

# **Risk management in electricity markets: hedging and market incompleteness**

**Bert Willems**

Tilburg Law and Economics Center, TILEC, Tilburg University, the Netherlands

Energy Institute, K.U.Leuven, Belgium

b.r.r.willems@uvt.nl

**Joris Morbee**

Center for Economic Studies, K.U.Leuven, Belgium

Joris.Morbee@econ.kuleuven.be

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## **PRELIMINARY RESULTS**

### **Abstract**

This paper aims at a better understanding of the origin of risk in electricity markets and studies how electricity firms should manage their risk exposure. The paper develops an equilibrium model of the electricity market with risk averse firms, and compares the aggregate welfare under different assumptions with respect to market completeness, i.e. the number and type of derivatives which are traded in the market. We show that aggregate welfare in the market increases with the number of derivatives offered in a particular market, but most of the benefits are achieved with a rather limited set of derivatives. Forward prices are biased estimates of future spot prices; however, this bias does not depend on the number of other derivatives which are traded in the market. This bias is eliminated when speculators (traders) are present in the market.

In a future version of the paper, we would like to study the incentives of firms to vertically integrate. Vertical integration will be modeled as a way to write complete contracts between retailers and generators.

## 1. Introduction

The electricity sector has been subject to major structural changes during the last decade. Liberalization policies all over the world required a separation of formerly vertically integrated monopolies into three parts: production, retail and network services. Network services are a natural monopoly and should remain regulated. Introducing competition is possible at the level of production and retail. In this paper we look at competition at the production level and the retail level, while assuming that prices in the retail market remain fixed in the short run.

The product electricity has special characteristics which greatly affects the way electricity markets are organized. Electrical energy *cannot be stored* economically, and therefore has to be produced the moment it is consumed. Intertemporal arbitrage is impossible, and the price for electricity is determined by the supply and demand conditions at each given hour. As *demand* for electrical energy is *very inelastic* and of a *stochastic nature*, and generators face production capacity constraints, spot prices are very volatile.

Liberalized electricity markets are therefore typically organized around regional spot markets for energy, which determine hourly spot prices, complemented with markets for long term contracts, which help coordinating the actions of the players and to hedge volume and price risks.

This paper aims at a better understanding of the origin of risk in electricity markets and studies how electricity firms should manage their risk exposure. In particular we look at two different management tools: (1) hedging market position by buying derivatives and signing contracts, (2) vertical integration as a way to write complete contracts.

The paper develops an equilibrium model of the electricity market, which includes the production process, the spot market trades and the trade of derivatives. One of the results of the model is that it shows how prices and price volatility are driven by the underlying market fundamentals, i.e. cost characteristics and demand uncertainty.

The results of the model are complementary to the classical valuation models for financial derivatives. Classical models assume that the prices of the underlying asset follow some stochastic process and that a risk free portfolio can be build using delta hedging to price the derivative. Here we explain how price volatility is driven by the market dynamics. The advantage of our approach is that pricing of all products will be consistent with the overall industry behavior and with the underlying physical reality.

## **2. Overview of the paper**

The first part of the paper studies how derivatives can be used to hedge risk in the electricity market, and at what prices these contracts will be traded. A simple partial equilibrium model of the spot market and the derivatives market is developed. It is assumed that firms are risk averse and have an incentive to sign financial instruments to hedge risks.

The equilibrium of the model will determine the spot price, spot price volatility and forward prices. The model is based on (Hendrik Bessembinder, Michael L. Lemmon 2002), but will allow for multiple financial products to be traded.

Our research focus in this part is to see how firms should hedge optimally their price and quantity risks and to check whether forward prices are biased predictors of prices in the future, and whether market completeness, i.e. the number of derivatives in the market, will change the bias in forward prices.

In the proposed second part of the paper (not yet covered in this extended abstract), we study vertical integration as an alternative way to hedge market risk. As electricity markets are typically incomplete, *i.e.* it is impossible to sign contracts which hedge against all possible risk factors, risk sharing between generators and retailers is incomplete as well.

By integrating vertically, firms improve their risk sharing. Vertical integration has some benefits but also some drawbacks. On the one hand, firms will be able to share the risk between retailers and generators in an optimal way. On the other hand, by not trading in the forward market, firms loose the opportunity to sell

their risk to outside investors, such as banks. We expect that there will be a trade-off between the different hedging mechanisms.

