Dynamics of Spatial Infrastructure with Application to Gas and Forest

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Abstract

Dynamics of spatial infrastructure development is considered theoretically. The main application are gas and forest industries. For these industries the price/weight ratio is low, and thus transport costy play an important role in total costs. This leads to a necessity of using spatial models, with explicit accounting for spatial structure of either production (here for forest) or distribution (for gas). The paper presents several mathematical models that deal with spatial structure explicitly. An interesting phenomenon is observed in gas industry: the consumption of gas in Germany was growing together with long term positive price trend. At the same time, gas pipeline network was also developing in EU. This can be expained as a positive externality effect of gas network development on demand. Two-dimensional spatial model allows to get the aggregate demand as the function of network expansion.There are two dynamic optimization models that assume different strength of externality. If it is moderate, then there is long term convergence to steady state despite monopoly power of gas supplier. While forest industry is completely different, the role of spatial infrastructure on market is also important there, and this is done by spatial derivation of corresponding dynamic optimization model.

1 Introduction

Economic theory typically deal with parameters that are unrelated to geography. In macroeoconomic models, for example, production function typically includes aggregated labour and capital. However, spatial structure of production and spatial distribution of inputs and/or markets is often quite important. Such problems are typically studied in regional science. An article of Beckmann and Thisse (1986) presents a good survey of the state of art in this field. Weber (1909) considered a problem of optimal plant location when geographical locations of inputs and market were given as well as unit distance transport costs for all items. Hotelling (1929) was the first to consider a problem of spatial competition between two firms, who can choose both location and price, in an environment of consumers continously distributed in one-dimensional space. Such a setting became the corner stone of many models in industrial organization. Yegorov (2000) has generalized the model of Hotelling for two-dimensional space, with the possibility of variable consumer density.

The method of dynamic optimization is frequently used in economic models that consider optimal growth; see Barro, Sala-i-Martin (1995) as an example. However, it is also used in much wider set of applications that often contain socio-economic models with externalities.[1]

This paper presents an attempt to combine these two trends in socio-economic literature and to apply them to some real problems. On one hand, it deals with dynamic optimization problems. On the other hand, it uses explicitly spatial structure of production or consumption including possible network externalities for the derivation of such dynamic models.

For the industries that produce the output with low price/weight ratio spatial structure is important. Especially in the case of land transportation. The first model is about the development of a network around new strategic gas pipeline. Special focus is on external effect for the dynamic change in the demand for gas.

[1]Feichtinger et al have considered such application as spread of drugs, social externalities in cities, etc.

The spatial structure is taken here in abstract form (without using particular geographical maps, that account for all types of spatial heterogeneities). At the same time, space is treated as two-dimensional and continuous. Although this assumption is naturally taken for granted in all natural science models it is not so common in economics (including new economic geography), with the exception of some models is regional science and industrial organization.

One of important results here is an influence of existing infrastructure on demand schedule. While in neoclassical economic literature demand is considered as constant, in business application there exist models, when it is affected by advertising or fashion; see Kort et al (2006) as an example.

Over the last decades we have observed a substantial increase of gas consumption (in European Union, in particular) that grew even faster than energy consumption. Prices for gas were moving up and down, following oil price shocks (German data are presented on Fig. 1), but at some periods (in the beginning of the 21-st century) both prices and volumes were increasing. At the same time, we see the development of pipeline network, both at strategic level (trans-continental, between Russia and EU) and at the local level (gas supply to houses). The paper focuses on the effect of positive externality from gas infrastructure development on the aggregate demand for gas.

The second application is an optimal exploitation of forest. Here we deal with a renewable natural resource with spatial distribution of production. Forest is a relatively cheap good, with low price/weight ratio, espacially in the form of round wood. According to FAO, in 2003 Russia has exported 37.7 mln. cum of round wood, with the value of 1.8 bln. USD (thus the average export price was about 50 USD/cum). Russia has the largest forest area in the world; it covers 809 mln. ha in 2005, or about 47 % of the territory. Fig. 2 shows the spatial distribution of forest. Most of it is located in Siberia, in the zones with poorly developed transport infrastructure. Contrary to Russia, Brazil (the second largest world forest holder, if territory is counted) has lower terrirory covered with forest (478 mln. ha) but large stock of wood due to higher density of forest. That is why, although Amazon area is also remote, Brazil may need less investment in infrastructure to access its forest in comparison to Russian.

