Intra-day power market balancing with network constraints cost allocation *

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Abstract

Mechanisms of infrastructure market clearing have to cope with a lot of physical constraints and caused by them costs. Presence of this costs reduce economic wealth, significantly complicate market procedures and cause that simple market balancing rules, which are used successfully on standard markets, do not work. In this paper we present, discus and compare various methods of the intra-day electric power market pricing, focusing on the problems of appropriate physical constraints cost allocation, offers pricing and market price determination. We propose some new constraints costs allocation mechanisms, based mainly on the game theory, as well as some new pricing methods, which take into consideration costs allocated previously to the constraints. We analyze proposed solutions and compare them to the widely applied in power systems LMP method. At the end of the paper we show basic features of analyzed methods on simple illustrative example.

Keywords: Power markets, Cost allocation, Market pricing

JEL-code: D40

1 Introduction

In case of infrastructure systems like power, telecommunication and transportation systems, designing efficient market procedures is much more complicated than in other cases. Main problem are infrastructure constraints, which usually significantly limit trade possibilities. In this way they reduce trade profits and disturb offers' acceptance rules. Therefore, market clearing rules for such systems have to take into consideration physical constraints and economic wealth

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reduction caused by them. Clearing mechanisms have to determine cost of each constraint i.e. divide loss of economic wealth between all constraints, allocate them to appropriate offers and determine market prices.

In this paper we consider the problem of designing and modeling pricing mechanisms for electric power intra-day market. We analyze locational marginal pricing method (LMP) [6], widely applied for price diversification along the limited transmission network, and show that it has some evidently undesirable or controversial properties. We also propose some new constraints cost allocation methods, based mainly on game theory, as well as mechanisms which allocate these costs to specific offers and determine market prices. We start from simple price determination rules, show that in case of infrastructure market they are insufficient and propose some more sophisticated pricing methods. We investigate properties of proposed methods on simple illustrative example and compare them with the LMP method.

2 Power market models

2.1 The "copper-plate" power market model

On many European national power markets the "copper-plate" market model is applied. In these models the energy is balanced in an aggregated way. There is only one energy balance equation in each period and one price for energy in the whole system. However, each offer on the market may be charged with transmission network charges, so final prices for particular market entities may be diverse.

For the purposes of this paper we consider a simple copper-plate intra-day balancing market model with limited network capacity constraints. A general formulation of a (single-period) quantitative balancing model can be stated as follows:

$$\widetilde{Q} = \max_{d,p}[Q = \sum_i c_i d_i - \sum_j c_j p_j] \tag{1}$$

$$\sum_i d_i - \sum_j p_j = 0 \tag{2}$$

$$\sum_v w_{ve}(\sum_{j \in J_v} p_j - \sum_{i \in I_v} d_i) \le Q_e \qquad \forall e \tag{3}$$

$$0 \le p_j \le p_j^{max} \qquad \forall j \tag{4}$$

$$0 \leq d_i \leq d_i^{max} \qquad \forall i \tag{5}$$

Variables p_j and d_i represent accepted volume for sell and buy offers respectively, c_j and c_i are offer prices, p_j^{max} and d_i^{max} are maximal offered volumes, and Q_e is capacity of power line e. Constraint (2) is the "copper-plate" energy balance for the system. The objective function (1) maximizes the economic wealth. Network capacity constraints (3) are modeled by using the power distribution factors (PTDF) [5], denoted as w_{ve} for node v and edge e. J_v and I_v are the sets of offers submitted in given node v by sellers and buyers respectively. The model can be easily extended to the multi-period case with respecting other system constraints, as it was shown e.g. in [7].

2.2 Locational marginal pricing

Locational marginal pricing (LMP) is widely applied method for the network constraints consideration, especially related to congestions and transmission losses. Pennsylvania-New Jersey-Maryland (PJM) [13] and New York ISO (NYISO) [11] power markets in United States as well as New Zealand Wholesale Electricity Market (NZEM) [12] relay on the LMP concept, moreover some European market operators consider introduction of the LMP methodology. The concept of LMP was introduced by Schweppe et al. [6] and extended by Hogan [2]. The LMP method allows one to calculate marginal prices of feeding (or withdrawing) energy in individual nodes of the infrastructure network. Price diversification between two adjacent nodes occurs when the transmission capacity between the two nodes is limited and exhausted, or loses appear. To formulate the LMP model, we have to replace balance equation (2) with the set of balance equations for all network nodes (7) and add some constraints representing physical laws concerning power flows (8) - (10).

$$\max_{d,p}[\sum_i c_i d_i - \sum_j c_j p_j] \tag{6}$$

$$\sum_{w=1}^{N} P_{vw} - \sum_{j \in J_v} p_j + \sum_{i \in I_v} d_i = 0 \qquad \forall v \tag{7}$$

$$P_{vw} = V_v V_w Y_{vw}(\theta_v - \theta_w) \qquad \forall (v,w) \in E \tag{8}$$

$$\theta_0 = 0 \tag{9}$$

$$-Q_{vw} \leq P_{vw} \leq Q_{vw} \qquad \forall (v,w) \in E \tag{10}$$

LMP method provides direct constraints cost allocation to the specific offers by setting diverse transaction prices for these offers. Dual variables - λ_v related to the energy balance equation for each network node (7) may be used as the energy prices in appropriate nodes. Cost of network constraints for given line is represented as a difference between energy prices in adjacent nodes.

