Market Design in Wholesale Electricity Markets

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Abstract

As European countries move towards complete unbundling in electricity markets, some issues regarding market design are still under discussion. In particular, which market configuration would give the right incentives to promote efficiency and reduce final prices. In this paper we analyze a design in which prices are binding for more than one market period (like in the former British system or in the Australian system) and we compare price equilibria and collusive incentives under proportional and efficient rationing. To do so, we build on Le Coq (2002) and Crampes and Creti (2003) framework to account for stochastic demand. Our results suggest that with stochastic demand, incentives for strategically withholding capacity are still present but incentives to agree on market share are mitigated by efficient rationing.

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1 Introduction

Many authors have used game theory to model wholesale electricity markets since monopolistic and vertically integrated environments left their place to deregulated environments where the objective is profit maximization instead of system-wide cost minimization. In particular, theory tried to find out if the market configuration chosen by different countries provided the right incentives for efficient production.

A first group of papers used Bertrand paradigm to model electricity markets taking capacity constraints as given. This is the case of Hobbs (1986) that used the previous paradigm applied to New York's electricity market. Similarly, others modeled power transactions as static and complete information games where information costs were shared among players and bidding prices were linked with generation output (e.g. Ferrero et al. 1997).

It has been observed that, while state monopoly of generation was accompanied by over-investment in capacity, in a liberalized market big generators use their available capacity strategically to enhance their market power. Patrick and Wolak (2001) studied British market for the period 1991-1995 and found evidence that nourish this affirmation[1]. Similar results were obtained by other econometrics based empirical work, such as Joskow and Kahn (2001) and Harvey and Hogan (2001).

Therefore, models that endogenize capacity choice gain particular relevance. This is the case of von der Fehr and Harbord (1997) that model a two-stages-game and assumed that utilities choose *investment* in a first stage and price in a second one working in a framework similar to the one used in the seminal paper of Kreps and Scheinkman (1983). The later had shown that, in a market with homogeneous products in which firms non-cooperatively pick capacities in the first stage and set prices in the second stage, the equilibrium outcome was that of a one-shot Cournot game. In the same line, Castro-Rodriguez et al. (2001) show that from a social welfare point of view, firms build low levels of generation capacity. Finally, Reynolds and Wilson (2000), analyzing investment and pricing incentives in a symmetric Bertrand-Edgeworth framework, find that a smaller firm has no incentive to expand its capacity as this expansion would reduce its expected revenue if demand is lower than expected.

We will not focus on long-run *investment* decisions but more precisely generator's "capacity declarations", i.e. their short-term capacity availability. In other words, this means that the strategic instrument

[1] The same has been observed in other countries like Spain, Norway, Sweden and Finland and of course California.

we will consider is not underinvestment per se but capacity withholding: when generators reach their declared capacity there is an incentive to bid a higher price. Moreover, given that demand in this market is highly inelastic, the incentive to bid a high price is even stronger.

Le Coq (2002) and Crampes and Creti (2003) are particularly interested in the phenomenon just described. Both papers, like Kreps and Scheinkman (1983), consider a two-stage game where in a first stage agents bid capacities *knowing* demand, and in the second one they bid prices. However, differently from the previous literature, they model the second stage as a uniform price auction (instead of Bertrand competition) to describe the most commonly observed electricity market design. The latter adds value to the former allowing for mixed strategy Nash equilibria for a specific cost configuration.

We build on their framework to analyze an alternative design that is the one present in the Australian system and in the former British system. In these cases generators must submit a single bid for the 48 thirty-minute day-ahead markets (and therefore facing a stochastic demand): in our model agents bid capacities *without knowing* the level of demand that will be realized. Then, we compare the results of our model in terms of price equilibria and collusive incentives under proportional and efficient rationing. Finally, we compare our results with the ones found with deterministic demand by Crampes and Creti (2003).

Our market configuration leads to the same results found in Crampes and Creti (2003) in that the equilibrium price is never the one that would result from a Bertrand game, that is, by withholding capacity agents succeed in maintaining high prices. But contrary to what happens in their market configuration, in our design we find that incentives to agree on market share are mitigated when demand is stochastic and the System Operator (SO) uses efficient rationing.

In Section 2 we state the assumptions and timing of the game. In Section 3 and 4 we present our model and compare it with previous findings. In Section 5 we conclude and suggest some further extensions.

2 Assumptions and timing of the game

We assume there are two generators G_a and G_b. It is common knowledge that one of them, lets say G_a, is bigger a priory ($K_{a\max} > K_{b\max}$) and more efficient[2] so the marginal costs satisfy: $c_a < c_b$.

We also assume there exists a price cap[3] fixed by the regulator de-

[2] This is generally the case in electricity markets where the different technologies used to produce energy have well known costs and returns.

[3] Usually the regulator sets the price cap after estimating the efficient marginal cost of generation.

noted by $\hat{p} > c_b$.

We proceed by backward induction so the last stage exposed is the first one played.

In the *second stage* of the game, we assume that generators play a capacity constrained game. Therefore, before bidding price, each generator j knows the capacity chosen by his competitor i and the level of inelastic demand[4] for the day-ahead market that in the first stage was unknown.

