 · 9 =) 54 units of time. If we now assume that a toll on motorways is introduced, say in terms of 10 monetary units (with one monetary unit being equivalent to a unit of time, for simplicity), the initial cost functions (2) and (4) would re-emerge. Yet, as demonstrated above, rational and payoff maximizing agents would reconsider their route choice and the new equilibrium flow distribution over the three possible routes would be: $R_1 = R_2 = R_3 = 2$. As noted, under these circumstances, i.e., $x_{II} = x_{III} = x_{IV} = 2$ and $x_I = x_V = 4$, each agent has travel costs of 20 units of time and total travel costs amount to (6 · 20 =) 120 units of time. But this means that an excess burden of up to 26 units has emerged, because total costs (120) minus total revenue (4 · 10=40) minus minimum costs (54) amounts to 26.

In the following section, we generalize the underlying model and show how to calculate a maximum road toll that does not cause Braess's Paradox and the associated excess burden, subject to the prevailing parameter values. From a welfare perspective, road tolls should not exceed such a level. In fact, the experimental design which we introduce here allows for testing whether such a maximum toll would really avoid an excess burden in practice, that is, if real human beings are faced with unregulated selfish routing in a road network where Braess's Paradox may emerge. Finally, it must be stressed that Braess's Paradox is neither driven by the linearity of the cost functions nor by the symmetry of the cost functions adopted here nor by the fixed demand for travel (number of agents), but simply by the fact that a basic network (Routes 1 and 2) is transformed into an augmented network (Routes 1, 2 and 3) (e.g. see Rapoport et al. 2005, p. 7; Roughgarden 2005, p. 51–52). According to Penchina (1997, p. 380) the necessary and sufficient conditions which such a network structure has to fulfill are: (i) the network (but not each link) must have both fixed and variable user costs, (ii) the two routes of the basic network must have an opposite order of appearance of the links dominated by fixed versus variable costs and, (iii) the fixed costs of the traverse link in the augmented network must be lower than the difference in fixed costs between the links dominated by fixed and variable user costs.

3 The Model

To discuss some essential aspects of the underlying model, we assume that each of the five links is associated with the following generalized, nonnegative, nondecreasing linear cost function: $\gamma_i + \delta_i \cdot x_i$, with $i = I, ..., V$ and $\gamma_i, \delta_i \in \Re_0^+$ and $x_i \in N_0$. As noted, we specify the transportation network shown in Figure 1 as a two-tier road network, where links *II* and *IV* represent motorways *M* and links *I*, *III* and *V* represent highways *H*. For simplicity, we assume that parameters γ and δ are identical on each motorway and take the value γ_M and δ_M, respectively. Likewise, we assume that parameter δ is identical on each highway and takes the value δ_H and that parameter γ is set equal to zero on highways. Typically, δ would be measured in units of time and would depend on a number of other parameters associated with link *i* (such as the number of lanes, surface conditions, etc.), but for simplicity we assume that δ can be expressed in monetary units. Blayac (2006) discusses some methods to express travel time in monetary units. Hence, the parameter δ captures the technical difference between the two tiers, highway and motorway. Under normal circumstances, $\delta_H > \delta_M$ would hold, indicating that a highway is less efficient than a motorway in terms of travel time or costs. In contrast, parameter γ is an intrinsically monetary measure that may be set by the provider of the road network, which could be either the government or a private firm.

Based on the generalized versions of (1) to (5) and the generalizations regarding the parameters γ and δ, the costs C_j of using route *j*, with *j* = 1, 2, 3, can be calculated by adding up the costs associated with each arrow involved in using a specific route (see Figure 1). Rearranging terms, route costs can be written as follows:

$$C_1(x_1, x_3) = \gamma_M + (\delta_M + \delta_H)x_1 + \delta_H x_3, \tag{6}$$

$$C_2(x_2, x_3) = \gamma_M + (\delta_M + \delta_H)x_2 + \delta_H x_3, \tag{7}$$

$$C_3(x_3) = \delta_H X + 2\delta_H x_3, \tag{8}$$

where x_j is the total number of agents traveling on route j and X is the total number of agents using the network, with $X \geq x_j$ and $X = x_1 + x_2 + x_3$. Route cost functions (6) to (8) describe the strategy space and, therefore, allow for analyzing endogenous route choice behavior in a road pricing environment in which Braess's Paradox may emerge. In any such analysis identifying allocations that either represent a Nash user equilibrium, a minimum of total costs, a revenue maximum, and a combination of these three, would be of interest because knowing such values would allow for identifying the excess burden associated with certain route choice allocations.

Let us first consider Nash user equilibria. Suppose that every *used* route has the same costs and every *unused* route has costs greater than or equal to those of the used routes. This situation characterizes a Nash user equilibrium because no rational, payoff maximizing agent has an incentive to change its route choice. Given the assumptions made so far, it follows from (6) and (7) that $C_1 = C_2$ holds, if $x_1 = x_2$. Therefore, it is sufficient to examine the following three route cost relations to derive the conditions for all conceivable Nash user equilibria: $C_1 = C_3$, $C_1 \leq C_3$, and $C_1 \geq C_3$. Regarding the latter two, it immediately follows that if $C_1 \leq C_3$ holds, allocation $x_1 = x_2 = X / 2$ and $x_3 = 0$ emerges in equilibrium, and that if $C_1 \geq C_3$ holds, allocation $x_1 = x_2 = 0$ and $x_3 = X$ results in equilibrium. Hence, applying (6) to (8) to the three relevant cost relations and rearranging yields the following three conditions of which one must hold, if the flow distribution is a Nash user equilibrium.

$$X \geq 2\frac{\gamma_M}{(\delta_H - \delta_M)}, \qquad \Rightarrow \qquad x_1 = x_2 = \frac{X}{2} \text{ and } x_3 = 0, \tag{9}$$

$$X \leq \frac{\gamma_M}{2\delta_H}, \qquad \Rightarrow \qquad x_1 = x_2 = 0 \text{ and } x_3 = X \quad , \tag{10}$$

$$\frac{\gamma_M}{2\delta_H} \leq X \leq 2\frac{\gamma_M}{(\delta_H - \delta_M)}, \qquad \Rightarrow \qquad x_1 = \frac{2\delta_H X - \gamma_M}{(\delta_M + 3\delta_H)}. \tag{11}$$