3 Constraint costs

Presence of system constraints (3), (10) in market balancing models causes the economic wealth decrease, which may be considered as the system constraints cost. The total costs of system constraints can be defined as $c(N) = \widetilde{Q}(\emptyset) - \widetilde{Q}(N)$, where $\widetilde{Q}(\emptyset)$ is the economic wealth, which can be achieved in case of constraints absence, whereas $\widetilde{Q}(N)$ is the reduced economic wealth achieved when respecting the whole set N of system constraints. The LMP method allocate implicitly network constraints costs to offers and determine market prices. In case of other constraints and other cost allocation methods, cost of the constraints have to be determined explicitly. In this section we present various methods of allocation cost $c(N)$ to individual constraints. Properties of allocations obtained using presented method were analyzed in [3]. Let $c(S)$ denote the joint costs of system constraints when only subset S of constraints is considered and rest are omitted. Generally, constraints' costs are not subadditive, so for some subsets of constraints $S1$, $S2$ usually $c(S1)+c(S2) \neq c(S1 \cup S2)$, that is sum of constraints contained in $S1$ cost and constraints contained in $S2$ cost is different from cost caused by all these constraints considered together. Therefore, problem of constraints' costs allocation is nontrivial.

3.1 SCRB

The Separable Costs Remaining Benefits (abbr. SCRB) method is an arbitrary method arising from civil engineering problems of allocating costs of multipurpose reservoirs [10]. In this method cost x_i allocated to constraint i is divided into *separable cost* - s_i and *remaining cost* - R. Separable cost of constraint is equal to cost of its inclusion to all other constraints $s_i = c(N) - c(N \setminus i)$. Remaining cost: $R = c(N) - \sum_{i \in N} s_i$ is allocated proportionally to remaining benefits $r_i = c(i) - s_i$, that is cost of respecting solely constraint i minus separable costs allocated so far to this constraint, thus finally cost x_i allocated to the constraint i may be determined as:

$$x_i = s_i + \frac{r_i}{\sum_{j \in N} r_j} R \qquad (11)$$

3.2 Shapley value

This method derives the approach from game theory. In our game each capacity constraint is considered as a player. The Shapley value represents the average player contribution into constraints cost for all orders of players joining into the game [10]. To compute Shapley value, the following equation can be used:

$$x_i = \sum_{S \subseteq N, i \in S} \frac{|S \setminus i|!|N \setminus S|!}{|N|!}[c(S) - c(S \setminus i)] \tag{12}$$

For each $S \subseteq N$ (each subset S of constraints set N) the joint cost $c(S)$ has to be calculated. Thus this method has much bigger complexity than SCRB, but it explores more possible dependencies between constraints.

3.3 Aumann-Shapley pricing

Aumann-Shapley pricing address the complexity problem arising with Shapley value [10]. Let vector $\mathbf{b}$ be values of right-hand sides of all network constraints (capacities) and $F(\mathbf{b})$ be constraints cost for the values $\mathbf{b}$. Aumann-Shapley price for player i is an average marginal cost of constraint i along the ray from $\mathbf{b}^0$ (all constraints completely relaxed) to the target $\mathbf{b}^*$ (nominal values of right-hand sides):

$$x_i = \int_0^1 [\partial F(t(\mathbf{b}^* - \mathbf{b}^0) + \mathbf{b}^0)/\partial b_i)]dt \tag{13}$$

As distinct from Shapley value and SCRB some fractional coalitions are taken into consideration, not only the cases of infinity or nominal capacities.

3.4 MASIT

In this approach an allocation which is "minimal" in some meaning and satisfies the incremental cost test (15) is searched. This condition states that the cost allocated to all constraints in subset S is not lower than the cost caused by introduction this constraint into the market. This condition has to be met for all subsets of constraints $S \subseteq N$. We may find this allocation by solving some auxiliary LP problem:

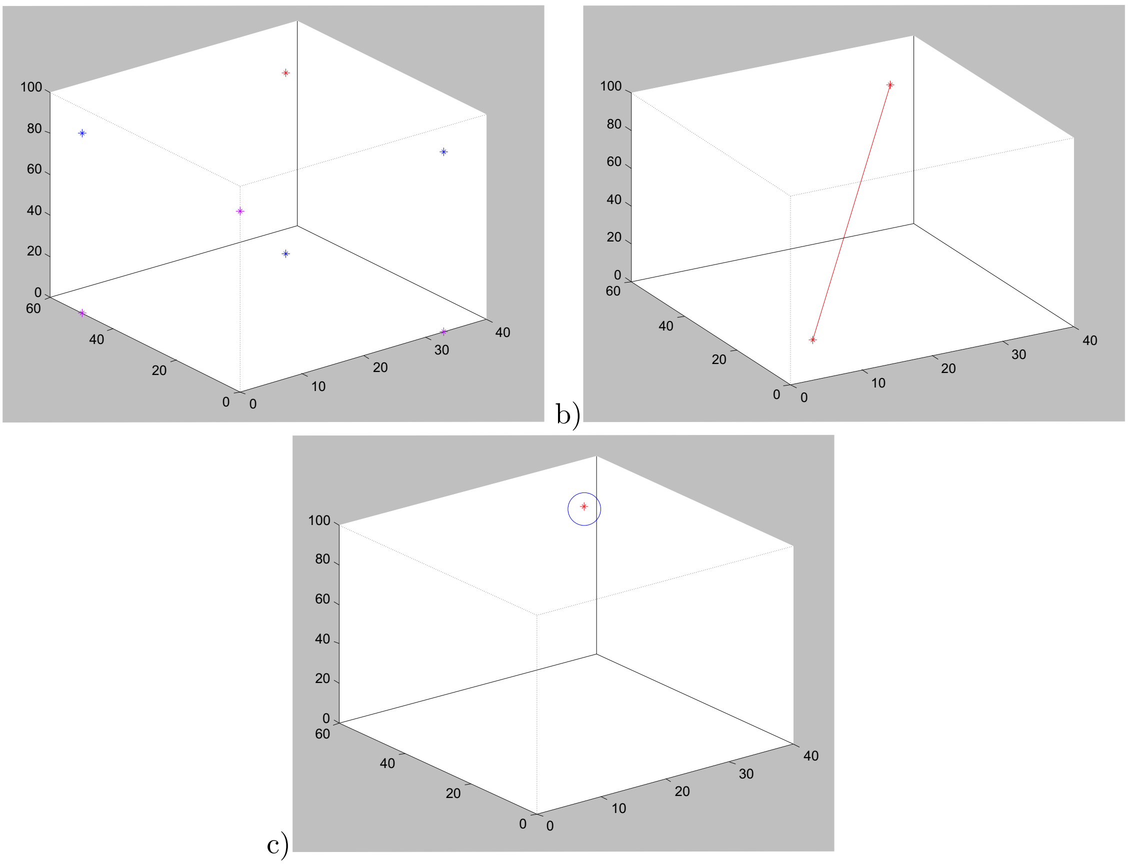
$$\min_{i \in N} x_i \tag{14}$$

$$\sum_{i \in S} x_i \geq c(N) - c(N \setminus S) \qquad \forall S \subseteq N \tag{15}$$

$$x_i \geq 0 \qquad \forall i \in N \tag{16}$$

Above problem is multi-criteria and has nice properties if equitable preferences model [4] is used to find the solution of (14)-(16). In this approach the total cost allocated to all constraints may be higher than $c(N)$.

Points in right-hand side values (RHS) space taken into consideration in all described above methods are presented in Fig. 1. SCRB, Shapley and MASIT take into consideration only nominal values of RHS and points where some constraints are complectly mitigated. However SCRB takes into consideration only points where one constraint is mitigated and points where all except one constraints are mitigated. While Shapley value and MASIT take into consideration all possible combinations of mitigated and unmitigated constraints. Aumann-Shapley method takes into consideration all points on the line between point where RHS have their nominal values and point where all constraints are completely mitigated. In contrast, the LMP method takes into consideration only the neighborhood of point with nominal RHS values.



a) b) c)

Figure 1: Points in right-hand side values space taken into consideration in: a) SCRB, Shapley and MASIT methods, b) Aumann-Shapley method, c) LMP method.

4 Power market pricing

To clear the market, all constraints cost should be properly allocated to the market entities. Proper allocation means, that cost of given constraint is allocated to all these generations and loads which cause it. Share of this cost should be proportional to influence of particular load or generation on these constraint. Constraints costs may be allocated to offers charging them by *node fees* for power flow from node j to the "cooper-plate" – virtual market node, in case of generation offer, and from the "copper-plate" to node i, in case of load offer, as it was shown in Fig. 2 and described in [9]. Price for a sell offer in node j can be calculated in such a case

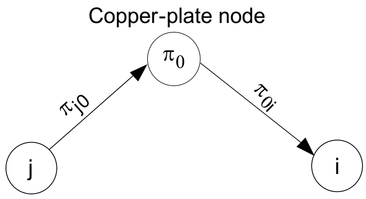


Figure 2: Prices for power flow to and from the copper-plate node

as: $\pi_j = \pi_0$ - π_{j0}, and for a buy offer in node i as: $\pi_i = \pi_0 + \pi_{0i}$, where π_0 is energy price in the "copper-plate" virtual node and π_{j0} and π_{0i} are transmission network charges, e.g. for power flow from node j to "cooper-plate" and from "copper-plate" to node i respectively. In fact π_{j0} and π_{0i} may have negative values, which increase prices for sell offers and decrease for buy offers.

We have to solve two problems: determine "copper-plate" energy price – π_0 and node fees – π_{j0}/π_{0j}. Additional problem is fact that node fees change minimal and maximal prices which do not generate losses for entities from offer values – c_i, c_j to $c_j + \pi_{j0}$ for buyer and to $c_i - \pi_{0i}$ for seller, what is presented in Fig. 3.

