Urban Structure and Demographic Development

Dr. Joachim Geske

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1 Preface

One way to analyse demographical change is to focus on its macroeconomic consequences. By doing so an implicit assumption is that organisational microstructures remain unchanged. But a central characteristic of the second demographic transition is its spatial heterogeneity. Therefore the macroeconomic assumption of spatial homogeneity is applicable.

This paper opens up the field of second generation model based demographic analysis by focussing on endogenous microstructures. Particularly the analysis is concentrated on the spatial organisation of life, which is the basis for urban structures. Since 80% of the German population lives in urban areas I use an urban structure model to understand population density distribution in the area of conflict between housing decisions and commuting to the Central Business District (CBD).

This model is dynamised in an overlapping generations context. While dynamic models of urban structure are frequent ([Miy87], [Pae70], [BW04]), there is to my knowledge no model that considers demographic development. This is due to the problem that urban size is not by itself internalised into the dynamic decision process. To overcome this difficulty I define the Dynamic Urban Structure Equilibrium (DUSE).

Using the DUSE I formulate a model and expose it to a decreasing population, which stabilises in the long term. I found out that in peripheral areas density reacts more dynamic than in central areas (buffering function). Initially this leads to faster population density decrease in central areas. Later on the trend reverses. Infrastructure planning is therefore challenged by heterogeneous developments regarding time and space.

The findings will contribute to formulate strategies for urban and infrastructure planning. They will help to improve the understanding of developments in real estate markets.

2 Dynamic Urban Structure Equilibrium - DUSE

In the following I will present a discrete dynamic and spatial adaption of the classic urban structure model ([Mut61], [Mil70]). The discrete spatial structure simplifies the numerical solution of the equilibrium.

The model consists of a generational structure of representative agents (overlapping generations structure, OLG), which maximise their lifetime utility respective to their budget constraint in every period. The income consists of capital and wage earnings. Wages are payed only until the agents reach their retirement age T_r.

$$
\begin{aligned}
& \max_{c_{i,j}} \sum_{i=1}^{I} \beta^{i-1} U\left(c_{i,j}\right) \\
s.t. \quad &: \quad \left\{ \begin{array}{cc} w_j & i < T_r \\ 0 & i \geq T_r \end{array} \right\} + r_j a_{i-1,j-1} = p_t^i \left(c_{i,j} + a_{i,j}\right) \\
s.t. \quad &: \quad a_{I,j} = a_{0,j} = 0 \\
s.t. \quad &: \quad a_{i,0} = a_i^{SS} \\
j \quad &= \quad t-1+i...t-1+i \\
i \quad &= \quad 1...I
\end{aligned}
$$

Every generation of size L_t^i consists of a continuum of consumers, which differ from each other as to their residence.

Any atomic consumer of generation i lives on a ring of distance $u \in U = (u_1, ..., u_U)$ to the Central Business District (CBD). He maximises the utility U^i, that he receives from the consumption of living space c_t^{wi} and residual consumption c_t^{ci} in consideration of the consumable income $p_t^i c_t^i$, that he obtains for working in the CBD. The Consumer commutes from his living space in ring u to the CBD. This causes mobility costs of $p_t^c tu$, consisting of the unit cost t and the distance u. Mobility cost is interpreted as service. The price of the residual consumption is p_t^c, the rental fee is p_t^w. Additionally every consumer owns land $L_t^w(u)$. This land is used as a factor in housing production. Every generation gets a quota of κ_i of the total rent on this land ($\sum_i \kappa_i = 1$).