For each market period, the market is cleared by a uniform price auction. This means that when agents bid different prices, the low bidder is dispatched first and then the higher bidder is dispatched the residual demand. The marginal price for electricity will be the one bided by the high bidder and both will be paid this price for the amount of electricity they are called for.

When two generators bid the same price we will assume for a start that, as in most countries, proportional rationing is used, that is, each generator G_i is dispatched a quantity equal to $\frac{K_i}{K_i+K_j}$. In Lemma 17 we will prove that results for the second stage of the game are invariant if efficient rationing is used instead[5].

The day is divided into market periods[6], usually hourly or for every 30 minutes and like in the case of the former British market or the Austrialian market, the price must be held fixed for more than one market period. In this case generators don't know demand when they decide their level of production (or capacity as we will call it). We capture this feature by assuming demand is stochastic in the first stage of the game.

Therefore, in the *first stage* to be modeled in section 4 generators choose to produce any amount lower than their maximum capacity $K_i \leqslant K_{i\max}$ without knowing the level of demand that can be at its peak level D_p with probability h or off peak, D_n, with probability $(1-h)$.To model this we will assume that only the output effectively produced will generate costs.

Moreover, we will assume that shortage can only be provoked by firms[7], that is $K_{a\max} + K_{b\max} > D_p$ and that there is a penalty if this occurs: both generators must pay a fixed amount denoted by S.

[4]Assuming inelastic demand is reasonable because, as explained by Joskow and Tirole (2004), in most of the countries consumers are metered only once a month or every few months having no incentives to change demand in response to changes in real time prices.

[5]Of course this is not the case for the first stage of the game where incentives to collude change greatly weather we assume efficient or proportional rationing.

[6]This period is usually called load.

[7]We have no interest in analyzing the case in which there is unintentional shortage, that is, we are not interested in long term investment decisions.

3 Second stage: price game

The choice of capacity made in the first stage of the game K_i will lead generators to one of the following ex-post demand cases.

3.1 *Case a) Low demand*

If once revealed demand is such that $D < K_i$; $i = a, b$, both generators have enough capacity to satisfy all the market by themselves if the SO dispatches it. If both enterprises had symmetric marginal costs we would be in Bertrand's classical price competition but, as we assumed that the bigger one is more efficient ($c_a < c_b$), G_a will win the game.

Proposition 1 *If the pair (K_a^{br}, K_b^{br}) chosen in the first stage determines that none of the generators is capacity constrained, the pair (p_a^{br}, p_b^{br}) that satisfies the following equation is a Nash equilibrium of the price subgame:*

$$\begin{aligned} p_a^{br} &= \max(c_b - \varepsilon, c_a) \quad \textit{which gives} \quad \Pi_a = (c_b - c_a)D \\ p_b^{br} &= c_b \quad \textit{which gives} \quad \Pi_b = 0 \end{aligned} \tag{1}$$

where ε tends to 0.

Proof. See Apendix. ■

The fact that ε tends to zero means that it is a bid smaller than c_b but as close to it as possible[8].

Therefore in this case of ex-ante low demand, in equilibrium, G_b will realize zero profits and G_a will serve all the demand.

3.2 *Case b) Intermediate demand*

If once revealed demand is $K_i < D < K_j$ there exists a continuum of equilibria where the price cap $\hat{p}$ is offered by the larger firm knowing that the low capacity firm will offer less to avoid being undercut. Both receive the price cap at the end (uniform auctioning).

Proposition 2 *When demand is lower than the larger capacity declared, if G_a, the most efficient generator, is the one with the capacity advantage, that is $K_b^{br} < D < K_a^{br}$, there are two possible families of equilibria for (p_a^{br}, p_b^{br}):*

[8] In real markets generator's price announcements are limited to a finite number of values lower than the price cap called ticks. We ignore this to avoid calculus complications that would not change our results significantly.

a)

$$\begin{aligned} p_a^{br} &= \hat{p} \qquad \textit{which gives} \qquad \Pi_a = (\hat{p} - c_a)(D - K_b) \\ p_b^{br} &\in [0, \alpha_a] \qquad \textit{which gives} \qquad \Pi_b = (\hat{p} - c_b)K_b \\ \alpha_a &= c_a + (\hat{p} - c_a)\frac{(D - K_b)}{D} \qquad \textit{where } \alpha_a > c_b \end{aligned} \tag{2}$$

b) Equilibrium defined in Case a if $\alpha_a < c_b$.

where α_a *is the threshold that ensures* G_b *to sell all his production when* G_a *bids at the price cap.*

Proof. See Appendix. ■

G_b will be dispatched first as he bids a lower price and he will sell all the capacity he made available. Then, G_a will be dispatched the residual demand. Finally both will receive the uniform price cap for each MWh of electricity.

It could also be the case that, even if $K_{a\max} > K_{b\max}$, in the first stage of the game G_a chose to make available less capacity than G_b. In that case where $K_a^{br} < D < K_b^{br}$, instead of the two families of equilibria described in the previous proposition we would have only one family of equilibria symmetric to the one described in (2) simply changing subindex a for b and vice versa. In this case, G_a would be dispatched first and G_b would be the one serving the residual demand. The only difference would be that the condition on the threshold, that in this case would be $\alpha_b > c_a$ is not needed any more. This is the case because as we assumed $c_a < c_b$, G_b is always better off bidding the price cap $\hat{p}$ than undercutting G_a's bid (strategy that gives negative profits).