After the beginning of transition, new Russian enterpreneours started to cut wood that is closely located to border and has less transport cost. It was exported to common-border countries: Finalnd, Japan, China. This activity was quite intense.[2] Moreover, this logging lead to non-sustainable exploitation of forest reserves, when the logging intensity in border areas have lead to deforestation. Even ecologists caring about survival of Siberian tiger were alarming the public with the fact that the area of tiger hunting in Primorie (area between Vladivostok and Khabarovsk) have reduced substantially due to deforestation.

Economic objective of new enterpreneurs was clear: to minimize export cost by exploiting forest areas close to the border. Due to poor development of forest property rights and poor ecological and legislative control in the beginning of transition period in Russia the damage for ecology and future sustainable use of forest was quite substantial. At the same time, there exist remote areas full of forest in Central Siberia that are practically not exploited due to lack of transport infrastructure. Again, at present the development of such infrastructure may not be profitable, given plenty of possibilities to develop forest industry in closer located areas. This shows a special importance of spatial consideration of economics of forest in Russia. In the model presented below it will be assumed that forest is exploited in a sustainable way (that is, annual logging intensity in a particular area equals to the growth of forest mass in the same region). The accent is on the dynamics of spatial penetration of such forest exploitation taking into account spatial heterogeneity of transport cost and investment in transport infrastructure.

2 Gas Network and Gas Demand

2.1 Assumptions

1. Consider two-dimensional space (x, y), with a pipeline growing along axis x from the point $(0, 0)$.

[2]According to FAO, in 1990 Russia removed 268 mln. cum of forest comparing to only 104 mln. cum in 2000. With internal consumption close to 100 mln. cum and changing slowly over time, this reveals predator and logging in early 1990-ies mostly oriented on export.

2. At time t, pipelines covers an interval $[0, x(t)]$, where

$$x(t) = aK(t), \tag{1}$$

with $K(t)$ denoting growing capital stock invested in a pipeline. In the simplest case of constant investment flow, $I(t) = \dot{K}(t) = c$, pipeline drows linearly with time: $x(t) = ct$.

3. The development of local infrastructure along pipeline is done by other capital (different from one of the investor in the main pipeline). Orhhogonal propagation of network is done with the speed v. That is, if at time t_0 the main pipeline ends at the point $x_0 = x(t_0)$, then at time $t_0 + t$ the vertical branch at $x = x_0$ will end at the points $y = \pm vt$.

Different additional assumptions can be made ro specify model completely.

The case of linear investment. For the case of constant investment flow we can find the coverage area at any time t:

$$-vt + \frac{v}{c}x \le y(t) \le vy - \frac{v}{c}x, \qquad 0 \le x \le ct. \tag{2}$$

The covered area is then $S(t) = vct^2$.

Additional demand. On the way of pipeline there are both strategic users of gas (large firms, that are typically located close to it) and small users that are spread over space. The development of local infrastructure makes them to shift for using gas as an alternative energy source. They create additional demand for gas. In general, we can write three components for gas demand (each depending on price p, like in typical economic models): a) independent on pipeline, b) dependent on its length X, c) dependent on covered area S:

$$D(p) = D_0(p) + D_1(X, p) + D_2(S, p) \tag{3}$$

Despite some less of generality from the assumption of linear investment it makes sense to keep this assumption in order to keep realtive mathematical simplicity of the model. However, it makes sense to modify the assumption about unbounded growth of the demand area influenced by pipeline, with a new assumption that involves its upper limit.