The utility maximisation problem is

$$
\begin{aligned}
& \max_{c_t^{wi}(u), c_t^{ci}(u)} U^i\left(c_t^{wi}(u), c_t^{ci}(u)\right) \\
s.t. \quad : \quad & p_t^i c_t^i - p_t^c tu + \kappa^i \frac{\sum_{u^*} r_t^l(u^*)\, Land_t^w(u^*)}{\sum_{u^*} n_t(u^*)} = p_t^c c_t^{ci}(u) + p_t^w(u)\, c_t^{wi}(u).
\end{aligned}
$$

A population of size $n_t(u)$ lives on each ring u. For an equilibrium distribution of consumers a stability condition is postulated that ensures nobody is willing to move among rings. The postulation is that consumers on any ring receive the same utility level. This stability condition is called "No Movement"-condition

$$V_t^i = U^i \left(c_t^{wi} (u), c_t^{ci} (u) \right) \quad \forall u. \tag{1}$$

Keeping this in mind it is possible to reinterpret the consumption good c_t^i from the dynamic problem independent of u as a composed good and as utility of the subproblem, consequently

$$V_t^i = c_t^i. \tag{2}$$

p_t^i is the price of one unit subutility of generation i.

Housing industry produces living space of size $f_t^w \left(K_t^w (u), L_t^w (u), Land_t^w (u) \right)$ on each ring u with production function f_t^w using the factors land $Land_t^w (u)$, capital $K_t^w (u)$ and labour $L_t^w (u)$. Factors are payed with interest r_t^k on capital, land price r_t^l and wage w_t. Each ring has an area of

$$Land_t^w(u) = 2\pi u.$$

The housing industry maximises its profits

$$\max_{K_t^w(u), L_t^w(u), Land_t^w(u)} p_t^w (u) f_t^w \left(K_t^w (u), L_t^w (u) \right) - r_t^k K^w (u) - r_t^l (u) L_t (u) - w_t L_t^w (u)$$

A second sector produces the residual consumption good. The enterprises in this sector also maximise their profits

$$\max_{K_t^c, L_t^c} p_t^c f \left(K_t^c, L_t^c \right) - r_t^k K_t^c - w_t L_t^c.$$

There are several markets which clear:

1. the housing markets,

$$\sum_i n_t^i(u) c_t^{wi}(u) = f_t^w(K_t^w(u), L_t^w(u), Land_t^w(u)) \quad u = 1...U$$

2. the market for residual consumption and

$$\sum_{u,i} n_t^i(u) c_t^{ci}(u) = f_t^c(K_t^c, L_t^c)$$

3. the factor markets for capital and labour.

$$\begin{aligned} \sum_i L_{it} a_{it} &= K_t^c + \sum_u K_t^w (u) \\ \sum_i L_{it} &= L_t^c + \sum_u L_t^w (u) \end{aligned}$$

Furthermore the adding up restriction

$$\sum_{u,i} n_t^i\left(u\right) = \sum_i L_{it}$$

is fullfilled. A dynamic urban structure equilibrium (DUSE) is a set of prices $\left(p_t^c, p_t^w(u), r_t^l(u), r_t^k, w_t, p_t^i\right)$ and a population distribution $n_t^i(u)$ so that

1. representative agents maximise their lifetime utility,
2. consumers of one generation maximise their utility from residual consumption and consuming housing space and
3. consumers receive the same amount of utility independent of the distance of their flat to the CBD,
4. producers maximise their profits,
5. the adding up restriction is fullfilled and
6. all markets clear.

To keep the model as simple as possible an ageless representative settlement-agent is defined for the subproblem. We thus drop out index i for the variables $c_t^c(u), c_t^w(u), n_t\left(u\right)$ and p_t. To connect the ageless subproblem to the to the dynamic problem the consumption of different age groups i is aggregated to a single consumption variable

$$\overline{c_t} = \frac{\sum_i L_{it} c_t^i}{\sum_i L_{it}}$$

which is the consumable income of the agless representative agent. It has to be kept in mind that differences between the generations concerning their choice of living space will thus be neglected. It is nevertheless useful to use this aggregation approach because

1. differences in settlement behaviour are exclusively the result of intergenerational income differences if $U_t^i = const$. The generations would therefore order around the CBD according to their income (e.g. [SVH76]). In a forthcoming paper it is argued that this result does not fit to the data and that these shortages can be overcome by introducing special preferences.
2. the dynamics of the simpler approach have to be analysed at first to understand the impact of more sophisticated age specific model features in setlement behaviour.
3. We don not need any assumptions on the distribution of the payments for land usage.