Proposition 3 *When demand is lower than the larger capacity declared, if* G_b*, the less efficient generator, is the one with the capacity advantage, that is* $K_a^{br} < D < K_b^{br}$*, there is only one possible family of equilibria for* (p_a^{br}, p_b^{br})*:*

$$\begin{aligned} p_b^{br} &= \hat{p} \qquad \textit{which gives} \qquad \Pi_b = (\hat{p} - c_b)(D - K_a) \\ p_a^{br} &\in [0, \alpha_b] \qquad \textit{which gives} \qquad \Pi_a = (\hat{p} - c_a)K_a \\ \alpha_b &= c_b + (\hat{p} - c_b)\frac{(D - K_a)}{D} \end{aligned} \tag{3}$$

Proof. The proof follows from the one of the previous proposition. ■

3.3 *Case c) High demand*

If once revealed demand is $K_i < K_j < D < K_i + K_j$ we find two sets of equilibria in pure strategies: one in which G_b bids the price cap and serves just residual demand and anotherone in which G_a is the one that bids the price cap while G_b undercuts this bid and is dispatched first.

This is the case as the production of both agents is needed to satisfy demand and therefore both know that they will be called into operation.

Proposition 4 *If the pair (K_a^{br}, K_b^{br}) chosen in the first stage determines that $K_i^{br} < K_j^{br} < D$, we find two set of equilibria in pure strategies for the pair (p_a^{br}, p_b^{br}):*

a)

$$\begin{aligned} p_a^{br} &= \hat{p} \qquad \textit{which gives} \qquad \Pi_a = (\hat{p} - c_a)(D - K_b) \\ p_b^{br} &\in [0, \beta_a] \qquad \textit{which gives} \qquad \Pi_b = (\hat{p} - c_b)K_b \\ \beta_a &= c_a + (\hat{p} - c_a)\frac{(D - K_b)}{K_a} \qquad \textit{where } \beta_a > c_b \end{aligned} \tag{4}$$

where β_a is the threshold for p_b^{br} that ensures that G_a is better off bidding the price cap $\hat{p}$.

b)

$$\begin{aligned} p_b^{br} &= \hat{p} \qquad \textit{which gives} \qquad \Pi_b = (\hat{p} - c_b)(D - K_a) \\ p_a^{br} &\in [0, \beta_b] \qquad \textit{which gives} \qquad \Pi_a = (\hat{p} - c_a)K_a \\ &\textit{with} \qquad \beta_b = c_b + (\hat{p} - c_b)\frac{(D - K_a)}{K_b} \end{aligned} \tag{5}$$

Proof. See Appendix. ■

If $\beta_a < c_b$, we would have *b)* as the unique set of equilibria in pure strategies but this particular case needs not to be satisfied neither in our model nor in reality. Therefore, when $\beta_a > c_b$, none of this two set of equilibria can be discarded by a Pareto dominance criterion as both generators would prefer to be the lower bidder in order to sell all the capacity they made available. This means that G_a is better off in equilibria *b)* while G_b is better off in equilibria *a)*. Then, the two set of pure-strategy equilibria described illustrate that when a firm chooses a price to bid, on one side, he has an incentive to bid the highest price possible as he knows that he will be called into operation anyway. On the other side, bidding low enough to avoid undercutting ensures to be called into operation first in the merit order and be able to sell all the capacity made available. The previous contraposition makes some authors, like Larson and Salant (2003), say that in the case of uniform auctions where prices can vary continuously[9] and all the players know they must be called into operation, is more likely that generators play in mixed strategies. Similarly, Binmore et al. (2004) say "...the equilibria may well be in mixed strategies whenever bidders do not have sufficient lots to satisfy all of the demand, but there is excess supply in aggregate."

Given the previous argument, we will also characterize the equilibrium in mixed strategies. In this equilibrium, for a given strategy of the competitor, each generator is indifferent between all the prices over which he randomizes.

Proposition 5 *If the pair* (K_a^{br}, K_b^{br}) *chosen in the first stage determines that* $K_i^{br} < K_j^{br} < D$ *and* $\beta_a > c_b$[10]*, we find the following mixed strategies equilibrium for* p_j^{br}*:*

$$p_j^{br} \sim F_i(p_i) \qquad on \qquad [c_j, \hat{p}] \tag{6}$$

where $F_i(p_i)$ *describes the cumulative distribution of probabilities for* G_i *bids in the support* $[c_j, \hat{p}]$*.*

Proof. See Appendix. ■

3.4 *Case d) Not served demand*

If demand is $D > K_a + K_b$, there is a shortage penalty $S \geq 0$ such that the profits for both generators lower than in the previous case: $\Pi_i = (\hat{p} - c_i)K_i - S$. This implies that generators will try to avoid paying this shortage penalty, that is, they will prefer to be in *Case b* or *Case c* than in this case.