The Nash equilibrium flow distribution is then calculated by first checking which of the three relations on the left hand side actually holds with respect to given parameter values. Next, x_1 is calculated by applying the relevant formula on the right hand side. Regarding (11) values for x_2 and x_3 follow from any value calculated for x_1, according to $x_1 = x_2$ and $x_3 = X - 2x_1$. We denote any such x_j as $\breve{x}_j$. In some cases, however, (11) may either lead to $\breve{x}_1 < 0$, or $\breve{x}_1 > X/2$ which implies $\breve{x}_3 < 0$. But as negative values for $\breve{x}_j$ are not permitted, in these cases either (9) or (10) must hold. In particular, for all $\breve{x}_1 > X/2$, implying $\breve{x}_3 < 0$, the next permissible solution is $\breve{x}_3 = 0$, which implies that condition (9) holds, and for all $\breve{x}_1 < 0$ the next permissible solution is $\breve{x}_1 = \breve{x}_2 = 0$, which implies that condition (10) holds. To summarize, for any set of given values of the parameters γ, δ, and X, a Nash user equilibrium flow distribution exists and can be calculated from (11). That is, if $0 \leq \breve{x}_1 \leq X/2$ holds, an interior solution results where in equilibrium all three routes are used (mixed strategy equilibrium), $\breve{x}_j > 0, \forall j$. Yet, if $\breve{x}_1 < 0$ holds, a corner solution results where in equilibrium only one route is used, with $\breve{x}_1 = \breve{x}_2 = 0$ and $\breve{x}_3 = X$. Likewise, if $\breve{x}_1 > X/2$ holds, another corner solution results where in equilibrium only two routes are used, with $\breve{x}_1 = \breve{x}_2 = X/2$ and $\breve{x}_3 = 0$. Moreover, it is worth noting that (11) compares to equation (4a) in Pas and Principio (1997, p. 269), once the necessary changes in notation and assumptions are made (in our terms Pas and Principio define the traverse link III as a motorway and not as a highway).

Next we derive the total cost minimum. In a first step, we calculate total costs C as the sum of costs incurred by each agent:

$$C(x_1, x_2, x_3) = x_1 \cdot C_1(x_1, x_3) + x_2 \cdot C_2(x_2, x_3) + x_3 \cdot C_3(x_3). \qquad (12)$$

Since $C(\cdot)$ assumes a minimum at $x_1 = x_2$ (see appendix A for a proof), we substitute equations (6) to (8) into (12) and replace x_2 by x_1. Because $x_1 = x_2$ implies $x_3 = X - 2x_1$, we can also replace x_3 by $X - 2x_1$. Hence, total costs are a function of x_1 only:

$$\hat{C}(x_1) = 2(\delta_M + 3\delta_H)x_1^2 + 2(\gamma_M - X4\delta_H)x_1 + 3\delta_H X^2. \qquad (13)$$

Assuming that (13) is twice continuously differentiable, Meinhold and Pickhardt (2007) derive the FOC for a local minimum as:

$$\frac{\partial \hat{C}}{\partial x_1} = 4(\delta_M + 3\delta_H)x_1 + 2(\gamma_M - 4\delta_H X) \overset{!}{=} 0 . \tag{14}$$

Rearranging yields:

$$\hat{x}_1 = \frac{4\delta_H X - \gamma_M}{2(\delta_M + 3\delta_H)} . \tag{15}$$

Inspection of (14) shows that the second order condition for a local minimum also holds, with $4(\delta_M + 3\delta_H) > 0$, because δ_H and δ_M are always positive. But again, equation (15) may either lead to negative values for x_1 or to $x_1 \geq X/2$, which implies negative values for x_3. Yet, it can be shown that the minimum of total costs occurs at $x_1 = x_2 = 0$ and $x_3 = X$ for negative values of x_1, or at $x_1 = x_2 = X/2$ and $x_3 = 0$, for negative values of x_3.

We now turn to revenues. Although each agent traveling on the road network may have to bear both the γ and δ cost component, only γ generates revenue for the network provider. Moreover, it is possible that an increase of γ induces route choice changes which result in a decrease of the δ cost component and an overall decrease of total costs (see section 2). To avoid such a situation, the total cost minimum must coincide with the minimum of the δ cost component. The latter can be calculated by setting $\gamma_M = 0$ in (15). Moreover, we now confine the analysis to cases in which $\delta_H > \delta_M$ holds. As already noted, this situation describes realistic cases because highways are usually less efficient than motorways in terms of travel time or costs. Under these circumstances, however, it follows from (15) that x_1 is larger than $X/2$, because of $2\delta_H / (\delta_M + 3\delta_H) > 1/2$. This implies that a Nash user equilibrium, which simultaneously represents a minimum of total costs and a minimum of variables costs (i.e. the δ cost component), requires $x_1 = x_2 = X/2$ and $x_3 = 0$. Hence, to calculate the maximum γ_M, (11) is to be solved for γ_M. Replacing x_1 by $X/2$ yields:

$$\gamma_M^* = \frac{X}{2}(\delta_H - \delta_M)\,. \tag{17}$$

According to (17), the maximum toll that can be charged without causing Braess's Paradox is determined by the efficiency gap between the two network tiers ($\delta_H - \delta_M$), and total traffic volume X. In this context, it is worth noting that the r.h.s of (17) indicates a certain similarity with the well known Samuelson condition for the optimal provision of public goods (e.g. see Batina and Ihori 2005, p. 10; Pickhardt 2006). Furthermore, it must be stressed that (17) applies only in cases where X and x_i are steady and large as in Meinhold and Pickhardt (2007; 2008, p. 152). If X and x_i are discrete and small, as in the experiment, the emergence of Braess's Paradox is avoided if $C_1(X/2, 0) < C_3(1)$ holds. Inserting these values in (6) and (8) and rearranging gives:

$$\gamma_M^* < \frac{X}{2}(\delta_H - \delta_M) + 2\delta_H\,. \tag{18}$$

Hence, for the parameter values $X = 6$, $\delta_H = 2$, $\delta_M = 1$, (18) yields 6 < 7, so that 6 is the largest integer that does not cause Braess's Paradox. Yet, if X is getting large, the first term on the r.h.s. of (18) dominates and the second term on the r.h.s. can be neglected, so that (17) emerges. Maximum revenue $R_{max.}$, subject to avoiding Braess's Paradox, is then given by:

$$R_{\max} = \gamma_M^* X\,. \tag{19}$$

We can now calculate the excess burden by first separating (13) with respect to the two cost components δ and γ, which yields:

$$\hat{C}_\delta(x_1) = 2(\delta_M + 3\delta_H)x_1^2 - 8\delta_H X x_1 + 3\delta_H X^2\,, \tag{20}$$

$$\hat{C}_{\gamma}(x_1) = 2\gamma_M x_1, \tag{21}$$

with (19) and (21) coinciding for $x_1 = X/2$. From a welfare perspective it is worth noting that costs associated with (21) may not, or may not fully, represent a welfare loss. This is because costs due to the γ component are fully transformed into revenue for the network provider and, therefore, may contain the profit of the network provider. The latter, however, does not represent a cost for society. In an extreme setting, as in our experiment, the entire revenue may be profit for the network provider. In practice, this could be the case if provision and maintenance of the road network is already financed via taxes and a road toll is charged simply because of budgetary needs. Under these circumstances, the excess burden is defined as the costs of the δ component that arise in excess of the minimum costs of the δ component that must be born for passing the network from origin (start) to destination (end). Now recall that for $\delta_H > \delta_M$ the minimum of the δ cost component occurs at $x_1 = x_2 = X/2$ and $x_3 = 0$ and, therefore, the maximum of the δ cost component occurs at $x_1 = x_2 = 0$ and $x_3 = X$. The maximum excess burden is then calculated from $\hat{C}_{\delta}(0) - \hat{C}_{\delta}(X/2)$, according to (20). Rearranging yields:

$$EB^{\max} = \frac{(5\delta_H - \delta_M)X^2}{2}. \tag{22}$$

Likewise, for a Nash equilibrium that represents an interior solution, the excess burden is calculated from $\hat{C}_{\delta}(\breve{x}_1) - \hat{C}_{\delta}(X/2)$, where $\breve{x}_1$ is a nonnegative integer calculated from (11). Rearranging gives:

$$EB = \frac{(5\delta_H - \delta_M)X^2}{2} - (2\gamma_M + 4\delta_H X)\breve{x}_1. \tag{23}$$

Further, from a theoretical perspective, it is interesting to see how our concept of excess burden is related to the ‘price of anarchy’ discussed by Roughgarden (2005, pp. 22–48) and others. The latter is defined “as the worst-possible ratio between the cost of a flow at Nash equilibrium and that of an optimal flow” (2005, p. 22). In other words, the ‘price of anarchy’ measures “the extent to which competition approximates cooperation” (2005, p. vii). In our terms, in the BPT-treatment, according to (12) it would be the ratio of $C^{Nash}(2,2,2)$ / $C^{*}(3,3,0)$, that is, 120/114 ≈ 1.05. Yet, because of our explicit distinction of two cost components and provided that the γ cost component contains a positive profit, this ratio does not give a good measure of the “worst-possible loss of social welfare from selfish routing” (Roughgarden 2005, p. 3). Rather, the worst-possible loss occurs by removing the profit element that may be contained in the γ cost component. Hence, in our extreme setting where all γ related costs are profit, removing the entire γ cost component in both cases gives (120-40) / (114-60) = 80/54 ≈ 1.48. Here the difference between 80 and 54 is the excess burden of 26 tokens and, therefore, the 20 tokens of costs that are shifted from the γ component (optimal solution, $C^{*}(3,3,0)$) to the δ component (Nash solution, $C^{Nash}(2,2,2)$) are now duly taken into account. In fact, this shows that the ‘price of anarchy’ might be much higher in practice and might even go beyond the 4/3 upper bound which is proofed for linear cost functions (see Roughgarden 2005, p. 51-57).

4 Experimental Design

All sessions reported here were conducted in a non-computerized ‘paper and pencil’ fashion. Each session was done in the same laboratory room and with six subjects participating in the experiment plus one additional subject that served as an assistant to the instructor. Subjects were drawn from an introductory economics class attended by about 600 first year students of economics, social sciences and engineering. Each subject received a show-up fee of five Euro plus a proportion of the token earnings accumulated over all rounds of the experiment. The proportions varied from three Eurocent per token for the two subjects with the highest token earnings, to two Eurocent for the two subjects with the third and fourth highest earnings and one Eurocent for the two subjects with the lowest earnings. Total Euro payoffs ranged from about eight Euro to 18 Euro. The assistant subject received ten Euro as a lump sum payoff. Subjects were paid individually right after their session. A typical session lasted approximately 75 minutes.

4.1 Procedure

Once all subjects were present, the instructor asked for a volunteer to act as assistant. Thereafter, the remaining six subjects were seated in a way that they would not be able to look at each other and it was made clear that any kind of communication was strictly forbidden during the entire session. Each subject received an introductions sheet, a filled-in earnings record sheet showing some examples, three playing cards with face values 1, 2, 3, and a pen. Subjects were told to read the introduction sheet quietly at their own pace. When everybody had finished readings the instructor summarized the introduction again and then asked some questions on cost and payoff calculations and about how to fill in the earnings record sheet. Once all answers were correct and subjects had no further questions, the first block of the session started with the distribution of an instructions sheet and an earnings record sheet. The introductions sheet and the instructions for the first block were identical in each session (see appendix B).