Depreciation rate	$\delta^c = 0.1$	$\delta^w = 0.06$
Cost shares	$\alpha^{wK} = 0.6$	$\alpha^{wL} = \alpha^{wLand} = 0.2$
	$\alpha^{cK} = 0.3$	$\alpha^{cL} = 1 - \alpha^{cK}$
Productivity	$z^c = 2.5$	$z^w = 1.5$
Discont factor	$\beta = 0.85$	
Utility parameter	$\gamma = 2$	
Residual consumption Share	$\mu = 0.8$	
Mobility cost	$t = 0.1$	

Table 1: Model parameter

At first specifying the subutility function as

$$U^i = c^{ci}(u)^{\mu}\, c^{wi}(u)^{1-\mu}$$

The simplified model will now be quantified and numerically solved within in following chapters.

3 Numerical Analysis

A time period in the model is defined to 5 years. Assuming a deterministic life expectancy of 80 years and a working entry age of 20 years gives the number of generations $I = \frac{80-20}{5} = 12$ periods. The retirement age is

$$T_r = \frac{2}{3}I + 1 = 9$$

periods. The number of rings is fixed to $U = 20$. The dynamic model will be solved up to period $T = 70$. The sectoral production functions and the Utility Function are

$$\begin{aligned} U(c) &= \frac{c^{1-\gamma}}{1-\gamma} \\ f^w &= z^w K^w(u)^{\alpha^{wK}} L^w(u)^{\alpha^{wL}} Land(u)^{\alpha^{wL}} \\ f^c &= z^c K^c(u)^{\alpha^{cK}} L^c(u)^{\alpha^{cL}}. \end{aligned}$$

The housing sector cost shares are taken from [WB06]. Parameter values are standard in literature. They are arranged in Table 1.

with regard to the demographic development it will be assumed that individuals live $I \cdot 5$ periods and die afterwards. The number of newborn in Period t is

$$N_t = 1 - \begin{cases} \min\left[0.5, \frac{60}{100I}(t - 2\cdot I)\right] & t > 2I \\ 0 & sonst \end{cases}$$

That means the number of newborn N_t declines after period $t = 2I = 24$ from 1 linearly to 0.5. This results in the cohort size L_{it}:

$$L_{it} = \sum_{t^*:t-t^*=i} N_{t^*} + \begin{cases} 1 & t < I \\ 0 & sonst \end{cases} .$$

So population decreases by 50% in the long run.

3.1 Steady State Analysis and Comparative Statics of Demographic change

In this section at first the spatial structure of the steady state will be analysed. Thereafter I will present an interpretation of the transition process from an initial steady state to a steady state with a different demographic structure. This interpretation will be used in the next chapter to understand the complete transitional dynamics between the steady states.

The spatial structure of the Steady State key variables are presented in figure 1. The "local" income $p_t\bar{c}_t - p_t^c tu$ decreases linear in the distance u from the CBD because of linear mobility costs. As soon as the price of residual consumption p_t^c is not spatially differentiated - in contrast to the rental fee $p_t^w(u)$- the only spatial influence on demand for residual consumption is the local income. That is why residual consumption decreases also linear with distance from the CBD.

The increase in housing demand is due to the "No Movement" condition as from equations (1 and 2) it can be deduced

$$\begin{aligned} const_t \cdot (p_t\bar{c}_t - p_t^c tu)^{\frac{1}{1-\mu}} &= p_t^w(u) \\ \frac{1}{1-\mu} &> 1. \end{aligned}$$

That means a linear decrease of consumable income will reduce the rental fee more than linear. This trend of the rental fee can be seen in the rental fee curve in Figure.:1. The decreasing rental fee increases demand for housing space by more, than the falling consumable income decreases it. This explains the over average increase in housing space with respect to the CBD.

The "No Movement" condition thus transforms linear spatial income differences to nonlinear spatial price differences, which result in nonlinear spatial consumption differences.

The course of the population distribution is singlepeaked. Most consumers live on ring 4, while population decreases on the more central and distanced rings. Population density (not shown here) decreases monotonously with distance to the CBD.