3.5 Key results from the second stage

Some remarks are worthwile before moving to the first stage of the game. In *Case b* of ex-post intermediate demand, even if demand could be completely served by one of the generators, say G_a, but exceeds the capacity declared by the other one, say G_b, the marginal price is equal to the price cap $\hat{p}$ and both firms benefit from high markups. The efficient outcome would be having G_a serving the whole market at a price equal to c_a[11].

[9] This implies that infinitesimal undercutting is possible.

[10] Remember that if this is not the case the set in *b)* is the unique pure nash equilibria.

[11] We will show in Section 4 that this will never be the case as *Case a* will be avoided by strategically withholding capacities.

In *Case c* of high demand, as none of the generators can satisfy the demand alone, the equilibrium price will never be lower than the highest marginal cost c_b, or in other words, as both must be dispatched to serve the demand, the least efficient firm is protected in any case against losses due to low bids. Moreover, the most efficient one knows he will never get a price lower than c_b, which ensures him a positive profit.

4 First stage: capacity game

In the previous Section we showed that different price equilibria can be reached depending on the capacities chosen in this first stage K_a and K_b in comparison with the level of demand realized. Therefore, in this Section we will model how the choice of K_a and K_b is made. As a preliminary result, we will present in the following subsection the case where this choice is made knowing the level of demand to be realized.

Then, from subsection 4.2 to 4.4 and in contrast with the existent literature, we will consider the case in which these capacity declarations are made *before* the true demand is known i.e. demand in this stage is considered *stochastic* and its distribution is assumed to be common knowledge[12].

To make our modeling easier we define a parameter $\delta = \frac{\hat{p}-c_b}{\hat{p}-c_a} < 1$ that measures the disadvantage of G_b in terms of costs. As we assumed $c_a < c_b$, this implies that $\hat{p} - c_a > \hat{p} - c_b$ which means that the new parameter is $\delta < 1$. Then, the condition on (2) that $\alpha_a > c_b$ is equivalent to $K_b < D\delta$ which ensures that $(\hat{p}-c_a)(D-K_b) > (c_b-c_a)D$, or in other words, that ensures that bidding the price cap is better than a fight in prices[13].

Similarly, the condition on (4) that $\beta_a > c_b$ is equivalent to $K_a \leq \frac{1}{1-\delta}(D - K_b)$, which in other words ensures that $(\hat{p} - c_a)(D - K_b) \geq (c_b - c_a)K_a$ is satisfied[14].

[12]Remember that D is distributed as follows:

$$D = \begin{cases} D_p \text{ with probability } h \\ D_n \text{ with probability } 1-h \end{cases}$$

[13]If $K_b < D\delta$, which is equivalent to $K_b < D\frac{\hat{p}-c_b}{\hat{p}-c_a}$. Multiplying by $(\hat{p} - c_a)$ and substracting $\hat{p}K_b$ to both sides we get $(\hat{p}-c_a)(D-K_b) > (c_b-c_a)D$ which we already showed is equivalent to $\alpha_a > c_b$.

[14]$K_a \leq \frac{1}{1-\delta}(D - K_b)$, given the definition of δ, means that $K_a \leq \frac{\hat{p}-c_a}{c_b-c_a}(D - K_b)$ equivalent to $(\hat{p} - c_a)(D - K_b) \geq (c_b - c_a)K_a$, dividing both sides by K_a and substracting c_a to both sides, the previous expresion is equivalent to $\beta_a > c_b$.

4.1 *Ex-ante low deterministic demand*

In this section we analyze the case where generators *know* the level of demand before deciding their level of production, i.e. demand is deterministic. Moreover none of them is a priory capacity constrained, i.e. $D < K_{b\max} < K_{a\max}$.

In this case the best response functions in the capacity game can be obtained as in the following Lemma 6 and 7:

Lemma 6 *The best response function of G_a in the case of deterministic low demand is:*

$$K_a(K_b) = \begin{cases} K_a > D - K_b & \text{if } K_b \leq D\delta \\ K_a = D - \varepsilon & \text{if } K_b > D\delta \end{cases} \tag{7}$$

where $\varepsilon > 0$ tends to zero.

Proof. See Appendix. ■

Lemma 7 *The best response function of G_b in the case of deterministic low demand is:*

$$K_b(K_a) = \begin{cases} K_b > D - K_a & \text{if } K_a < D \\ K_b = D\delta - \varepsilon & \text{if } K_a \geq D \end{cases} \tag{8}$$

Proof. The best response for G_b is derived computing Π_b in each of the cases in Lemma 6. ■

Given Lemmas 6 and 7, the equilibria is stated in Proposition 8.

Proposition 8 *There are three families of equilibria in the case of deterministic low demand:*

$$\begin{gathered} i)\ (K_a, K_b) = \{K_a, K_b \text{ s.t. } K_a < D, K_b \leq D\delta, K_a + K_b \geq D\} \\ ii)\ K_a \geq D, K_b = D\delta - \varepsilon \\ iii)\ K_a = D - \varepsilon, K_b \geq D\delta \end{gathered} \tag{9}$$

Proof. See Appendix. ■

4.2 *Case 1) Ex-ante low stochastic demand*

As we already said, in many countries generators have to decide the capacity they will make available *before knowing* the actual level of demand. In this subsection we derive the equilibrium assuming that just the stochastic distribution of demand is known. Moreover, we consider the general case where none of the generators is capacity constrained, i.e. $D_n < D_p < K_{b\max} < K_{a\max}$.