### 3. Relevant Literature

*To be added in the final version of the paper.*

(A. Porchet 2007)

(Hendrik Bessembinder, Michael L. Lemmon 2002)

(Nils-Henrik Von Der Fehr 2007)

(R. Green 2007)

(R. Green 2003)

(Y. Oum *et al.* 2006)

(D. Duffie, R. Rahi 1995)

(F. Allen, D. Gale 2004)

(R. Elul 1995)

### 4. Model description

We extend the competitive market equilibrium model of the forward and spot markets developed by (Hendrik Bessembinder, Michael L. Lemmon 2002). The main difference with their model is that we allow for multiple financial products to be traded on the market. We start with a description of the spot market and continue with a description of the forward markets.

Demand for electricity  $D$  is inelastic and stochastic. The total production costs of the industry is the sum of a fixed cost  $F$  and a variable cost

$$C(Q) = F + \frac{a}{c} Q^c \quad (1)$$

where  $F, a$  and  $c$  are parameters that determine the shape of the cost function.

The spot market is perfectly competitive, and the wholesale price for electricity  $P$  is determined by market clearing.

$$P = C'(D) = a D^{c-1} \quad (2)$$

As demand is random, also the spot price is a random variable.

The generator's profit is equal to spot market revenue minus production costs:

$$\pi_g = P \cdot D - C(D) \quad (3)$$

Retailers buy the energy on the spot market and sell their energy at a regulated retail rate  $R$  to consumers. Their profit is equal to:

$$\pi_r = (R - P)D \quad (4)$$

Both retailers and generators' profit are affected by the stochastic nature of demand.

In the forward market, a derivative  $i \in \{1, \dots, I\}$  is traded at a price  $F_i$  which promises a payment  $T_i(P)$  which is conditional on the spot price  $P$ . This paper assumes that the only derivatives which are traded are call options. Hence:

$$T_i(P) = \max(P - S_i, 0) \quad (5)$$

with  $S_i$  the strike price of option  $i$ .

The total profit  $\Pi_j$  that a firm  $j = r, g$  makes when it buys  $k_i^j$  derivatives in the forward market is equal to:

$$\Pi_j = \pi_j(P) + \sum_{i=1}^I k_i^j \cdot (T_i(P) - F_i) \quad (6)$$

The firm's profit is the sum of the profit it makes in the spot market, and the profit it makes on the derivatives it has bought. Both terms are stochastic as they depend on the realization of the demand level.

We assume that the retailers and generators are risk averse, and that their utility can be described by profit - variance utility with a risk aversion parameter  $A$ :

$$j = r, g \quad U_j = E(\Pi_j) - \frac{A}{2} \text{Var}(\Pi_j) \quad (7)$$

The risk aversion parameter  $A$  measures the risk aversion of the generator and the retail sector as a whole, which we assume to be identical across sectors.<sup>1</sup>

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<sup>1</sup> If there are  $N$  identical firms in the generation sector, then each of the firms own  $1/N$ th of the generation capacity, and has a risk aversion parameter equal to  $nA$ .

In the contracting stage, firm  $j$  will maximize its utility  $U_j$ , by choosing the amount of derivatives  $k_1^j, \dots, k_i^j, \dots, k_I^j$  it will buy. The equilibrium quantities contract positions are given by

$$\vec{k}^j = \Sigma^{-1} \frac{E(\vec{T}) - \vec{F}}{A} - \Sigma^{-1} \text{cov}\{\pi_j, \vec{T}\} \quad (8)$$

with  $\vec{k}^j = (k_1^j, \dots, k_I^j)$ , the vector of equilibrium quantities bought by player  $j$ ,  $\Sigma = \text{cov}\{\vec{T}, \vec{T}\}$  the  $I$  by  $I$  covariance matrix of the contracts  $\vec{T} = (T_1, \dots, T_I)$ ,  $\vec{F} = (F_1, \dots, F_I)$  the derivative price vector, and  $\text{cov}\{\pi_j, \vec{T}\}$  the 1 by  $I$  covariance matrix of contracts and firm  $j$ 's profit.

Equation (8) shows that the amount of contracts firm  $j$  buys is the sum of two terms. The first term is the pure speculative amount of contracts a firm would like to buy. If a financial derivative has an expected positive return, then the firm will buy some of it, as long as it does not increase the variance of its portfolio too much. The second term is the pure hedging demand by the firm. A firm  $j$  will buy derivatives in order to hedge its profit risk. It will buy more of a certain derivative, if it is more correlated with the profit it wants to hedge, and if the impact on the variance of the portfolio is smaller.