All these spatial structures have aequivalents in the classical urban structure model. The central question of this article is the reaction of the urban struc-

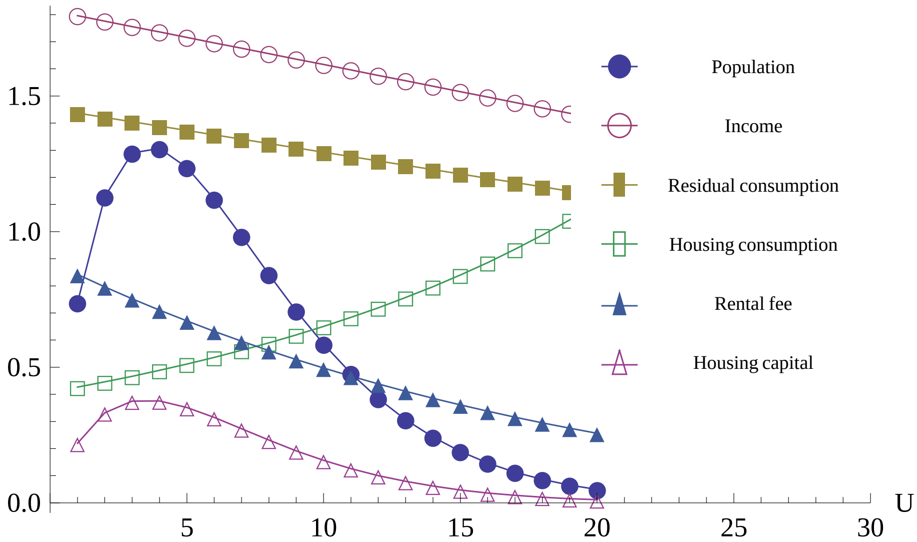

Figure 1: Steady State Spatial Structure of population, income, residual consumption, housing demand, rental fee, housing capital

ture to demographic changes. First of all we will compare two Steady States quantitatively by using population (growth) elasticities. This approach equals a comparative static analysis. The following arguments can be followed in Fig. 1.

Starting from a hypothetical population increase of 1%, the price p increases by 0.06% in equilibrium. The increase of the price level reduces c.p. real income, which reduces average savings by 0.08% and consumption c by 0.09%. In spite of the increase in p the decrease of consumption reduces consumable income $pc - tu$ by $(0.02\%, 0.03\%)$[1]. The reaction of the residual consumption demand and housing demand is different:

- While consumable income decreases slightly, the decrease of consumption c dominates. This causes via the "No Movement" condition a substantial increase in rental fee p^w by $(0.29\%, 0.26\%)$. This increase and the reduction of consumable income $p_t\bar{c}_t - p_t^c tu$ decrease housing demand c^w by $(0.36\%, 0.34\%)$.

- As $p^c = const$ the demand for residual consumption goods decreases with the rate of consumable income of $(0.02\%, 0.03\%)$. Without the "amplifying" mechanism of the "No Movement" mechanism the reactions of the residual consumption markets stay inconsiderable.

Because population distribution is equal on both marktes and housing demand decreases stronger than demand for residual consumption, production on

[1]The notation (x, y) describes a change of a variable in the closest CBD ring by $x\%$ and in the last ring by $y\%$.