The best response functions under stochastic demand are obtained in Lemmas 9 and 10 for each agent respectivelly.

Lemma 9 *The best response function of G_a in the case of ex-ante low stochastic demand is:*

$$K_a^{br}(K_b) = \begin{cases} K_a > D_p - K_b & \text{if } K_b \leq D_n\delta \\ K_a = D - \varepsilon & \text{if } K_b > D_n\delta \end{cases} \tag{10}$$

Proof. See Appendix. ■

Lemma 10 *The best response function of G_b in the case of ex-ante low stochastic demand is:*

$$K_b^{br}(K_a) = \begin{cases} K_b > D_p - K_a & \text{if } K_a < D_n \\ K_b = D_n\delta - \varepsilon & \text{if } K_a \geq D_n \end{cases} \tag{11}$$

Proof. The best response for G_b is derived computing Π_b for each zone as in Lemma 9. ■

Proposition 11 *There are three families of equilibria in the case of ex-ante low stochastic demand:*

$$\begin{gathered} i)\ (K_a^{br}, K_b^{br}) = \{K_a, K_b \text{ s.t. } K_a < D_n, K_b \leq D_n\delta, K_a + K_b \geq D_p\} \\ ii)\ K_a^{br} \geq D_n, K_b^{br} = D_n\delta - \varepsilon \\ iii)\ K_a^{br} = D_n - \varepsilon, K_b^{br} \geq D_n\delta \end{gathered} \tag{12}$$

Proof. See Appendix. ■

Each of these families of equilibria in the capacity game will lead us to a different outcome in the next step of the game described in Section 3. Comparing the payoffs for each realization of demand we can derive the following propositions:

Proposition 12 *If once revealed demand is D_p, type ii equilibria is preferred by G_b to type i and iii in that order and type iii is preferred by G_a to type i and ii in that order.*

Proof. See Apendix. ■

As profits of G_i are decreasing with K_j, in *type i* the case where the restriction on the sum of capacities is satisfied with equality $K_a + K_b = D_p$ Pareto dominates the case where $K_a + K_b > D_p$. This point is feasible when D_p is realized as, on one side, $K_a < D_n$ and therefore the following inequality is satisfied $K_a + K_b < (1+\delta)D_n$, and on the other side, given the condition on the distance between D_n and D_p mentioned in the proof of (12), the following inequality is true: $D_p < (1+\delta)D_n$.

Proposition 13 *If once revealed demand is D_n, although total payoffs differ from the ones verifyed when D_p is realized, still type ii $\succ_b$type i $\succ_b$ type iii and type iii $\succ_a$type i $\succ_a$ type ii.*

Proof. See Apendix. ■

In this case, when demand is D_n, in *type i* equilibria the point where $K_a + K_b = D$ is satisfied cannot be reached as they did not know that demand was going to be low and, to avoid the penalty, they played $K_a + K_b \geq D_p$. Therefore, when demand is low, both get lower profits than in the case of high demand.

Propositions 12 and 13 mean that each generator prefer to be the first one dispatched selling all the capacity they made available and being paid the price cap bidded by the other player. Therefore, none of the three families of equilibria in (12) can be discarded by a Pareto dominance criterion.

A final remark is worthwile to better understand the following section: in type *ii* equilibria G_a is better off when D_p is realized. Similarly, G_b is better off when D_p is realized in type *iii* equilibria as he would realize positive profits. Finally, in type *i*, as we already said, both are better off when D_p is realized and their production satisfies $K_a + K_b = D_p$.

4.3 Comparison between stochastic and deterministic demand

We will focus our comparison in two issues of interest: on one side, on the possibility of strategic capacity withholding, and, on the other side, on the incentives for players to agree on market share under either deterministic or stochastic demand. It is important to state clearly that our objective is just to discuss, in the light of our previous results, the incentives for agreeing on market share and not to model how the collusion could be attained or how the agreement could be reinforced. Modelling collusion is beyond the objective of this work and left to further extensions.

From the result in (9) that describes the equilibrium for the case where generators bid capacities *knowing* they are in an ex-ante low deterministic demand case, it can be seen that there are strong incentives to strategically withhold capacities to avoid being in *Case a.* Unfortunately, the following proposition states that the same can be concluded in the case of stochastic demand.

Proposition 14 *Introducing uncertainty with respect to the level of demand is not enough to induce players not to* ***strategically withhold capacity****.*

Proof. See Apendix. ■

Given that, as we already said, profits of G_i are decreasing with K_j, the case where the restriction on the sum of capacities is satisfied with equality $K_a + K_b = D$ pareto dominates the case where $K_a + K_b > D$ in type i equilibria. This could represent an incentive for generators to agree on market share as stated in the following Proposition:

Proposition 15 *When demand is* ***known,*** *if G_a and G_b want to avoid the worst possible outcome (type ii and type iii equilibria respectively), they can agree on sharing the demand in type i equilibria satisfying the Pareto dominant condition $K_a + K_b = D$.*

Proof. See Apendix. ■

Assuming risk averse individuals that prefer an intermediate outcome with certainty would be enough to conclude that an agreement will take place[15].