In equilibrium the demand and supply of derivative products should be equal. Hence, if there are no speculators active in the market  $i$  we find:

$$k_i^r + k_i^s = 0 \quad (9)$$

and using equation (8) the equilibrium price of derivative  $i$  is given by

$$F_i = E(T_i) - \frac{A}{2} \text{cov}\{\pi_g + \pi_r, T_i\} \quad (10)$$

Hence, the price of a derivative is equal to expected pay-off of the derivative minus a term which reflects that the derivative is used to hedge the risk of the individual firms. The last term depends on the risk aversion of all the firms and the covariance of industry profit with financial instrument  $i$ .

It is worth noting that the price of the derivative does not depend on the amount of products which are traded in the market.

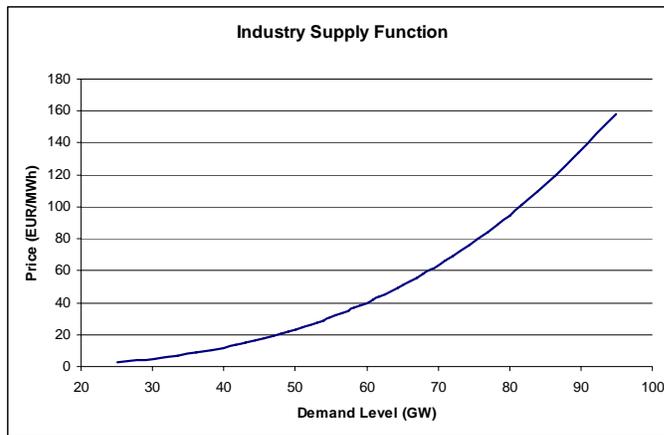
If there are risk neutral speculators active in derivatives market  $i$ , the risk premiums become zero, and we find that the price of the derivative should be equal to the expected value of the derivative:

$$F_i = E(T_i) \quad (11)$$

### 3. Model Data

The model is loosely calibrated on the German electricity market. Demand is assumed to be normally distributed with a mean of 60 GW and a standard deviation of 17GW.

The parameters of the aggregate production cost function are  $c = 4$ ,  $a = 1.852 \cdot 10^{-4}$  where price, quantities, and profits are expressed in (EUR/MWh), (GWh), and (1000 EUR/h). Figure 1 shows the industry marginal cost function with these calibration parameters.



**Figure 1 Industry marginal cost**

Retailers and generators have the same risk aversion parameter  $A = 0.0025$  which has the unit (h/ 1000 EUR).

Given the assumption on the supply and the demand side we can derive the wholesale price distribution. The distribution has a mean of 48 EUR/MWh and a standard deviation of 35 EUR/MWh. (Hendrik Bessembinder, Michael L. Lemmon 2002) show that as the industry marginal cost function is convex, the price distribution is skewed, which can also be seen in Figure 2.



**Figure 2 Wholesale price distribution**

Further, we assume that the fixed cost parameter  $F = 1200$ , and that retailers sell their energy at a fixed price of 58 EUR/MWh.

#### **4. Simulation Results**

In this section, we calculate the optimal hedging strategy of the generator and the retailers. In the first part of the simulations we assume that there are no speculators active on the market, and the supply and demand of financial contracts comes only from retailers and generators. We consider four scenarios with a different number of derivative markets present. In the first scenario only forward markets exist. In the second to fourth scenario, there operate, next to the forward market, respectively one, three and eleven additional option markets.

Table 1 shows the simulation results for all scenarios. It shows for each of the twelve derivative contracts, the net amount traded by generators and retailers. The option contracts have strike prices ranging from 0 to 143 EUR / MWh, where the zero strike price corresponds with the forward contract.