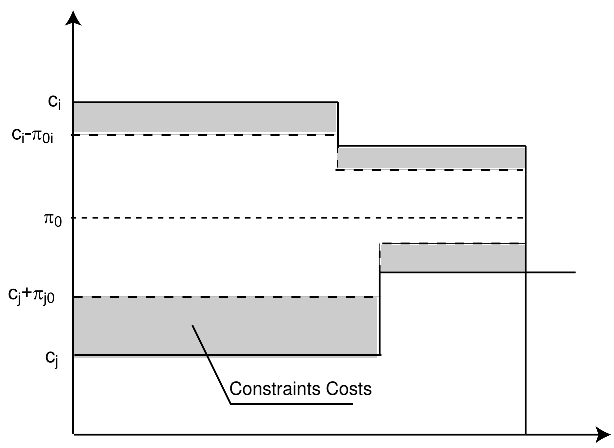


Figure 3: Prices changes caused by constraint costs allocated by node fees – π_{j0}/π_{0j}.

4.1 Power flow prices – π_{j0}/π_{0i}

Node fees should derive from constraint costs – x_i, which may be determined using one of presented above methods. In case of some constraints, eg. minimal generation or ramps [8], causing them offers and their share in constraint cost may be easy to determine. Although in case of network constraints it is not obvious what is influence of given load or generation to particular line flow and thereby to the constraint cost. To calculate node fees, first we have to determine the influence of the load or generation in node j on flow in line e. To do this we use tracking method proposed by Bialek in [1], which assumes that inflows are shared proportionally between outflows - the network node is a perfect "mixer". After determining the influence of net load/generation in given node on the flow in each line we distribute it among all loads and generators in this node proportionally to their production or consumption. Then we allocate cost of each network constraint x_i to specific offers according to calculated shares in each flow.

4.2 Compensations

If the system operator charges market entities by the transmission network charges he gets some surplus in amount $c(N)$ using SCRB/Shapley/Aumann-Shapely mechanisms, or higher in case of using LMP or MASIT mechanisms. If we do not consider network losses operator should not turn a profit on network constraints, so this surplus should be redistributed among market entities. Therefore, market entities which suffer a lose due to network constraints should get form operator's surplus some compensation, determined for seller j as:

$$R_j^0 = (\pi_0 - c_j)(p_j^0 - p_j) \tag{17}$$

and for buyer i as:

$$R_i^0 = (c_i - \pi_0)(d_i^0 - d_i) \tag{18}$$

where π_0 is price on market respecting constraints, c_j is unit j offer price, p_j^0 and d_i^0 are volume accepted on unconstraint market and p_j is volume accepted on market with constraints. Finally payment for seller j can be determined as:

$$O_j = (\pi_0 - \pi_{j0})p_j + R_j^0 \tag{19}$$

and amount to be payed by buyer i as:

$$I_i = (\pi_0 + \pi_{0i})d_i - R_i^0 \tag{20}$$

4.2.1 Compensations granting

Compensations should not be granted automatically to every entity, whose offer were reduced in comparison to unconstraint market. Operator has to carefully analyze every reduction and determine if it was really caused by system constraints or if it is speculative attempt of taking over some undue trade profits. System operator should verify if generation or load on unconstraint market – p_j^0/d_j^0 is justified and if costs are not speculatively inflated or underpriced. In other situation compensations mechanism creates broad possibilities of abuses. Extreme, but illustrative, example may be generator, which is not connected to the network. If such market entity submit offer with low price, it will be accepted on unconstraint market, and completely reduced on constraint market. Therefore entity gets large compensation, without necessity of doing anything, because it is sure that offer will be completely reduced.

4.3 Market price determination

On market without constraints market price – π_0 may be unequivocally determined as a offer price of the last accepted – marginal offer (sell offer with highest price or buy offer with lowest price). Such a price gives clear rules of offers' acceptance: all sell offers with offer price below π_0 and buy offers with offer price above π_0 are fully accepted, offers with price equal π_0 may be accepted in any part and all other offers are rejected. In case of market with constraints determination of π_0 is not so easy, among other things because due to system constraints some competitive offers may be fully or partially rejected whereas some uncompetitive offers may be fully or partially accepted [9].

In this paper we analyze few approaches to market price determination, in the first we calculate π_0 as a price of marginal sell offers (with highest offer price). Another solution may be formulation of the problem of market price determination in a form of a multi-criteria LP problem. In first attempt we simply maximize market entities income – z_n, determined according to equations (19) and (20) and entities' offer prices as (22) and (24), using following LP model:

$$\max_{n \in B \cup S} z_n \tag{21}$$

$$z_j = [(\pi_0 - \pi_{j0})p_j + R_j^0] - c_j p_j \qquad \forall j \in S \tag{22}$$

$$R_j^0 = \max(0, (\pi_0 - c_j)(p_j^0 - p_j)) \qquad \forall j \in S \tag{23}$$

$$z_i = c_i d_i - [(\pi_0 + \pi_{0i})d_i - R_i^0] \qquad \forall i \in B \tag{24}$$

$$R_i^0 = \max(0, (c_i - \pi_0)(d_i^0 - d_i)) \qquad \forall i \in B \tag{25}$$

We denote set of all sellers as S and set of buyers as B is. If we assume that load is negative generation ($p_i = -d_i$) and that $\pi_{0i} = -\pi_{i0}$, what is feature of all considered cost allocation methods, after simple transformations we get:

$$\max_{n \in B \cup S} z_n \tag{26}$$

$$z_n = [(\pi_0 - \pi_{n0})p_n + R_n^0] - c_n p_n \qquad \forall n \in S \cup B \tag{27}$$

$$R_n^0 = \max(0, (\pi_0 - c_n)(p_n^0 - p_n)) \qquad \forall n \in S \cup B \tag{28}$$

We solve above multi-criteria problem using equitable preferences model [4].