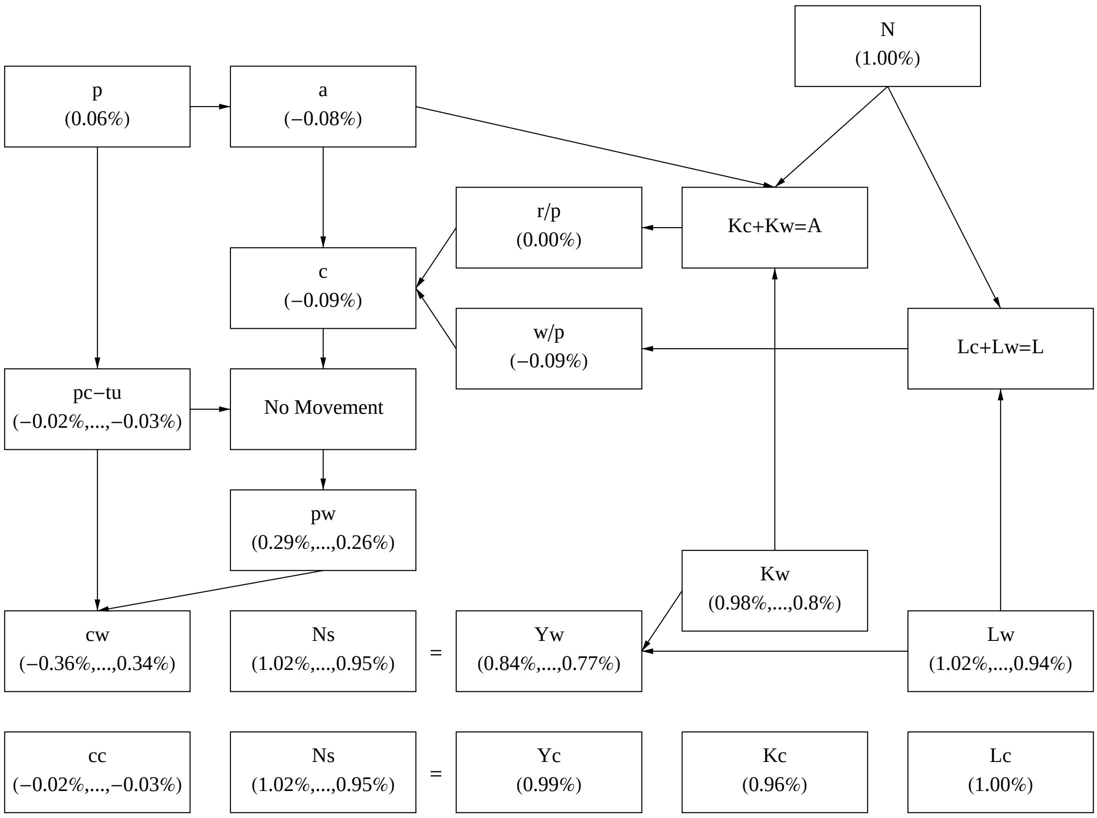

Figure 2: Schema to describe the influence of variables as a consequence of a 1.00% population increase

housing markets decreases more $(0.84\%, 0.77\%)$ than production of the residual consumption (0.99%). This production asymetry affects factor markets differently:

- Capital demand of housing producers increases less $(0.98\%, 0.90\%)$ than capital demand of the residual consumptions sector 0.96. The less than 1% increase in overall capital demand and the decrease of the average capital supply (-0.08%) and the assumed population increase of (1.00%) alltogether increase the interest rate by (0.06%) - which means there is almost no real interest rate increase.

- Individual labour supply is inelastic. Therefore population increase rises labour supply by (1.00%). As soon as production increases by less than (1.00%) there is an excess labour supply. To keep the markets clear wages fall by (0.15%), which also reduces real wages by (0.09%).

Both factor price movements explain the reduction of savings and consumption from the beginning of this analysis. It further shows that production has become less capital intense.

Assumed that price level p was fixed, then demand and supply were perfectly scalable, but numerical analysis shows

$$\sum_i L_i > \sum_i n(u).$$

As a consequence there would be not enough flats for the population. You could interpret this by saying the price of one unit composite consumption is too high. Only as consequence of an increase in p a new equilibrium can arise but price elasticity of a population shock is not 0 anymore.

The effect stems from the fact that indirect utility is determined dynamically. In general a given indirect utility level can only be reached by a change in population size. Only the floating price level ensures consistence of exogenously given population and endogenously produced settlement possibilities. In other words, excess supply of settlement possibilities is a kind of excess capital equipment -in the housing sector-, which reduces as a consequence of an increase in the price level.