The previous reasoning is not immediately applicable to our case of study: with stochastic demand, generators do not know a priory if demand will be D_p or D_n and this makes it impossible to agree on bidding capacities such that $K_a + K_b = D$ for every play. Moreover, the probability for G_b to end up with $\Pi_b = 0$ is reduced as, even in type *iii* equilibria, he has a probability h of getting $\Pi_b > 0$ if D_p is realized. Similarly, the probability for G_a to end up with $\Pi_a = (\hat{p} - c_a)(1 - \delta)D_n$ is reduced making his incentives to agree on market share weaker. However, incentives have not yet completely disappeared as stated in the following proposition:

Proposition 16 *Introducing uncertainty with respect to the level of demand is not enough to make incentives to* ***agree on market share*** *completely disappear.*

[15] We could even suggest that, it is likely that G_a gets the higher portion of the market by threatening G_b to bid all his capacity at c_b leaving him outside the market. It would be rational for G_b to believe this threat as $\Pi_a = (\hat{p} - c_a)(1 - \delta)D_n = (c_b - c_a)D_n$ in type *ii* equilibria.

Proof. See Appendix. ■

Some could already argue that an agrement under stochastic demand is more difficult to accomplish than in the case of deterministic demand where agents just needed to agree on fixed quantities.

As the agreement with stochastic demand must be based on proportions of ex-post demand we will now investigate the consequences of imposing efficient rationing instead[16] of proportional rationing. In particular, we investigate wheather the previous proposition remains true. To do so we prove in the following lemma, as we already suggested in Section 2, that equilibria in the second stage do not depend on the fact that we used proportional rationing.

Lemma 17 *The price game equilibria described from (1) to (6) and the derived payoffs (profits) are unchanged under efficient rationing.*

Proof. See Appendix. ■

Given the above lemma, the efficient rationing rule does not change the equilibria reached in the second stage but, as we will see, has a significant effect on the possibility of collusion when demand is stochastic.

Proposition 18 *In the case of stochastic demand,* ***incentives to agree on market share are mitigated*** *by the efficient rationing rule.*

Proof. See Apendix. ■

A final comment is worthwhile: the families of equilibria found are independent of the distribution of probability of demand. Then, it is straightforward to extend our reasoning for a continuous distribution, lets say a uniform distribution between D_p and D_n. In this case we would be in a continuum of cases between the ones exposed here.

Here we have analyzed the most general case in which none of the generators is capacity constrained. Now we describe the other possible equilibria when one or both generators are ex-ante capacity constrained. The propositions stated in this section are still valid for the following cases as they are particular cases of this general unconstrained case.

4.4 *Case 2) Smaller generator is capacity constrained*

In this section we assume that the smaller generator is capacity constrained that G_a is not, i.e. $D_p < K_{a\max}$. From Figure 4 in the Appendix, the equilibria described in (12) still holds if the smaller generator is capacity constrained in peak demand, even in the case in which

[16] Efficient rationing in an auction means that, when two players bid the same price, the one with lower marginal cost c_a is dispatched first.

$K_{b\max} < D_p\delta$. It also holds if he is constrained in off peak demand when it is the case that $D_n\delta < K_{b\max} < D_n$. Therefore, the conclusions extracted in the previous section can be applied immediately to this case.

On the other hand, if the capacity constraint of the smaller generator is such that $K_{b\max} < D_n\delta$, the equilibria in (12) are not longer possible and the following proposition is true:

Proposition 19 *If the capacity constrain of the smaller generator is such that $K_{b\max} < D_n\delta$, the only possible equilibria is:*

$$(K_a^{br}, K_b^{br}) = \{K_a, K_b \ s.t. \ K_a \leq D_n, K_b \leq K_{b\max}, K_a + K_b \geq D_p\} \quad (13)$$

Proof. This is a particular case of the unconstrained case proved in Lemma 9 and Proposition 11. ■

If revealed demand is D_n, this implies that in the next stage, we will be in *Case c*, where $E(\Pi_i) = (\hat{p} - c_i)(D_n - K_j)$.

On the other hand, if demand is D_p we will be in *Case c* but with a higher demand to satisfy that will determine higher expected profits: $E(\Pi_i) = (\hat{p} - c_i)(D_p - K_j)$. Pareto optimality will be achieved in this case if $K_a + K_b = D_p$ where K_a must increase as the constrain on K_b is more severe. This becomes an advantage for G_a because even without negotiation he will serve a greater part of the market at the price cap $\hat{p}$.

In other words, in a market where proportional rationing is used, negotiation on market share is still possible in this *Case 2* but the ¨negotiation set¨ is reduced to the advantage of G_a.Moreover, capacity withholding is also possible but the outcomes among which the players can choose are reduced by a real constraint: $K_{b\max} < D_n\delta$.

On the other hand, with efficient rationing the result stated in Proposition 18 is still true for this case.