According to the instructions subjects assume the role of freight forwarding agents and it is their job to forward some good *G* from the start node to the end node as indicated in Figure 1, where the two nodes may represent different cities. By forwarding good *G* each agent can earn 40 tokens per round. Hence, each agent maximizes its payoff by choosing the route which is associated with the lowest costs (i.e. route 1, 2, or 3). Subjects indicate their route choice by placing a playing card with the corresponding face value (i.e. 1, 2, 3) face down in front of them. The instructor then collects all cards and announces the number of agents traveling on each route. At this point it is the job of the assistant to verify and supervise this procedure. Next, subjects fill-in their earnings record sheets by noting their own route choice, the overall route choices, the costs associated with the route they have chosen themselves and their net payoff (see appendix B). While the subjects complete their earnings record sheets, the instructor returns to them their own cards, again face down. Thereafter a new round starts. In each block six rounds were played one after the other. Then a new block started with new instructions and earnings record sheets distributed at the beginning. In all sessions the first block was identical and represented a NOT-treatment.

In the second block a toll on motorways was introduced. In sessions A to E this toll was ten tokens, which *c.p.* represents a BPT treatment and in sessions F to J the toll was six tokens, which *c.p.*

represents a MAT treatment. In the third block, sessions A to E, the toll was reduced to six tokens and in sessions F to J it was increased to ten. Hence, in sessions A to E the treatment order was NOT-BPT-MAT, but NOT-MAT-BPT in sessions F to J.

4.2 Treatments

All three treatments are based on the generalized route costs functions (6) to (8). The following parameter are fixed in all three treatments and take the values: $X = 6$, $\delta_H = 2$, $\delta_M = 1$. The toll on motorways, however, may be either: $\gamma_M = 0$ (NOT-treatment), $\gamma_M = 6$ (MAT-treatment), or $\gamma_M = 10$ (BPT-treatment). To facilitate the understanding of route choice consequences, as well as cost and payoff calculations, each instructions sheet contains not only Figure 1, but also a table of costs, showing the costs associated with every conceivable route choice constellation, and the relevant route cost formulas. Table 1 shows the costs for the first block, NOT-treatment. The construction of Table 1 reflects the fact that the costs of route 3 are independent of how many agents have chosen either route 1 or route 2 (see (8)), whereas the costs of route 1 and 2 depend on the number of agents who have selected route 3 (see (6) and (7)). For example, say three subjects choose route 1, another three route 2, and nobody chooses route 3. It follows from (9) that this allocation represents a Nash-equilibrium. Every agent has costs of 9 tokens (first column, third row), which also represents the minimum of total costs per round ($6 \cdot 9 =$) 54 tokens and the minimum of the δ cost component. Hence, unilateral deviation would not be beneficial (see Table 1). In fact, if two subjects choose route 1, three route 2 and one route 3, costs are 8, 11, and 16 tokens, respectively, and total costs amount to 65 tokens. Similar tables can be constructed for the MAT- and BPT-treatments (see appendix B).

In the BPT-treatment, for example, it follows from (11) that $R_1 = R_2 = R_3 = 2$ is now the Nash-equilibrium allocation. But because of (15) the minimum of total costs continues to occur at $R_1 = R_2 = 3$. Therefore, an excess burden emerges, which in equilibrium amounts to 26 tokens per round, that is, ($6 \cdot 20 =$) 120 tokens total costs, minus total revenue ($4 \cdot 10 =$) 40, minus the minimum of total costs of the NOT-treatment ($6 \cdot 9 =$) 54 tokens. Alternatively, the excess burden may be calculated directly from (23).

Table 1: Costs associated with route choices in the first block (NOT treatment)

		Route 1 and 2						Route 3
		Number of persons who chose Route 3						
		0	**1**	**2**	**3**	**4**	**5**	
Number of persons who chose the same route	**1**	3	5	7	9	11	13	16
	2	6	8	10	12	14		20
	3	9	11	13	15			24
	4	12	14	16				28
	5	15	17					32
	6	18						36

Notes: Bold numbers in the row represent the number of agents on route 3 (x_3), bold numbers in the column on the far l.h.s. represent the number of agents traveling on route 1 (x_1) or 2 (x_2), all other numbers denote costs in terms of tokens for given parameter values X=6, $\gamma_M = 0$, $\delta_H = 2$, $\delta_M = 1$.

Regarding the MAT-treatment, (11) and (15) indicate that the Nash-equilibrium allocation, the minimum of total costs and the minimum of the δ cost component again coincide at $R_1 = R_2 = 3$. Hence, in equilibrium no excess burden emerges, because total costs are (6 · 15 =) 90 tokens, minus total revenue (6 · 6 =) 36 tokens, minus the minimum of total costs of the NOT-treatment (6 · 9 =) 54 tokens. As noted, for the given parameter values, $\gamma_M = 6$ is the maximum toll on motorways that can be charged without causing some excess burden due to Braess's Paradox (see (18)) and 36 tokens is the associated maximum revenue according to (19).

Table 2: Nash-Equilibrium values per treatment and block

	Nash	**Costs**	**Revenue**	**Excess Burden**
Treatment	(x_1/x_2/x_3)	(tokens)	(tokens)	(tokens)
NOT	18/18/0	324	-	0
BPT	12/12/12	720	240	156
MAT	18/18/0	540	216	0

Notes: (x_1/x_2/x_3) *denotes the number of agents on route 1, 2, and 3,respectively; all other numbers denote costs, revenue or excess burden in terms of tokens; all figures apply on a per block basis, i.e., are aggregated over 6 rounds.*

Table 2 summarizes relevant equilibrium values per block of six rounds for the three treatments. In fact, these results would be expected if all agents are fully informed, rational and payoff maximizing individuals.