2.2 Linear Investment and Demand Satiation

Suppose not that only a neighbourhood of radius R around a pipeline is affected. There my be several justifications for that. First, the land is finite. Moreover, neighbouring territories are limited by geographical and national borders. But there is another argument, and it is related with opportunity cost. Consider an optimal decision about use of gas done by a potential consumer at some distance r from a new pipeline as a separate subproblem. It may happen that this agent already consumes gas from another pipeline that is located closer. Thus, one of the reasons for spatial finiteness of demand area is in spatial competition (a la Hotelling [1]) with other pipelines. The second reason may be use of alternative fuels that may be preferred because of substantial distance from a pipeline. This is also a typical model of industrial organization, but now in a mix of real and product differentiation space (see [2] as an example). Again, assumping transport cost for delivery, there exists some threshold distance R, such that only for lower distances consumers would shift their demands towards using gas from this particular pipeline.

As before, we assume now that a new pipeline is expanded linearly, i.e. $X(t) = ct$, and that demand area around it expands linearly in the orthogonawl direction y with the speed v. But now this expansion is not unbounded, but is limited with the distance R in the orthogonal direction.

Area dynamics. Now we can calculate the dynamics of covered areas of new demand for gas. First of all, it is easy to find that orthogonal expansion takes the time $\tau = R/v$ before satiation. At any time moment $t > R/v$, there are two subareas:
a) a rectangular neighbourhood of the old part of pipeline (built R/v years ago or earlier, they have area S_1),
b) new, triangular area around the opints of pipeline constructed not more than R/v years ago (they have area S_2).

It is easy to find that

$$S_1(t) = 2Rc(t - \frac{R}{v}), \qquad S_2(t) = vc\tau^2 = c\frac{R^2}{v}. \tag{4}$$

Demand dynamics. Let us assume that all demand components are linear: $D_i = c(i)D_0(1-p)$. Then, using formulae (3) and (4), we can write:

$$D(p,t) = D_0(1-p) + \gamma ct(1-p) + \delta(2Rct - c\frac{R^2}{v})(1-p). \tag{5}$$

2.3 Economic Implications

If we look at the dynamics of prices for gas and its consumption, wee can see that in some countries (Germany, for example - to present data) both prices and volumes can grow simultaneously. At the first glance, this seems to contradict classical microeconomic theory, where the demand has a negative relationship with price. An exception is the so called "Giffen good", but does it really exist?

The formulae of the previous section show that a careful consideration of dynamic investment and decisions of spatially distributed actors can give rise to the aggregate demand where the coefficients depend on time explicitly. Thus, the demand grows over time, even when the price is fixed.

3 Dynamic Models with Gas Network

3.1 Model 1: High Externality

Here we neglect the fact that gas is non-renerwable resource. This is justifiable if the typical time horizon of convergence to steady state infrastructure happens much faster than gas extinction.

Consider a gas monopoly which invests in pipeline infrasrtructure, thus shifting thdemand for gas, according to the formula $D(p,t) = a(K(t)) - b(K(t))p$. At each time moment a monopoly selects an optimal price $p(t)$ and decides about investment $I(t)$. Its objective is to maximize the discounted consumption stream

$$max_{I(t),p(t)} \int_0^\infty e^{-rt}C(t)dt, \tag{6}$$

$$C(t) = D(p(t), K(t))[p(t) - c(K(t))] - I(t) - \mu I^2, \tag{7}$$

$$\dot{K}(t) = I(t) - K(t)\delta. \tag{8}$$

Consider the demand function

$$D = (D_0 + \gamma K)(1 - p), \tag{9}$$

which describes the positive externality from investment into pipelies. Assume that $c(K) = a + bK$, that is, longer pipelines require higher variable costs (for transporting gas). Now we can express intertemporal profit as the function of price and capital stock:

$$\pi(p, K) = (D_0 + \gamma K)(1 - p)(p - a - bK). \tag{10}$$

Since price has no historical influence, at any moment it can be chosen independently. Maximization of monopolistic profits occurs at

$$p^* = \frac{1 + a + bK}{2}. \tag{11}$$

Intertemporal consumption $C(t)$ is given by the difference between profit and investment cost. Calculated at $p = p^*$, it becomes the function of only one variable - capital stock $K(t)$:

$$C(K, p^*) = \frac{1}{4}(D_0 + \gamma K)(1 - a - bK)^2 - I - \mu I^2. \tag{12}$$