3.2 Transitional Dynamics

Now we turn to a concrete demographic transition, namely a population decrease. To understand transitional dynamics of this population decrease, the whole process will be partitioned into four phases:

Phase	Description	Period
A	initial population Steady State	1-8
B	accelerating population decrease	8-17
C	decelerating population decrease	17-27
D	decreased population Steady State	27-40

The processes described in this section are graphically represented in figures (3,4 and 5).

Phases A and D correspond to the initial population steady state (A) and the decreased population steady state (D). The relation of these two steady states have been analysed carefully in the previous chapter.

During phase B the population decreases accelerating as more and more less populated (young) cohorts enter. On the capital market there are on the one hand relatively few young consumers trying to save for their retirement later on and on the other hand relatively many old retired consumers who finance their consumption by resolving their savings and receiving interest rates. Furthermore population decreases, which lowers capital demand if the depreciation is low and rises capital intensity. All in all there is an excess capital supply which causes a decline of the interest rate.

At the same time labour supply decreases. Labour becomes scarce, wages rise.

If a price adoption of p was impossible, excess supply of capital would cause an excess supply of living space. As it was derived in the last chapter in this case the price level p decreases. Compared to the initial steady state real interest rate r/p declines and real wages w/p rise. Because the decline in real interest exeeds the increase of wages, the composite consumption $\bar{c}$ declines. As the price level p and $\bar{c}$ decline, consumable income $p\bar{c}$ decreases more than c. Under these circumstances the "No Movement" condition generates a strong decline of the rental fee p^w, resulting in a strong increase in the individual demand for housing c^w. Thereby excess supply of livingspace reduces. The urban area approaches equilibrium.

We can further deduce the spatial structure of the transition. Mobility costs form a different fraction of the fixed income in dependence of the distance to the CBD. Thus an increase in income affects the local consumable income with different growth rates. That is why the rental fee decreases the stronger the greater the distance of the flat to the CBD. Peripherial areas react more dynamic, the urban area centralises.

The development slows down in phase C. Price level p recovers and the new Steady State is reached in phase D.

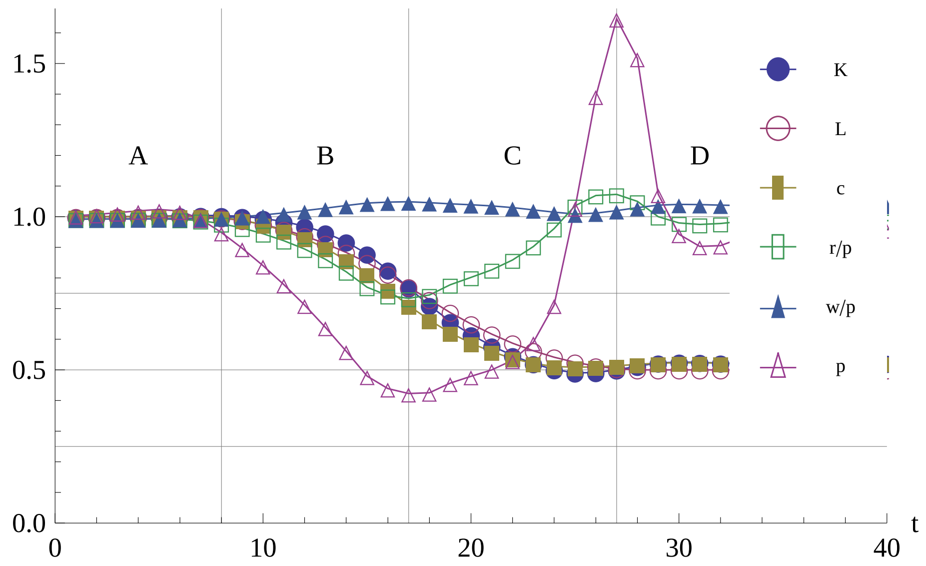

Figure 3: Demographic Transition: Development of aggregate capital (K), Population (L), average consumption (c), real factor prices (r/p, w/p) and the price level