Solving problem (26) - (28) we may obtain negative income (lose) for some entities. Introduction of strict constraint ($z_n \geq 0 \; \forall n \in B \cup S$) leads to unfeasible model for some cases, therefore we may prevent such situation by adding soft constraint, in a form of penalty for market entities loses. In such a case problem of market price determination may be formulated as follows:

$$\max_{n \in B \cup S} z_n^M \tag{29}$$

$$z_n = [(\pi_0 - \pi_{n0})p_n + R_n^0] - c_n p_n \qquad \forall n \in S \cup B \tag{30}$$

$$R_n^0 = \max(0, (\pi_0 - c_n)(p_n^0 - p_n)) \qquad \forall n \in S \cup B \tag{31}$$

$$z_n = z_n^+ - z_n^- \qquad \forall n \in B \cup S \tag{32}$$

$$z_n^M = z_n^+ - \alpha z_n^- \qquad \forall n \in B \cup S \tag{33}$$

where parameter $\alpha \geq 1$ determine how strong the penalty for loses is.

This method of market price – π_0 determination try to ensure nonnegative incomes for all market entities (see Table 3). However in some specific situations it is impossible, e.g. in case when marginal buyer and seller has the same (or very similar) offer pice – c_m. Pricing mechanism with only one market price can not distribute economic wealth is such a way, that all market participants do not incur losses (see Table 4). If part of constraints costs is allocated to the marginal entities, minimum price which gives nonnegative profit to the marginal seller increases to $c_m + \pi_{j0}$, while maximum price, which marginal buyer may pay without incurring losses decreases to $c_m - \pi_{0i}$. Thus price of marginal seller is higher than price of marginal buyer, and for any value of π_0 both of them can not achieve nonnegative profit. This situation was depicted on the Fig. 4a. In this case market procedures generate some system's costs – R,

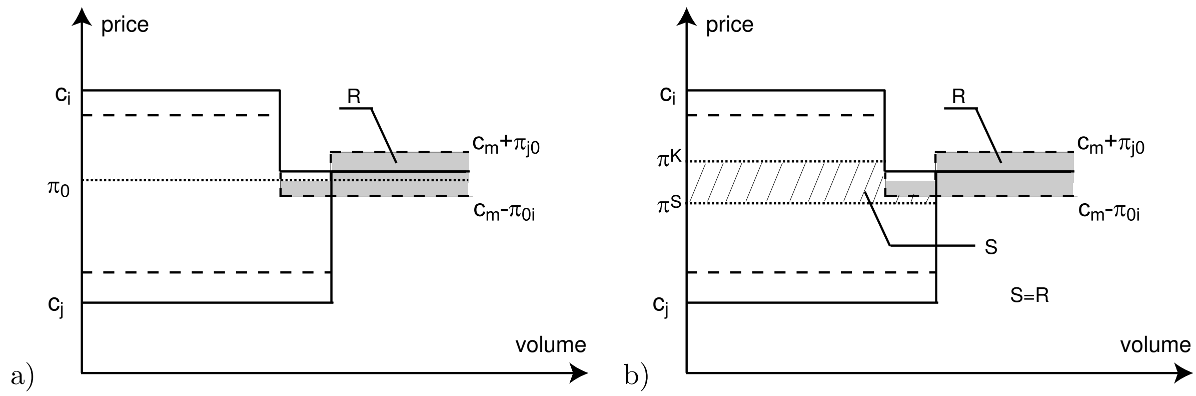


Figure 4: System cost - R caused by constraints cost, and market sell and buy price diversification

which result from fact that offers of marginal players were in fact uncompetitive, respecting constraints costs, and were accepted only because during quantity balancing market operator do not take these cost into consideration. However it is desirable that such markets players achieve nonnegative profits, because in other case they would have incentives to submit untruth offers. To compensate all system costs market operator may diverse prices for buyers and sellers, determining selling price - π^S and buying price – π^K. Difference between these prices generate some surplus - S for operator, which may be used to compensate system costs - R, see Fig. 4b. To determine market prices as well as compensations granted to market entities operator may use following LP model:

$$\max_{n \in B \cup S} z_n \tag{34}$$

$$\lambda_j^+ - \lambda_j^- = \pi^S - (\pi_{j0} + c_j) \qquad \forall j \in S \tag{35}$$

$$\lambda_i^+ - \lambda_i^- = c_i - (\pi^K + \pi_{0i}) \qquad \forall i \in B \tag{36}$$

$$R_n = \lambda_n^- |p_n| \qquad \forall n \in B \cup S \tag{37}$$

$$R_n^0 = \lambda_n^+ (|p_n^0| - |p_n|) \qquad \forall n \in B \cup S \tag{38}$$

$$z_n = (\lambda_n^+ - \lambda_n^-)|p_n| + R_n + R_n^0 \qquad \forall n \in B \cup S \tag{39}$$

$$\sum_{i \in B} (\pi^K + \pi_{0i})|p_i| - \sum_{j \in S} (\pi^S - \pi_{j0})|p_j| - \sum_{n \in B \cup S} (R_n + R_n^0) = Q^0 \tag{40}$$