Finally, it is worth noting how our design differs from the three designs of Rapoport et al. (2005) and the design of Rapoport et al. (2006). Experiment 1 of Rapoport et al. (2005) and Rapoport et al. (2006) are close to our design because they use the same four-node, five links network topology. In the first block of Rapoport et al. (2005), first experiment, subjects are faced with the basic network (Figure 1 without link III) and in their second block Braess's Paradox is generated by *adding* link III (see Figure 1). In three groups of 18 subjects did this 'ADD' condition with 40 rounds per block and another three groups of 18 subjects played the two blocks in reverse order (DELETE condition). In contrast, Rapoport et al. (2006) use just the five link topology and generated Braess's Paradox with different group sizes. Hence, apart from differences in the cost structure, the number of participating subjects and the number of rounds per block, there are three major differences: (i) Rapoport et al. do not distinguish between a NOT and a MAT treatment, which in their notation both compare to the basic network, (ii) Rapoport et al. focus primarily on whether Nash equilibrium flow distributions actually emerge over time and on aspects of learning, whereas our focus is on the actual excess burden that emerges in each treatment, (iii) Rapoport et al. either use a given cost structure for each link and then generate Braess's Paradox by adding or removing a link, as in Braess's original version (Figure 1, link III), or by a variation of the group size (network demand), in contrast, we use a given four-node, five links network with given network demand and generate Braess's Paradox by changing the cost structure on certain links. Hence, the Rapoport et al. (2005) design is suitable to analyze the effects of changes to the network infrastructure, whereas our design is more suitable to analyze the effects which the introduction or removal of user charges may have. Further, in their experiments 2 and 3, Rapoport et al. (2005) use a more complex topology (see also Roughgarden 2005) and asymmetric endowments (experiment 3) to test for the robustness of Braess's Paradox in more complex decision environments.

5 Results

Table 3 summarizes the results of sessions A to J on a per block basis, where the last five sessions are displayed in italics to indicate the different order of treatments. Total actual costs represent the actual costs per block, which are calculated by applying the route choice allocation ($x_1/x_2/x_3$) is each of the six rounds to the relevant cost table or cost function. Actual revenue is calculated by multiplying the relevant toll on motorways (6 (MAT), or 10 (BPT)) with the number of agents using route 1 and 2. The actual excess burden emerges from calculating actual costs minus actual revenue minus minimum of total costs (i.e. 324 tokens).

Table 3: Summary of session results

Session	Block	x_1	x_2	x_3	Actual Costs	Actual Revenue	Actual Excess Burden
	#1 (NOT)	18	16	2	430	-	106
A	#2 (BPT)	12	11	13	769	230	215
	#3 (MAT)	21	12	3	585	198	63
	#1 (NOT)	17	19	0	402	-	78
B	#2 (BPT)	12	12	12	774	240	210
	#3 (MAT)	13	21	2	626	204	98
	#1 (NOT)	16	19	1	425	-	101
C	#2 (BPT)	14	12	10	744	260	160
	#3 (MAT)	11	18	7	607	174	109
	#1 (NOT)	17	17	2	394	-	70
D	#2 (BPT)	14	13	9	725	270	131
	#3 (MAT)	16	15	5	641	186	131
	#1 (NOT)	15	21	0	426	-	102
E	#2 (BPT)	8	16	12	792	240	228
	#3 (MAT)	19	13	4	648	192	132
	#1 (NOT)	*11*	*24*	*1*	*407*	-	*83*
F	*#2 (MAT)*	*19*	*13*	*4*	*648*	*192*	*132*
	#3 (BPT)	*10*	*14*	*12*	*746*	*240*	*182*
	#1 (NOT)	*20*	*13*	*3*	*379*	-	*55*
G	*#2 (MAT)*	*17*	*14*	*5*	*583*	*186*	*73*
	#3 (BPT)	*11*	*18*	*7*	*727*	*290*	*113*
	#1 (NOT)	*12*	*22*	*2*	*412*	-	*88*
H	*#2 (MAT)*	*19*	*13*	*4*	*598*	*192*	*82*
	#3 (BPT)	*14*	*10*	*12*	*818*	*240*	*254*
	#1 (NOT)	*13*	*23*	*0*	*414*	-	*90*
I	*#2 (MAT)*	*17*	*13*	*6*	*598*	*180*	*94*
	#3 (BPT)	*17*	*8*	*11*	*765*	*250*	*191*
	#1 (NOT)	*15*	*18*	*3*	*387*	-	*63*
J	*#2 (MAT)*	*21*	*13*	*2*	*598*	*204*	*70*
	#3 (BPT)	*17*	*13*	*6*	*716*	*300*	*92*

Notes: ($x_1/x_2/x_3$) *denotes the number of agents on route 1, 2, and 3, all other numbers denote costs, revenue or excess burden in terms of tokens.*

Inspection of Table 3 makes it clear that in none of the sessions a Nash-equilibrium has prevailed over an entire block of six rounds, because in each block the actual total costs differ from those calculated in Table 2. Moreover, with the exception of one case (session B, #2) the actual aggregate route choice allocation differs from the aggregate Nash-allocation, which is (18/18/0) for the NOT- and MAT-treatment and (12/12/12) for the BPT-treatment. However, in all sessions aggregate route choices are fairly close to the expected results. To this extent, the results compare well to those of Rapoport et al. (2005), experiment 1, who did 40 rounds per treatment and also find that equilibrium allocations are approached. Subjects in the Rapoport et al. (2006) design reached equilibrium even twice as quickly as in the 2005 design. Further, in their more complex experiment 3, Rapoport et. al. (2005) find that the majority of their subjects learn to avoid non-equilibrium routes after less than 10 rounds, which also compares well to our findings, as they are based on 6 rounds plus the two introduction cases.

Deviations from the aggregate Nash-allocation also explain why a positive actual excess burden emerges in both the NOT- and MAT-treatment. As in both cases the Nash-allocation (18/18/0) coincides with the minimum of total costs and the minimum of the δ cost component, any other allocation necessarily generates higher costs and, therefore, an excess burden. Hence, the actual excess burdens in the NOT- and MAT-treatments arise from imperfections in route choice coordination. In contrast, the excess burden of the BPT-treatment shown in Table 2 is due to a reallocation induced by an inappropriately high toll. Moreover, as the Nash-allocation of the BPT-treatment does not coincide with the minimum of total costs, imperfections in route choice coordination may lead to either a higher or a lower actual excess burden in comparison with the excess burden of the Nash-allocation, that is, 156 tokens (see Table 2). Indeed, Table 3 shows that in three sessions (D, G, J) the actual access burden has been lower than 156 tokens, but higher in the remaining seven sessions. For this reason, it cannot be ruled out entirely that some overlapping occurs in the sense that the actual excess burden of either the NOT- or MAT-treatment exceeds the excess burden of the BPT-treatment.