Contract		Price	Net Contract Position			
Nr	Strike Price		Forward	Forward + 1 Option	Forward + 3 Options	All Contracts
1	0	-45.3	68.0	52.0	32.1	-5.3
2	13	-33.6				37.9
3	26	-25.2				16.2
4	39	-19.1			37.7	11.1
5	52	-14.5				8.3
6	65	-10.9				6.9
7	78	-7.9		45.4	15.9	5.9
8	91	-5.6				5.2
9	104	-3.8				4.3
10	117	-2.4			14.5	6.6
11	130	-1.3				-3.7
12	143	-0.7				13.1
<b>Welfare</b>			<b>768.4</b>	<b>1224.1</b>	<b>1321.7</b>	<b>1337.0</b>

**Table 1: Market Equilibrium without speculation**

The results show that if there are only forward contracts, generators will over hedge their position. They sell 68GW forward, while in expected terms they will only sell 60 GW. The reason for this is that the generators and retailers want to hedge their quantity risk, and as price and quantity are positively correlated, they can do this by selling, respectively buying more contracts. The price of the forward contract is 45.3 EUR/MWh, which is below the expected price of 48 EUR/MWh in the market.

In scenarios 2 to 4, extra financial instruments are added to the market. Table 1 shows that once more instruments become available, generators will reduce the amounts of standard forward contract they sell and substitute these contracts with option contracts. The generator and the retailer reduce their supply and demand of forward contracts. Although both demand and supply functions shift, the price of the forward contract remains 45.3 EUR/MWh as shown in derivation (10).

The last row in Table 1 is the aggregate market welfare, measured in certainty equivalents (1000 EUR/h). Increasing the number of contracts traded clearly increases market efficiency. The introduction of one option contract, when none existed before, increases welfare with approximately 50%. Adding extra markets for option contracts increases welfare, but to a lower extend. For instance, increasing the number of option markets from 3 to 11, increases welfare with 1.2

% . Hence risk sharing between generation and retail is close to optimal once a few option contracts are traded.

For the second part of the simulations, we assume that speculators can actively participate in the market, by taking positions in the electricity derivative markets and financially closing their position in the spot market. They will trade away the risk premia in the market: the price of the derivatives becomes equal to the expected value of the derivative. As speculators provide extra liquidity to the market, the supply of derivatives by generators does no longer need to exactly balance the demand by retailers. The difference of generators' supply and retailers' demand is the position speculators take in the market. For the same four scenarios as before, Table 2 gives the net position of generators and retailers. In scenario 1, only forward contracts exist, and generators sell 69.1 GWh forward, retailers buy 67 GWh, and speculators buy 2.1 GWh. The results indicate that as more derivative markets are introduced, the larger the gap between the supply and the demand for forward contracts, and the larger the role that speculators play. In scenario four, in which there are one forward market and eleven option markets, generators sell 34.2 GWh, retailers *sell* 44.8 GWh and speculators buy 79 GWh. The introduction of speculators increases welfare, as the players can share their risk with players outside the market, the speculators.

Contract		Price	Net Contract Position							
Nr	Strike Price		Forward		Forward + 1 Option		Forward + 3 Options		All Contracts	
1	0	-48.4	69.1	67.0	58.7	45.3	48.0	16.2	34.2	-44.8
2	13	-36.0							13.2	62.6
3	26	-25.9							8.8	23.6
4	39	-18.1					20.5	55.0	6.6	15.7
5	52	-12.4							5.5	11.2
6	65	-8.3							4.7	9.1
7	78	-5.5			29.3	61.5	12.2	19.7	4.2	7.6
8	91	-3.5							3.8	6.6
9	104	-2.1							3.2	5.4
10	117	-1.2					10.5	18.4	4.9	8.2
11	130	-0.7							-2.7	-4.7
12	143	-0.3							9.9	16.4
<b>Welfare</b>			<b>771.6</b>		<b>1284.6</b>		<b>1402.8</b>		<b>1423.5</b>	

Table 2: Market Equilibrium with speculation

## 5. Conclusion

This paper derives an equilibrium model for spot and derivative markets in the electricity market.

We show that markets for electricity derivatives are important, as they allow firms to hedge both quantity and price risks: Firms do not only need futures/forward contracts, but also some option contracts to hedge quantity risks. Increasing the number of contracts increases the gains of trade in the market, but the marginal improvement of one extra market quickly becomes negligible. Most gains of trade are achieved with a relatively small amount of derivative markets.

Allowing speculators to actively trade in the market, eliminates the risk premium in the forward market, and increases aggregate welfare in the market. The welfare effect of adding speculators to the market increases with the number of financial products which are traded in the market.

*In a future version of the paper, we would like to study the incentives firms to integrate vertically between retail and generation. Vertical integration will be modeled as a way to write complete contracts between retailers and generators.*

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