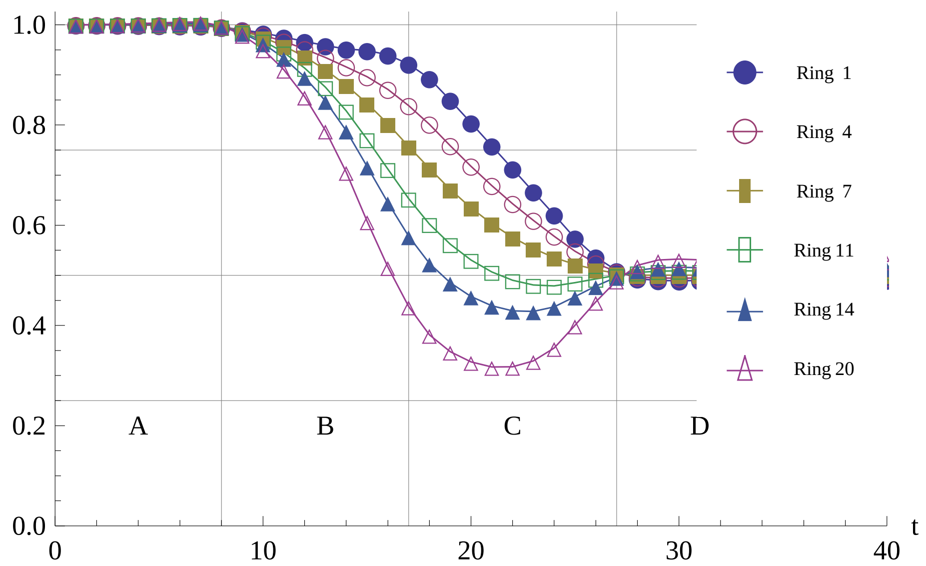

Figure 4: Demographic Transition: Development of the distance specific population index (Period 1 = 1) for selected rings

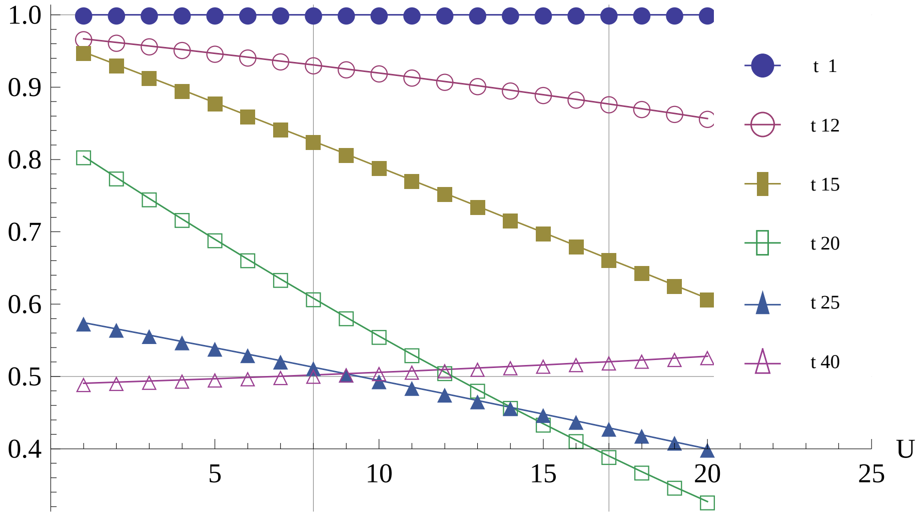

Figure 5: Demographic Transition: Spatial distribution of ring specific population index (Period 1 =1) for selected periods

4 Implications for Infrastructure Planning

Although infrastructure is not explicitly modeled in a way that size and distribution influence the functioning of the economy, some implications can be derived from the simple model . To do so it has to be assumed, that infrastructure elasticity of the population distribution is low. In this chapter I will present a method how spatial and dynamic tendencies of infrastructure demand can be deduced from population distribution.

We will focus only on gridbound infrastructures that enable the spatial distribution of goods (water, electricity, labour force,...) from or to the CBD[2]. If the population distribution is given, and a constant per capita demand is assumed, it can be calculated how many goods have to be transported to each point in the urban area.