Where λ_n^+ is surplus for offer n above offer price and transmission service price, λ_n^- is shortfall for offer n, R_n^0 is compensation for offer reduction due to constraints, R^n is compensation for forced uncompetitive offer realization and Q^0 is surplus, which market operator wants to obtain on the market. In many cases $Q^0 = 0$, but sometimes operator may need some resources to cover system costs, which are not explicitly priced on the market, or simply to cover costs of his activities. Above model relays on balancing models proposed in [9], we added prices for energy transmission service – π_{0j}/π_{i0} and changed objective function to maximize profits of all market entities.

If we substitute (37) and (38) into (39) and (40) after simple transformations we get:

$$\max_{n \in B \cup S} z_n \tag{41}$$

$$\lambda_j^+ - \lambda_j^- = \pi^S - (\pi_{j0} + c_j) \qquad \forall j \in S \tag{42}$$

$$\lambda_i^+ - \lambda_i^- = c_i - (\pi^K + \pi_{0i}) \qquad \forall i \in B \tag{43}$$

$$z_n = \lambda_n^+ |p_n^0| \qquad \forall n \in B \cup S \tag{44}$$

$$-\sum_{n \in B \cup S} (c_n p_n + \lambda_n^+ |p_n^0|) = Q^0 \tag{45}$$

We may realize that profits for entity n depends only on surplus of market price above offer price and transmission service price and volume accepted on unconstraint market, whereas payment for entity is equal to its profit plus value of offer according to its offer price c_n. Therefore appropriate determination of p_n^0 is very important. This value has great influence on entities profits and often may be subject of speculations, therefore it has to be very carefully verified as it was mentioned earlier .

Solving model (41) - (45) we may observe some undesired features. First of all model abuse compensation mechanism and very often price π^S is equal 0, while sellers get very large compensations. This results from fact that only objective of LP problem is to distribute economic wealth as evenly as it is possible. If we add as additional objective – minimization of difference between π^K and π^S mechanism try to get even economic wealth distribution using similar selling and buying prices. We usually get the same profits distribution like in previous case, but with other, much more similar, values of π^S and π^K. Therefore we may suppose that LP problem has a lot of various solutions with the same value of the objective function.

Another problem is fact that entities income which result from proposed mechanism are very often not diverse enough. In fact if it is only possible every market entity would get the same income, because in such situation we get the best value of the objective function (see Table 5). Whereas we want to diverse offers, and reward most the best of them. Using equitable preferences model we can not apply directly weight coefficients for each offer, therefore instead of maximizing entities income we decided to minimize difference between entity's income gained on the constraint market (z_n) and income gained on the unconstraint market[1] ($(\pi_\emptyset - c_n)p_n^0$) minus allocated constraint costs ($\pi_{n0}p_n$). Thus objective function may be formulated as:

$$\min_{n \in B \cup S} \{|[(\pi_\emptyset - c_n)p_n^0 - \pi_{n0}p_n] - z_n|, (\pi^K - \pi^S)\} \tag{46}$$

Value $[(\pi_\emptyset - c_n)p_n^0 - \pi_{n0}p_n]$ is in such a case some kind of reference point for market entity's incomes - z_n. We may notice one more time that volume accepted on unconstraint market – p_n^0 has big influence on profits distribution and may be very often subject of speculations. Therefore it should be very carefully verified. Simple solution may be usage of p_n as verified value of p_n^0 using objective function:

$$\min_{n \in B \cup S} \{|(\pi_\emptyset - c_n - \pi_{0n})p_n - z_n|, (\pi^K - \pi^S)\} \tag{47}$$

5 Methods comparison

We prepared framework system for testing and comparison various approaches to constraints cost allocation and market entities pricing and used it for experiments. At the beginning we consider simple example with three nodes presented in Fig. 5. Let us assume existence of two producers, located in the first and in the second node and one consumer located in the third node. Every market participant submits one elastic offer, values of particular offers are presented in Fig. 5. Network has three edges, every one is constrained in the respect of maximum acceptable flow. After market quantity balancing we get generation and loads: (118, 16, 134), power flows: (34, 50, 84) and economic wealth $\widetilde{Q} = 3700$. In case of constraints absence we get generation and loads: (120, 30, 150), power flows (30, 90, 60), and economic wealth $\widetilde{Q} = 3900$, so it follows

[1] We assume that $\pi_\emptyset$ is market price determined on market without constraints

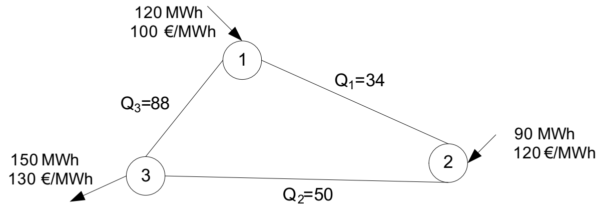


Figure 5: Considered network

that $c(N) = 200$.