Now the problem of dynamic optimization of gas monopoly growth can be formalized as:

$$\max_{I(t)} \int_0^\infty e^{-rt}[\frac{1}{4}(D_0 + \gamma K)(1 - a - bK)^2 - I - \mu I^2]dt \tag{13}$$

$$s.t. \qquad \dot{K} = I - \delta K.$$

The net present value Hamiltonian is:

$$H = \frac{1}{4}(D_0 + \gamma K)(1 - a - bK)^2 - I - \mu I^2 + \lambda(I - \delta K). \tag{14}$$

The first order conditions lead to the system of 2 differential equations and one algebraic equation:

$$\dot{K} = \frac{\lambda - 1}{2\mu} - \delta K, \tag{15}$$

$$\dot{\lambda} = \lambda(\delta + r) - \frac{\gamma}{4}(1 - a - bK)^2 + \frac{b}{2}(D_0 + \gamma K)(1 - a - bK), \tag{16}$$

$$I = (\lambda - 1)/(2\mu). \tag{17}$$

Isoclines. Let us define $P_2(K) \equiv \frac{\gamma}{4(\delta+r)}(1-a-bK)[1-a-2bD_0/\gamma-3bK]$. Then the isoclines in K, λ space are:

$$\dot{K} = 0 \Rightarrow \lambda = 1 + 2\mu\delta K, \tag{18}$$

$$\dot{\lambda} = 0 \Rightarrow \lambda = P_2(K). \tag{19}$$

In the generic case, there are two steady states: (K_1, λ_1) and (K_2, λ_2), $K_2 > K_1$. However, numerical calculations show that for a wide range of parameters none of them is a saddle. For example, for $a = 0.3, b = 1, r = 0.05.\delta = 0.05, D_0 = 1, \gamma = 10, \mu = 0.1$, there are two roots, $K = 0.142, \lambda = 1.0014$ and $K = 0.724, \lambda = 1.0072$, but both of them are vortices with positive real parts of eigenvalues. This result is rather intuitive: here positive externality is rather strong. This means that in a monopolistic setting and unlimited demand externality there is no convergence to any finite state of capital. In reality, demand externality matters only in the middle run, because both the world is limited as well as the gas resources.

3.2 Model 2: Moderate Externality

It is possible to consider a model modification with some satiation in demand. A possible functional form is satiation with negative exponent. Quadratic function is less realistic because of declining branch, but it can be simpler for modelling. A model presented below is written for square root dependence of demand factor on capital.

Let us assume now that the postive demand externality arising from the investment in pipeline capital has the following form:

$$D = (D_0 + \gamma\sqrt{K})(1 - p). \tag{20}$$

This shape refelct the following properties of the assumed demand for gas:
a) the demand elasticity is $p/(1-p)$, so that the demand is unit elastic at the optimal price of a monopolist without cost, i.e. for $p = 1/2$;
b) there exists some demand even in the absence of pipeline infrastructure (it may be covered by LNG or other close gas substitutes);
c) capital investment in pipeline has a positive effect on demand, but with diminishing marginal returns, that vanish to zero as capital goes to infinity.

The property c) makes the model different from the case 1. Mathematically it ensures the possibility of a steady state with finite level of capital. From the practical point of view, it reflects the satiation effect that is always persent, because the Earth is finite and because the neighboorhood of pipeline that can be affected are also finite.