From a policy perspective, it is therefore important to test whether the theoretically expected result, that is, the BPT-treatment generates a higher excess burden than both the NOT- and MAT-treatment (see Table 2), still holds if selfish route choices of real human beings are considered. In particular, three questions are of interest:

(i) is the actual excess burden of the BPT-treatment higher than that of the NOT-treatment,

(ii) is the actual excess burden of the BPT-treatment higher than that of the MAT-treatment,

(iii) are the excess burdens of the NOT- and MAT-treatment identical?

To answer these questions, we apply a Wilcoxon signed ranks test to sessions A to E and F to J. Moreover, we test for the presence of path dependencies in the two sets of sessions with a Wilcoxon-Mann-Whitney rank-sum test. Regarding the latter, Table 4 provides the rank-sums for each set of sessions, where the excess burden (EB) per session and treatment is taken from Table 3. In particular, we test the null hypothesis, *the NOT (MAT, BPT) excess burdens of the two populations A to E and F to J have the same distribution (H0)*, against the alternative hypothesis, *the NOT (MAT, BPT) excess burden of the A-E population is higher than that of the F to J population (H1)*. According to Siegel and Castellan (1988, Table J, p. 341), for m=5 and n=5, the rank-sums 34, 32 and 31 yield a probability of 0.11, 0.21 and 0.27, respectively, for a one-sided test. Hence, for each treatment we cannot reject H0 at the ten percent level and conclude that in each case the two populations have the same distribution. Therefore, we can also conclude that the results of the BPT and MAT treatments do not depend on the order in which they are performed. This in turn implies that we can analyze the two session pairs both separately and jointly.

Table 4: Wilcoxon-Mann-Witney rank-sum test

Session	**A**	**B**	**C**	**D**	**E**	*F*	*G*	*H*	*I*	*J*	**Rank-sum**
EB (NOT)	106	78	101	70	102	*83*	*55*	*88*	*90*	*63*	-
Rank	10	4	8	3	9	*5*	*1*	*6*	*7*	*2*	34 (*21*)
EB(MAT)	63	98	109	131	132	*132*	*73*	*82*	*94*	*70*	-
Rank	1	6	7	8	9.5	*9.5*	*3*	*4*	*5*	*2*	31.5 (*23.5*)
EB (BPT)	215	210	160	131	228	*182*	*113*	*254*	*191*	*92*	-
Rank	8	7	4	3	9	*5*	*2*	*10*	*6*	*1*	31 (*24*)

Notes: EB denotes excess burden.

We now address the three questions mentioned above. Table 5 summarizes the differences between the excess burdens of the three relevant comparisons (BPT vs. NOT, BPT vs. MAT and MAT vs. NOT) on a per session basis and provides the rank-sums.

Table 5: Wilcoxon signed ranks test

Session	**A**	**B**	**C**	**D**	**E**	*F*	*G*	*H*	*I*	*J*	**Sum of Positive Ranks**	
(BPT-NOT)	109	132	59	61	126	*99*	*58*	*166*	*101*	*29*	A to E	*F to J*
Rank	3	5	1	2	4	*3*	*2*	*5*	*4*	*1*	15	*15*
Rank	7	9	3	4	8	*5*	*2*	*10*	*6*	*1*	55	
(BPT-MAT)	152	112	51	0	96	*50*	*40*	*172*	*97*	*22*	A to E	*F to J*
Rank	4	3	1	-	2	*3*	*2*	*5*	*4*	*1*	10	*15*
Rank	8	7	4	-	5	*3*	*2*	*9*	*6*	*1*	45	
(MAT-NOT)	-43	20	8	61	30	*49*	*18*	*-6*	*4*	*7*	A to E	*F to J*
Rank	(4)	2	1	5	3	*5*	*4*	*(2)*	*1*	*3*	11 (4)	*13 (2)*
Rank	(8)	6	4	10	7	*9*	*5*	*(2)*	*1*	*3*	45 (10)	

Notes: (BPT-NOT) denotes excess burden of the BPT-treatment minus the excess burden of the NOT-treatment. (BPT-MAT) and (MAT-NOT) denote corresponding differences. Ranks in parenthesis denote negative ranks or the sum of negative ranks.

To answer the first question of interest, we test the null hypothesis, *the excess burdens of the BPT and NOT treatments are equivalent (H0),* against the alternative hypothesis, *the BPT treatment generates a higher excess burden than the NOT treatment (H1).* According to Siegel and Castellan (1988, Table H, p. 332), for N=5, rank-sum 15 yields a probability of 0.03 for a one-sided test. Hence, we can reject H0 for both the A to E and the F to J set at the five percent level and conclude that the BPT treatment generates a higher excess burden than the NOT treatment. Further, if we consider the entire set of sessions A to J, for N=10, rank-sum 55 yields a probability of 0.001 for a one-sided test. Therefore, we can reject H0 at the 1 percent level and confirm that the BPT treatment generates a higher excess burden than the NOT treatment.

Regarding the second question, we test the null hypothesis, *the excess burdens of the BPT and MAT treatments are equivalent (H0),* against the alternative hypothesis, *the BPT treatment generates a higher excess burden than the MAT treatment (H1).* In this case, however, we get different results for the two sets. For the A to E set, with N=4 due to the tie rule, rank-sum 10 yields a probability of 0.06

for a one-sided test, whereas for the F to J set, with N=5, rank-sum 15 yields again a probability of 0.03 for a one-sided test. Hence, we can reject H0 for the A to E set only at the ten percent level, but at the five percent level for the F to J set and, thus, conclude that the BPT treatment generates a higher excess burden than the MAT treatment. Moreover, if we consider the entire set of sessions A to J, for N=9, rank-sum 45 yields a probability of 0.002 for a one-sided test. Hence, despite the somewhat weaker result of the A to E set, we can indeed reject H0 at the 1 percent level and conclude that the BPT treatment does generate a higher excess burden than the MAT treatment.