Pricing results for each pricing method are presented in Tables 1 – 6. For each cost allocation method there are shown: operator's surplus, market price – π_0, or prices π^S and π^K, summary trade profit as well as profits, prices, allocated constraints costs and compensations – R, R^0 for each offer. In Table 1 there was presented results for the LMP method, which are used to compare with other methods.

	LMP Surplus: 3700		
offer.	profits	price	al. cost
1	0	100	2360
2	0	120	0
3	0	130	1340
sum	0		

Table 1: Profits for each offers, using LMP method

	SCRB/Shapley Surplus: 0, π_0=120				MASIT Surplus: 50, π_0=120				Aumann-shapley Surplus: 0, π_0=120			
o.	prof.	price	cost	R^0	prof.	price	cost	R^0	prof.	price	cost	R^0
1	2323.2	119.69	76.8	40	2297.4	119,47	102.6	40	2364	120.03	36	40
2	**-23.2**	118.55	23.2	0	**-22.4**	118,6	22.4	0	**-64**	116	64	0
3	1400	119.55	100	160	1375	119,74	125	160	1400	119.55	100	160
$\sum$	3700				3650				3700			

Table 2: Profits for each offers, using price of marginal sell offer

Table 2 presents results obtained using price of marginal sell offer as market price – π_0. We may notice, that costs allocated to the market entities in the LMP method are slightly higher than in other methods. In our example operator gain whole economic wealth in amount 3700. The operator receives large trade surplus, so he has no incentives to mitigate system constraints, because he may profit on them. On the other hand, using game theory methods and market price based on marginal offers some entities may achieve negative profits (in our example marginal offer (no. 2), with $c_2 = \pi_0 = 120$).

If we determine market price – π_0 using LP problem (29) – (33) marginal offers do not achieve negative profits, if offer price of marginal buyer is higher enough than offer price of

	SCRB/Shapley Surplus: 0, π_0=120.77				MASIT Surplus: 50, π_0=120.74				Aumann-shapley Surplus: 0, π_0=122.13			
o.	prof.	price	cost	R^0	prof.	price	cost	R^0	prof.	price	cost	R^0
1	2416	120.47	76.8	41,55	2387	120.23	102.6	41.49	2620	122.2	36	44.27
2	**0**	120	23.2	10,83	**0**	120	22.4	10.45	**0**	120	64	29.87
3	1284	120.42	100	147.63	1263	120.57	125	148.05	1080	121.94	100	125.87
$\sum$	3700				3650				3700			

Table 3: Profits for each offers, using market price calculated by LP model

marginal seller, see Table 3. Income of marginal entities is equal zero for each method, what is positive feature. Economic wealth is distributed among market entities, only in case of MASIT cost allocation algorithm operator get small surplus. However if offer prices of marginal seller and marginal buyer are the same or close enough, they achieve negative profits what is shown in Table 4.

	SCRB/Shapley Surplus: 0, π_0=120				MASIT Surplus: 40, π_0=120				Aumann-shapley Surplus: 0, π_0=120			
o.	prof.	price	cost	R^0	prof.	price	cost	R^0	prof.	price	cost	R^0
1	2383.2	120.20	16.8	40	2366.4	120.05	33,6	40	2412	120.44	-12	40
2	**-3.2**	119.8	3.2	0	**-6,4**	119.6	6.4	0	**-32**	118	32	0
3	**-20**	120.15	20	0	**-40**	120.30	40	0	**-20**	120.15	20	0
$\sum$	2360				2320				2360			

Table 4: Profits for each offers, using market price calculated by LP model, with equal prices of marginal buyer and seller (120)

If we use pricing model with two prices (41) – (45) all entities always get nonnegative profits. However mechanism try to distribute economic wealth as equal as it is possible, therefore we often get equal incomes for all entities, what is undesired feature, because we want to diverse market entities and reward the most competitive offers. In Table 5 there are shown results obtained maximizing profits of all players and minimizing difference between π^K and π^S. If we omit minimization of price gap, we get exactly the same profits distribution, but with π^S equal 0, and large system cost compensation – R.