Consider the following dynamic optimization problem:

$$max_{I(t),p(t)} \int_0^\infty e^{-rt} C(t) dt, \tag{21}$$

$$C(t) = (D_0 + \gamma\sqrt{K})(1-p))[p(t) - a - bK(t)] - I(t) - \mu I^2,$$

$$\dot{K}(t) = I(t) - K(t)\delta.$$

Again, like in the case 1, the current price can be chosen in every period independently. Moreover, this new specification does not change it: $p^* = \frac{1+a+bK}{2}$. The new Hamiltonian that emerge after substitution of p^* has the form:

$$H = \frac{1}{4}(D_0 + \gamma\sqrt{K})(1 - a - bK)^2 - I - \mu I^2 + \lambda(I - \delta K). \tag{22}$$

The first order condition $dH/dI = 0$ gives the same algebraic equation as in case 1, $\lambda = 1 + 2\mu I$, but the system of two differential equations emerging from other first order conditions is different now:

$$\dot{K} = \frac{\lambda - 1}{2\mu} - \delta K, \tag{23}$$

$$\dot{\lambda} = \lambda(\delta + r) - \frac{\gamma}{8\sqrt{K}}(1 - a - bK)^2 + \frac{b}{2}(D_0 + \gamma\sqrt{K})(1 - a - bK). \tag{24}$$

Setting $\sqrt{K} \equiv X$, it is possible to convert this system into polynomial. Then we have

$$\dot{X} = \frac{\lambda - 1}{4\mu X} - \delta X/2, \tag{25}$$

$$\dot{\lambda} = P_3(X). \tag{26}$$

The numerical calculations show he following results. For parameter set, $a = 0.2, b = 1, r = 0.05, \delta = 0.05, D_0 = 1, \gamma = 2, \mu = 0.1$, there are 4 roots, but only two of them are real. Real roots $X_{1,2}, X \equiv \sqrt{K}$ correspond to the positive values of capital: $X_1 = 0.220, \lambda_1 = 1.00058$ and

$X_2 = 0.932, \lambda_2 = 1.01044$. The lower root X_1 corresponds to a saddle with eigenvalues $(+6.84, -6.80)$, while the higher root has the eigenvalues $(0.02 + i2.74, 0.02 - i2.74)$, and thus is an unstable vortex.

In these parameter values (and for a wide range of other parameters) we have a convergence to a unique steady state with a positive level of capital.

4 Forest Model

Consider an olipolistic problem of forest exploiter. This setting is chosen because the world forest market is more oligopolistic (even in local sense) than the gas market. First of all, many gas importing countries (including most of EU members) have substantial forest resources and production. Second, the technology assumes transportation by lorries or railroad, which more competitive in comparison with pipelines. Finally, monopolistic set-up has been already considered, and it makes sense to consider a different setting. However, contrary to typical game setting, the problem is again reduced to dynamic optimization. Such a setting is believed to not only provide a useful mathematical simplification (since dynamic games allow for fewer closed form solutions even in the simpest set-up), but also to portray better the reality. Only few countries in the world have now vast and underexploited forest resources. The greatest resources are in Brazil (Amazonian selva), but Russian resources in Siberia represent another example. Due to high distance between Brazil and Russia and high importance of tranport costs in this sector, they cannot compete at most of the local markets for forest.

There is Cournot competition with the rest of the world, but only one producer can technically expand capacity. The rest operate at the border of capacities and typically not interested in cutting the output, while the first player may expand it under investment.

The spatial structure of forest is as follows. It covers a rectangular area with one side b, that lies along the border (where it can be sold) and endogeneous deepness of exploitation $X(t)$. Several parallel roads are constructed into the direction orthogonal to the border. At time t the deepness of penetration is $X(t)$ and the area of forest that can be harvested is $S(t) = bX(t)$. Only a fraction of this forest is harvested (for sustaniability reasons); that is why the maximal possible output is $Q(t) = \nu S(t)$. It is optimal to expand X in such a way, that the production is at the frontier of production possibilities. The cost of cutting forest is σ, while the transport cost per unit distance is τ. The total transport cost of all forest can be calculated as an integral $\int xdx$. Taking into account the linear density Q/b, the total transport cost is $\tau X^2 Q/(2b)$.

The total world demand for forest is given by a function

$$p(Q, q) = A - Q - q. \tag{27}$$

For this function the maximal revenue is reached when $Q + q = A/2$ (for zero cost). If intiially $Q(0) + q(0) < A/2$, everybody is interested in the expansion of production, but there arer no spare capacities for it. We assume that this interest remains even if production costs are taken into account, at least for some time interval.