4.5 *Case 3) Both generators are capacity constrained*

This case implies that $K_{a\max} < D_n < D_p < K_{a\max} + K_{b\max}$. As in the previous case, the equilibria described in (12) still holds if K_a (and consequently[17] K_b) are lower than D_p but still higher than D_n. If it is not the case, the following proposition applies:

Proposition 20 *If $K_{b\max} < K_{a\max} < D_n$ the equilibria becomes:*

$$(K_a^{br}, K_b^{br}) = \{K_a, K_b \ s.t. \ K_a \leq K_{a\max}, K_b \leq \min(D_n\delta, K_{b\max}), K_a + K_b \geq D_p\} \quad (14)$$

[17] Remember that by assumption is always true that $K_{a\max} > K_{b\max}$.

Proof. This is a particular case of the unconstrained case proved in Lemma 9 and Proposition 11. ■

In this last case, if demand is D_n, we then will be in *Case c*, where $E(\Pi_i) = (\hat{p} - c_i)(D_n - K_j)$.

On the other hand, if demand is D_p we will be in *Case c* but with a higher demand to satisfy where Pareto optimality can be achieved if $K_a + K_b = D_p$.

In this case capacity withholding could happen but the constraints on capacity are already severe. An agreement to ensure $K_a + K_b = D_p$ could still be possible under proportional rationing but the incentives are few given that the negotiation set is already reduced. Moreover, with efficient rationing the result stated in Proposition 18 is also true for this case.

5 Conclusion

This work provides some insights into market design in electricity markets. We have modeled how price and quantity are determined in the day-ahead wholesale market for electricity as a two stages game where demand is stochastic in the first stage. Then, we have compared our results with the case of deterministic demand already modeled by Le Coq (2002) and Crampes and Creti (2003). Moreover, this comparison has been done taking into account alternatively two commonly used rationing rules: efficient and proportional rationing. Finally, we have discussed the incentives for agreeing on market share that emerge from our model results.

We have shown that incentives to agree on market share are strong in the case of deterministic demand but they are mitigated when demand is stochastic and the System Operator uses efficient rationing. This result holds for any distribution of probabilities of demand. In any case, even in this last case where an agreement is not possible, the equilibrium price is never the one that would result from a Bertrand game. Instead, by withholding capacity, agents succeed in maintaining the price higher than the efficient one and usually equal to the price cap.

As we explained in these few lines, our analysis introduces in a theoretical framework some features of real electricity markets unattended until today and in this sense constitutes a first step for further extensions. An immediate extension would be to actually model collusion under stochastic demand in order to investigate to which extent the agreement discussed here could be attained and the possible ways to reinforce it.

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7 Appendix

Only proofs of key resuls are displayed here. Other proofs are available upon request to sanin@core.ucl.ac.be.

7.1 Proof of Proposition 5

We proof the statement in relation to G_a as the solution in relation to G_b is symmetric. For any given strategy of G_b, that is, for a distribution of probabilities f_b played by G_b, G_a is indifferent between all the prices over which he randomizes, i.e. any price belonging to $[p_{a\min}, \hat{p}]$. To find f_b we maximize the expected profits of G_a with respect to p_a:

$$E(\Pi_a(p_a, f_b(p_b)) = \int_{p_{b\min}}^{\hat{p}} \Pi_a(u) f_b(u) du \tag{15}$$

$$\int_{p_{b\min}}^{p_a} (p_a - c_a)(D - K_b) f_b(u) du + \int_{p_a}^{\hat{p}} \Pi_a(u) f_b(u) du =$$

$$= (p_a - c_a)(D - K_b) F_b(p_a) + \int_{p_a}^{\hat{p}} \Pi_a(u) f_b(u) du$$

The F.O.C. is then obtained by deriving:

$$\frac{\partial E(\Pi_a(p_a, f_b(p_b))}{\partial p_a} = 0 \tag{16}$$

$$(D - K_b)[(p_a - c_a) f_b(p_a) + F_b(p_a)] + (\hat{p} - c_a) K_a f_b(\hat{p}) - (p_a - c_a) K_a f_b(p_a) = 0$$

$$(D - K_b - K_a)(p_a - c_a) f_b(p_a) + (D - K_b) F_b(p_a) = 0$$

as $f_b(\hat{p}) = 0$ given that *punctual* value of any continuous density function is zero.

The solution of the differential equation obtained gives us the distribution we were looking for[18]:

$$F_b(p_b) = \left[\frac{p_b - c_a}{A_b}\right]^{\gamma_b} \qquad \text{where} \qquad \gamma_b = \frac{(D - K_b)}{(K_a + K_b - D)} \tag{17}$$

where $A_b = \hat{p} - c_a$, the value in (15) of $p_{b\min} = c_a$. It can be determined taking into account that $F_b(p_b)$ accumulates all the probability between $[p_{b\min}, \hat{p}]$ which means that $F_b(\hat{p}) = \left[\frac{\hat{p} - c_a}{A_b}\right]^{\gamma_b} = 1$. Similarly,

[18] Is easy to verify that (17) is the solution for the last row of (16). Rearranging to find $\frac{f_b}{F_b}$ in function of all the other parameters we can derive $\frac{\partial F_b(p_b)}{\partial p_b} = \frac{\gamma_b}{A_b}\left[\frac{p_b - c_a}{A_b}\right]^{\gamma_b - 1}$ and substituing its value in the diferencial equation we find $\frac{f_b}{F_b} = \frac{A_b}{A_b}\gamma_b(p_b - c_a)^{-1}$ that is exacly what the third row of (16) describes.

as the minimum value of $F_b(p_b)$ is zero it must be the case that the minimum value for $p_{b\min} = c_a$. Also, $\gamma_b > 0$ when $K_b < D$. Of course $D < K_a + K_b$ as the case of rationing is described in the following section.