	SCRB/Shapley Surplus: 0 $\pi^K = 121.03$ $\pi^S = 110.93$					MASIT Surplus: 0 $\pi^K = 120.84$ $\pi^S = 111.15$					Aumann-shapley Surplus: 0 $\pi^K = 121.03$ $\pi^S = 110.58$				
o.	prof.	price	cost	R	R^0	prof.	price	cost	R	R^0	prof.	price	cost	R	R^0
1	1233.33	110.45	76.8	0	20,56	1233.33	110.45	102.6	0	20.56	1233.33	110.45	36	0	20.56
2	1233.33	197.08	23.2	826.12	575,56	1233.33	197.08	22.4	821.82	575.56	1233.33	197.08	64	872.45	575.56
3	1233.33	120,8	100	0	131.56	1233.33	120.8	125	0	131.56	1233.33	120.8	100	0	131.56
$\sum$	3700					3700					3700				

Table 5: Profits for each offers, using market price calculated by LP model with two prices, maximization of entities incomes

If we minimize difference between entity's income gained on the constraint market and on the unconstraint market using objective function (46), we get diverse market entities incomes, what was shown in Table 6. Incomes of all market entities are very similar to their reference

	SCRB/Shapley Surplus: 0		$\pi^K = 119.998$ $\pi^S = 119.914$			MASIT Surplus: 0		$\pi^K = 119.8$ $\pi^S = 120.13$			Aumann-shapley Surplus: 0		$\pi^K = 120.13$ $\pi^S = 119.74$		
o.	prof.	price	cost	R	R^0	prof.	price	cost	R	R^0	prof.	price	cost	R	R^0
1	2311.6	119.590	76.8	0	38.53	2311.2	119.586	102.6	0	38.52	2332	119.76	36	0	38.87
2	0	120	23.2	24.57	0	0	120	22.4	20.33	0	0	120	64	68.19	0
3	1388.4	119.639	100	0	148.1	1388.8	119.636	125	0	148.14	1368	119.79	100	0	145.92
$\sum$	3700					3700					3700				

Table 6: Profits for each offers, using market price calculated by LP model with two prices, minimization of income deviation from reference point

profits – incomes on unconstraint market minus allocated constraint costs (2364, -64, 1400) and close to profits, which could be reached on the unconstraint market (2400, 0, 1500). At the same time each market entity get nonnegative profit, market price gap is relatively small, and all economic wealth could be distributed among market entities. Even in case of MASIT cost allocation algorithm, which may allocate on entities cost higher than $c(N)$ operator surplus may be returned to entities. Unless operator does not set high value of his own surplus – Q^0. Returning of surplus gained by operator to market entities may result in market price inversion ($\pi^S > \pi^K$) as it may be noticed in Table 6.

Weak point of this method may be big influence of p_n^0 value on pricing result, which may be often subject of speculations. We may tray to verify this value, e.g. using p_n instead of p_n^0, like in objective function (47). Result for honest offers may be in such a case a little bit disturb, but mechanism should be less susceptible to speculations.

We also examined how various pricing methods react to small changes of parameters. If we change slightly line 1 capacity from 34 to 35 results of the LMP method changes significantly, both from the system operator as well as from the individual entities points of view, see Table 7. Operator's surplus is decreased by 2950, while profits for offer no. 1 are increased by 3000. In case of pricing model (42) – (45) with objective function (46) changes were appreciably lower for all cost allocation algorithms, see Table 8. Therefore we can suppose that the LMP method seems to be notably less stable than other considered methods.

	LMP Surplus: **750**		
offer.	profits	price	al. cost
1	**3000**	**125**	**-600**
2	0	120	0
3	0	130	1350
sum	0		

Table 7: Profits for each offers, using LMP method, with line 1 capacity increased from 34 to 35

	SCRB/Shapley $\pi^K = 120.00$ Surplus: 0 $\pi^S = 119.92$					MASIT $\pi^K = 119.99$ Surplus: 0 $\pi^S = 119.94$					Aumann-shapley $\pi^K = 120.02$ Surplus: 0 $\pi^S = 119.91$				
o.	prof.	price	cost	R	R^0	prof.	price	cost	R	R^0	prof.	price	cost	R	R^0
1	2334	119.45	57	0	0	2331.75	119.43	61.5	0	0	2336.25	119.47	52.5	0	0
2	0	120	18	19.125	0	0	120	13.5	14.34	0	0	120	22.5	23.91	0
3	1416	119.51	75	0	141.6	1418.25	119.49	75	0	141.83	1413.75	119.53	75	0	141.38
$\sum$	3750					3750					3750				

Table 8: Profits for each offers, using market price calculated by LP model with two prices, minimization of income deviation from reference point. Line 1 capacity increased from 34 to 35.

6 Summary

In the paper we proposed and analyzed few methods of power market pricing. We focused on physical constraints and proper allocation of costs that they contribute to the market system.

Appropriate cost allocation is crucial to design efficient market system for such commodity like electrical energy, because it may motivate market entities to submit sincere offers and to reveal their true costs as well as to mitigate constraints. Proposed methods were compared with widely applied LMP method which, as it was shown, has some negative features. Large surplus is gained by market operator, which do not have incentives to mitigate constraints. Obtained results are unstable and seems to be susceptible to speculation, especially that there is marginal player in each node. Other considered methods seem to have some better features. Firs of all they distribute economic wealth among market entities and not allocate most of it to the operator. On the other hand operator may ensure some necessary surplus Q^0 for himself. Proposed pricing methods allow to use various constraint pricing method, therefore one can chose method which fits best to given model. Obtained results are more stable than in LMP method and may be less susceptible to speculations.

Proposed methods have to be further analyzed and developed. Important future research subjects are incentive compatibility and market power mitigation on the assumption that some market entities speculate with their offer prices.

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