Here the capital investment can be measured by the deepness of penetration X. Hence, capital dynamics is given by the equation $\dot{X} = I - \delta X$, where δ denotes depreciation. Again, as in the model with gas network, investment costs have two components, linear and quadratic, and are given by the expression $I + \mu I^2$.

The firm (country) with a possibility of expansion strategically takes into account the dynamics of price, imposed by its investment and strategy. It solves the following dynamic optimization problem:

$$max_{I(t)} \int_0^\infty e^{-rt} C(t) dt, \tag{28}$$

$$C(t) = Q(X(t))[P(Q(X)) - \sigma Q - \tau X^2 Q/(2b)] - I(t) - \mu I^2, \tag{29}$$

$$\dot{X}(t) = I(t) - X(t)\delta. \tag{30}$$

Let us denote $\nu b \equiv v$. Then $Q = vX$, and

$$C = vX[A - vX - q - \sigma vX - \frac{1}{2}\tau v X^3] - I - \mu I^2. \tag{31}$$

The problem of optimal investment in the expansion of forest exploitation is associated with the NPV Hamiltonian:

$$H_2 \equiv vX[A - vX - q - \sigma vX - \frac{1}{2}\tau v X^3] - I - \mu I^2 + \lambda[I - \delta X]. \tag{32}$$

The first order conditions lead to one algebraic and two differential equations:

$$I = (\lambda - 1)/(2\mu) \tag{33}$$

$$\dot{\lambda} = \lambda(r+\delta) - v(A-q) + 2v^2(1+\sigma)X + 2\tau v^2 X^3, \tag{34}$$

$$\dot{X} = \frac{\lambda - 1}{2\mu} - \delta X. \tag{35}$$

Isoclines and steady states. Although we have a cubic equation $P_3 = 0$ for steady states (emerging from $\dot{\lambda} = P_3(X)$) along with linear positively sloped relation between X and λ, $2\mu\delta X = (\lambda - 1)$, these lines have not more than one intersection in the economically meaningful area $(X > 0, \lambda > 0)$.[3] Typically there is one intersection of saddle point type.[4] Thus, we typically have a convergence to a unique saddle point with positive capital stock.

This result has the following economic interpretation. In a competitive environment in the world market for round forest, some countries (like Russia) are still underexploiting their production capacity because of too little access to remote areas of forest deposits in Siberia. Capital investment in road infrastructure in a necessary pre-condition for such exploitation. But given the level of unit distance transport costs and world prices for forest, it might happen that some remote areas should not be still exploited. The model solution presents an optimal investment path in infrastructure. The saddle point corresponds to such penetration level X that should be the long run objective of infrastructure investment.

5 Conclusions

1. The paper has several goals having both theoretical and paractical applications. On theoretical level, the paper combines the ideas if spatial modelling from regional science (using some methods from physics about spatial aggregation) with the approach of dynamic optimization.

2. The idea about positive externality of capital investment in infrastructure on the demand for good is also developed here. Here it results as an influence of the development of gas infrastructure on the global demand for

[3] This comes from the fact that $P_3'(X) < 0$ for $X > 0$.

[4] There may be no intersection when $v(A-q) < r + \delta$.

gas. This conclusion does not contradict empirics.

3. The paper presents a spatial approach in the derivation of temporary evolving economic objective from the investment in spatial infrastructure. In the end it leads to dynamic optimization problems.

4. In the first model, the focus is on the evolution of demand for gas driven as a positive externality from the development of spatial gas network and caused by pipeline construction. Dynamic optimization problem about optimal investment path in infrastructure is considered in a monopolistic setting. In the case 1 of high externality this leads to unbounded expansion. In the more realistic case 2 of moderate positive externality there exists a unique stable steady state (saddle) with an optimal path converging to it.

5. In the second model, the focus is on the development of forest harvesting area caused by investment in road infrastructure. Typically, there exists a unique saddle point corresponding to an optimal lond run zone of sustainable forest explooitation. It depends paramatrically on transport cost and world price of forest, and thus can change in future.