Then, the expected profits of G_a for any play of G_b can be found:

$$\int_{p_a}^{\hat{p}} \Pi_a(u) f_b(u) du = (\hat{p} - c_a)(D - K_b) - (p_a - c_a)(D - K_b) \left[\frac{p_b - c_a}{\hat{p} - c_a} \right]^{\frac{(D-K_b)}{(K_a+K_b-D)}} \tag{18}$$

That is:

$$E(\Pi_a(p_a, f_b(p_b)) = (\hat{p} - c_a)(D - K_b) \tag{19}$$

As required, is independent from p_a : this is the expected profits for G_a from playing, in this high demand case, any price belonging to $[c_a, \hat{p}]$.

Using the same reasoning the expected profits that G_b would derive from bidding any price belonging to $[c_b, \hat{p}]$ for any play of G_a would be:

$$E(\Pi_b(p_b, f_a(p_a)) = (\hat{p} - c_b)(D - K_a) \tag{20}$$

Equations (19) and (20) characterize the profit derived from the equilibrium in mixed-strategies when demand is higher than the larger capacity declared available: $K_i < K_j < D < K_i + K_j$.

7.2 Proof Proposition 11

Intersecting both best response functions we find the families of subgame perfect equilibria for the case of stochastic demand distributed between D_n and D_p. These equilibria constitute the shaded grey area in the following figure that is the intersection between the black area that represents the best response of G_a and the dotted area that represents the best response of G_b.

To ensure the existence of type *i* equilibria (the triangular part of the grey area) we assume that the distance between D_n and D_p is such that $(1+\delta)D_n > D_p$. Otherwise only equilibria *ii* and *iii* would be possible as the triangular area in the figure would disappear. It is a strict inequality because if it was the case that $(1+\delta)D_n = D_p$, the constraint $K_a + K_b \geq D_p$ could never be satisfied as the equilibrium is defined for $K_a < D_n$.

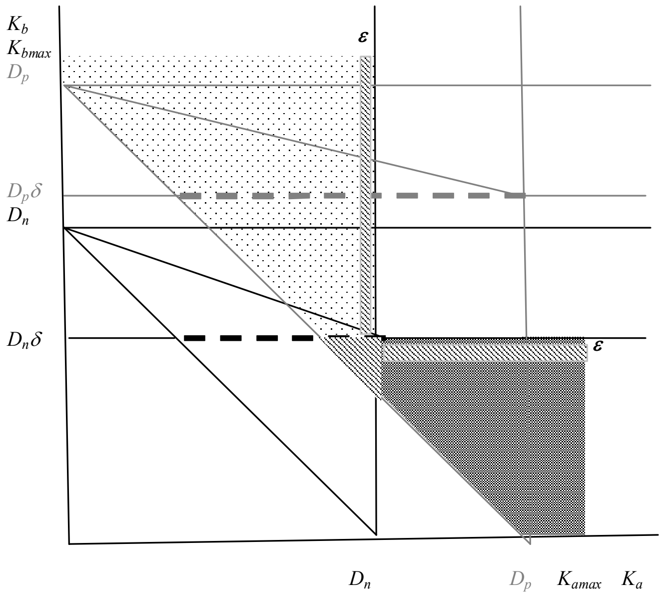

Figure 1

7.3 Proof of Proposition 12

Type *ii* equilibria lead us to *Case c* or, if $K_a > D_p$, to *Case b part a*[19] where G_a has the capacity advantage. Then, type *ii* equilibria when demand is D_p implies the following payoffs:

$\Pi_a = (\hat{p} - c_a)(D_p - \delta D_n)$ and $\Pi_b = (\hat{p} - c_b)D_n\delta$

Symmetrically, type *iii* equilibria lead us to the same *Case c* or Case *b* where G_b has the capacity advantage if $K_b > D_p$*:*

$\Pi_a = (\hat{p} - c_a)D_n$ and $\Pi_b = (\hat{p} - c_b)(D_p - D_n) > 0$

On the other hand, in type *i,* if demand is D_p, we fall in *Case c* where $E(\Pi_i) = (\hat{p} - c_i)(D_p - K_j)$.

Using the condition on the distance between D_n and D_p imposed in the proof of equation (12), i.e. $(1+\delta)D_n > D_p$, we can directly compare the payoffs of each player for each equilibria and conclude that:

a) when D_p is realized G_b prefers type *ii* equilibria to type *iii* as in the first case he gets $\Pi_b = (\hat{p}-c_b)D_n\delta$ for sure while in the second case he gets something lower that tends to the previous Π_b when $D_p \to (1+\delta)D_n$. Moreover, type *i* is prefered to type *iii,* as in type *i* he expects to sell $D_p - K_a$ where $K_a < D_n$.

b) when D_p is realized G_a prefers type *iii* equilibria to type *i* equilibria that is prefered to type *ii.*

[19]We will never be in *part b* as K_b cannot be higher than $D_n\delta$ which is equivalent to say that α_a cannot be lower than c_b.