Next, with respect to the third question, we test the null hypothesis, *the excess burdens of the NOT and MAT treatments are equivalent (H0)*, against the alternative hypothesis, *the MAT treatment generates a higher excess burden than the NOT treatment (H1)*. Results for the two sub-sets now reveal some contradictions. For the A to E set, with N=5, rank-sum 11 yields a probability of 0.2188 for a one-sided test. Thus, we cannot reject H0 at the ten percent level and conclude that the excess burden of the MAT- and NOT-treatments are equal. In contrast, for the F to J set, with N=5, rank-sum 13 yields a probability of 0.09 for a one-sided test. Hence, in this case we can reject H0 at the ten percent level and conclude that the MAT treatment generates a higher excess burden than the NOT treatment. In fact, this difference in the results of the two sub-sets may be explained by the toll prevailing in the previous treatment. If the MAT toll represents a decrease compared to the toll of the preceding treatment, H0 is not rejected, but if the MAT toll represents an increase, H0 is rejected. This conjecture, however, needs to be tested in additional sessions or experiments as it is not in line with the previous result of path-independence. Yet, regarding the entire set of sessions A to J, for N=10, rank-sum 45 yields a probability of 0.04 for a one-sided test. Therefore, for the entire set of sessions we can reject H0 at the five percent level and conclude that the excess burden of the MAT is higher than the excess burden of the NOT treatment.

In the light of this result, it may be of interest to run the experiment *ceteris paribus* with the following alternative toll structures: $\gamma_M = 0$ [0] (NOT-treatment), $\gamma_M = 4$ [3] (MAT-treatment), and $\gamma_M = 8$ [7] (BPT-treatment). In these cases, the maximum toll difference between the steady and the discrete case (see (17) and (18)), i.e., 3 versus 6, is less [not] exploited and the BPT toll would be just above [or equal] to the level of indifference (i.e. 7), while the distance of 4 between the MAT and BPT toll would be maintained. We conjecture that the actual excess burdens of the 4 [3] MAT-treatments do not significantly differ from the actual excess burden of the NOT-treatment. Also, it would be interesting to see how results from these toll structures compare to the present ones. But this rather delineates a future research agenda.

Finally, it is of interest how the average actual price of anarchy compares to the theoretically expected results (see section 3). To see that, we need to consider the BPT-treatment and calculate the average actual price of anarchy as the ratio of actual costs over optimal costs,

$$\sum_{n=A}^{J} C^{Actual}(x_1,x_2,x_3) \;/\; \sum_{n=A}^{J} C^{*}(3,3,0),$$

where costs are summarized over all ten sessions A to J, with $n = A, \ldots, J$. According to Table 3 the sum of actual costs in the BPT-treatment amounts to 7576 tokens and the sum of optimal costs amounts to (114·6·10 =) 6840 tokens. Hence, these figures yield an average actual price of anarchy of 7576 / 6840 ≈ 1.11, which is higher than the theoretically expected price of ≈ 1.05. Moreover, if we consider our amended version of the price of anarchy, we get (7576-2560) / (6840-3600) ≈ 1.55, which is also higher than the theoretically expected one of ≈ 1.48.

To summarize, the experimental evidence provided here clearly supports the theoretical expectation that a BPT-treatment generates a higher excess burden than either the NOT- or MAT-treatment. Therefore, the introduction of road tolls that may generate Braess's Paradox should be avoided. Further, in contrast to theoretical expectations, experimental evidence suggests that the introduction of a maximum road toll, which subject to given parameter values does not generate Braess's Paradox, may or may not generate a higher excess burden than the alternative scenario in which no road tolls are charged. Yet, overall experimental evidence indicates that the MAT-treatment generates a higher excess burden than the NOT-treatment. Moreover, the average actual price of anarchy associated with the Braess Paradox treatment is higher than the theoretically expected one.

6 Concluding Remarks

As road pricing is bound to play an increasingly important role in road infrastructure provision, designing optimal road pricing schemes is a crucial policy issue. In particular, selfish route choice behavior, which is typically disregarded in existing road pricing schemes, may have an important impact on overall economic efficiency. In this paper, we have shown that in a two tier road network introducing tolls for just one tier (i.e. motorways) may generate Braess's Paradox and, therefore, an excess burden. However, other things being equal, a maximum toll exists that does not generate Braess's Paradox. But because revenue from a Braess's Paradox generating toll would *ceteris paribus* be higher, profit maximizing road providers (private firms or leviathan governments) would typically prefer to charge such tolls, whereas benevolent providers may not, as they wish to avoid the associated excess burden.

From a policy perspective, it is therefore important to test whether the theoretically predicted results do prevail if real human beings are faced with the relevant incentive structures and strategy spaces. Experimental evidence provided here clearly shows that real humans do respond to the incentive structures in the predicted way, although it seems hard to establish stable equilibrium positions. In particular, we have shown that the excess burden associated with a Braess's Paradox generating toll (BPT) is significantly higher than excess burdens resulting from alternative treatments (MAT and NOT) that do not generate Braess's Paradox. This indicates that road user behavior should be taken into account in road pricing schemes and that road tolls should not exceed the MAT level. Moreover, our results suggests that both the theoretical and the actual loss of social welfare from selfish routing, known as the price of anarchy, may well be higher than previously expected. The policy relevance of our findings is further reinforced by the fact that the demand for traffic (i.e. X) may in practice fluctuate on a daily, weakly or even annual basis, because these fluctuations may *ceteris paribus* cause Braess's Paradox. In fact, for given parameter values, Braess's Paradox occurs only within an integer interval of traffic demand X, which is [3, 20] in the experiment according to (11) l.h.s. (see also Penchina 1997). Hence, Braess's Paradox might be a permanent issue if traffic demand fluctuates just within the interval, or might occur temporarily because traffic demand either grows from below into the interval or falls from above into the interval.

Finally, it is important to emphasize that our findings are not confined to the type of road network we have introduced here. For example, Meinhold and Pickhardt (2008, p. 153) point out that Braess's Paradox may exist in local, regional, national and international transportation networks. Moreover, Korilis et al. (1999, p. 215) and Roughgarden (2005, pp. 6–9) have already stressed that Braess's Paradox may emerge in a variety of networks, for example, in distributed computational systems such as the World Wide Web, hydraulic systems, and so on.