6 Literature

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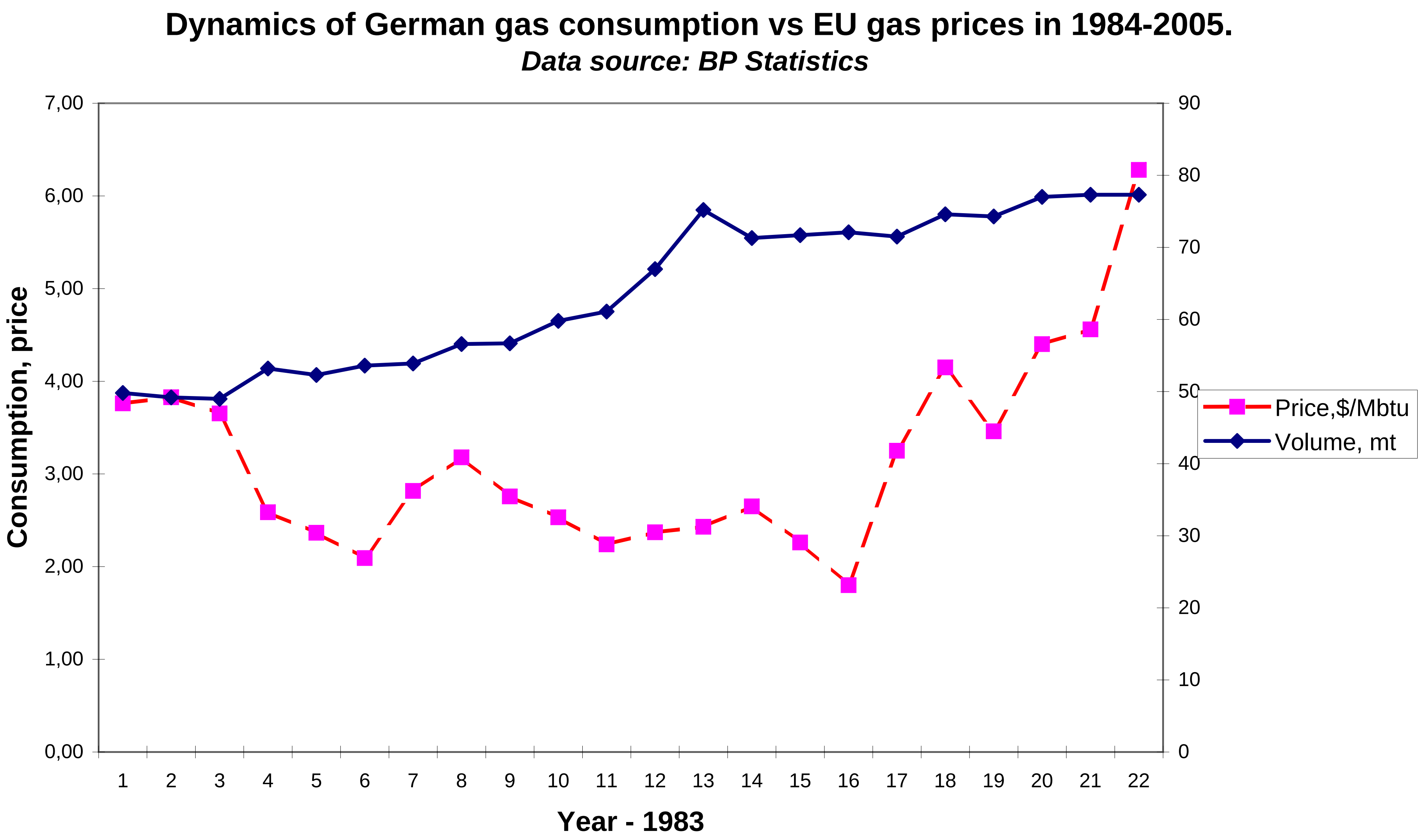
Dynamics of German gas consumption vs EU gas prices in 1984-2005.
Data source: BP Statistics
Consumption, price
7,00
6,00
5,00
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Year - 1983
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