To do so it will be assumed that an infrastructure at a certain point $x(u')$ in ring u has to "conduct" all the goods demanded at points on a ray $x(u)$ $u > u'$ from the center of the CBD through $x(u')$. We can therefore define the flow of goods at a point in ring u as

$$I(u) = \sum_{u'>u} \frac{n(u)}{2\pi u}.$$

A grid infrastructure demand is defined as a vector of flow of goods $(I(1), ..., I(U))$. Using this definition we can measure the grid infrastructure demand in depen-

[2]It is not necessary but makes things much easier.

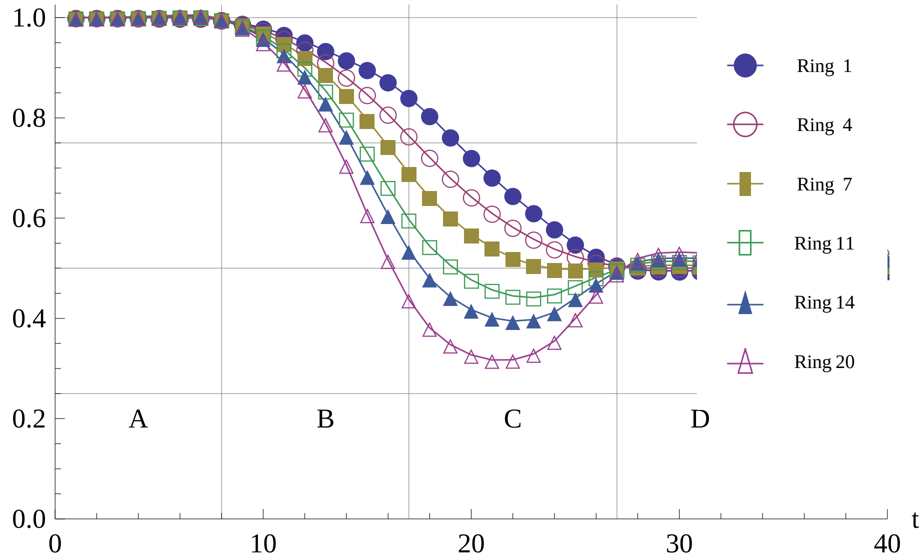

Figure 6: Demographic Transition: Development of infrastructure demand for selected rings

dence on time for the example of the previous chapter. The results are shown in Figure 6.

The Development of Infrastructure demand has the same dynamic and spatial properties as the development of population distribution. That means during phase B infrastructure demand decreases faster in peripheral areas than in central areas. In the first part of phase C peripheral areas show an "undershooting" of infrastructure demand from which they recover in the second part before transition to the Steady State (D).

Infrastructure demand shows a significant spatial and dynamical heterogeneity. Furthermore peripheral areas react in the early phase more sensitive and for the complete transition in non monotone way.

This heterogeneity makes infrastructure planning a difficult task. Furthermore it becomes clear that utilisation sensitive infrastructures (water) might have to suffer from repeated qualitatively different adjustments. This might make it necessary to replace infrastructure long before it is broken down. As a consequence, it could be useful to develop more flexible infrastructures instead of purely focussing on longevity.

5 Conclusion

In this paper the author presents a way, how to introduce demographic change into the classic urban structure model. This method has a big potential to become a powerful tool for economic analysis concerning real estate markets, energy demand pattern, infrastructure analysis... Within this documentation the method was not yet developed completely. To do so detailed data have to be gathered and analysed to improve the underlying static model.

A very interesting application of the model is the possibility to model infrastructure as a transport facility, that influences settlement behaviour. A consequent step is to use this model and characterise a dynamic optimal infrastructure policy, for example in order to derive an optimal infrastructure policy in the wake of demographic transition. This would be an substantial contribution to the theory of dynamic infrastructure planning. Thereby it has to be considered that infrastructure policy has next to its technical focus also a city- and regional planning aspect.

Nevertheless, this early analysis shows the substantial spatial and dynamic heterogeneity that infrastructure planning has to face when dealing with demographic changes.

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