Appendix A:

Proposition: Equation (12), that is, *C(·)* assumes a minimum at $x_1 = x_2$. Substituting equations (6) to (8) into (12) and rearranging yields:

$$C(\cdot) = (\delta_M + \delta_H)(x_1^2 + x_2^2) + \gamma_M (X - x_3) + \delta_H (X - x_3)x_3 + X\delta_H x_3 + 2\delta_H x_3^2 \,. \tag{12'}$$

Proof: Assume that x_3 is fixed. Minimizing (12') then coincides with minimizing the first term in (12'). Further, assume that $(X - x_3) = 2z$, so that $x_1 = x_2 = z$. The first term in (12') can then be rewritten as $\tilde{C}(\cdot)$:

$$\tilde{C}(x_1, x_2) = 2z^2(\delta_M + \delta_H)\,. \tag{12''}$$

Now assume that $x_1 \neq x_2$ and that $x_1 > x_2$. Then, an $s > 0$ exists, with $x_1 = z + s$ and $x_2 = z - s$, so that:

$$\tilde{C}(x_1, x_2) = [(z+s)^2 + (z-s)^2](\delta_M + \delta_H) = 2(z^2 + s^2)(\delta_M + \delta_H)\,. \tag{12'''}$$

Thus, for $s > 0$, it follows that $\tilde{C}(z, z) < \tilde{C}(z + s, z - s)$, which proofs that $\tilde{C}(\cdot)$, and consequently *C(·)*, assume a minimum at $x_1 = x_2$. *q.e.d.*

Appendix B:

Instructions for Introduction Round and #1 Block [NOT-treatment]

Assume you are a freight forwarding agent and your job is to forward some good *G* from the start node to the end node as indicated in the figure below, where the two nodes may represent different cities. By forwarding good *G* you earn 40 tokens per round. You can maximize your payoff by choosing the route which is associated with the lowest costs (i.e. route 1, 2, or 3). You indicate your route choice by placing the card with the corresponding face value (i.e. 1, 2, 3) face down in front of you. Once the instructor has collected all cards, the number of agents traveling on each route is announced. At this point you can fill in your earnings record sheet by noting your own route choice, the overall route choices, the costs associated with the route you have chosen (see the table below) and your net payoff. While you fill in your earnings record sheet, the instructor returns to you your own card, again face down.

Thereafter a new round starts. A total of six rounds will be played one after the other. Then a new block starts. Do you have any further questions?

Insert Figure 1 here

Cost functions of individual routes:

$C_1 = 3x_1 + 2x_3$;

$C_2 = 3x_2 + 2x_3$;

$C_3 = 12 + 4x_3$.

Insert Table 1 here

Instructions #2 Block [BPT-treatment]

All instructions from the first block remain valid. The only change is that a toll of 10 tokens is introduced on links II and IV. This toll increases the costs of route 1 and 2. However, the toll has no impact on route 3. Again, you indicate your route choice by placing the card (…)

Insert Figure 1 here

Cost functions of individual routes:

$C_1 = 10 + 3x_1 + 2x_3$;

$C_2 = 10 + 3x_2 + 2x_3$;

$C_3 = 12 + 4x_3$.

Table of costs [*BPT-treatment*]

		Route 1 and 2 Number of persons who chose Route 3						**Route 3**
		0	**1**	**2**	**3**	**4**	**5**	
Number of persons who chose the same route	**1**	13	15	17	19	21	23	16
	2	16	18	20	22	24		20
	3	19	21	23	25			24
	4	22	24	26				28
	5	25	27					32
	6	28						36

Instructions #3 Block [MAT-treatment]

All instructions from the second block remain valid. The only change is that the toll charged on links II and IV is reduced to 6 tokens, which changes the costs of route 1 and 2 accordingly. However, the toll still has no impact on route 3. Again, you indicate your route choice by placing the card (…)

Insert Figure 1 here

Cost functions of individual routes:

$C_1 = 6 + 3x_1 + 2x_3$;

$C_2 = 6 + 3x_2 + 2x_3$;

$C_3 = 12 + 4x_3$.

Table of costs [*MAT-treatment*]

		Route 1 and 2 Number of persons who chose Route 3						**Route 3**
		0	**1**	**2**	**3**	**4**	**5**	
Number of persons who chose the same route	**1**	9	11	13	15	17	19	16
	2	12	14	16	18	20		20
	3	15	17	19	21			24
	4	18	20	22				28
	5	21	23					32
	6	24						36

Cash Payoff Scheme [to be added to Introduction Sheet]

At the end of this experiment your earnings in terms of tokens will be cumulated over all rounds played. The instructor will then rank the token earnings from highest to lowest by using the ID numbers of the participants. This ranking will not be made public. You will learn your own rank only. The two participants with the highest token earnings will receive 3 Eurocent per token, those two with the third and fourth highest token earnings will receive 2 Eurocent per token and the remaining two participants will receive 1 Eurocent per token. Payments will be made immediately after the experiment. *Example 1*: Assume for simplicity your payoff is 24 tokens in each of the 18 rounds, in total 432 tokens. Further, with your total earnings you are ranked second. Hence, you get: 432*0.03 = 12.96 Euro, plus 5 Euro show-up fee, which amounts to a total of 17.96 Euro. *Example 2*: Assume again for simplicity your payoff is 8 tokens in each of the 18 rounds, in total 144 tokens. Now you are ranked fifth. Hence, you get: 144*0.01 = 1.44 Euro, plus 5 Euro show-up fee, which amounts to a total of 6.44 Euro.

[Add to each instruction sheet for #1, #2 and #3 Block]

Your earnings in terms of tokens will be added-up right after the experiment and will be translated into cash payments according to the payoff scheme introduced in the introduction.

Earnings Record Sheet									
Date:		Gender:			My-ID:		Session:		
	My Route Choice			Total Number of Route User			Revenue (R)	Costs (C)	Payoff (P = R – C)
	Route 1	Route 2	Route 3	Route 1	Route 2	Route 3			
Example1	X			4	2	0	40	12	28
Example2			X	1	2	3	40	24	16
Round 1							40		
Round 2							40		
Round 3							40		
Round 4							40		
Round 5							40		
Round 6							40		

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