Competition Schemes and Investment in Network Infrastructure under Uncertainty*

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Abstract

This paper compares two specific types of competition schemes (i.e., service-based competition and facility-based competition) by focusing on a firm's incentive to invest in network infrastructure. We show that when monopoly rent is large, facility-based competition makes the initial introduction of infrastructure earlier than service-based competition. However, when not only monopoly rent but the degree of uncertainty are small, service-based competition makes the initial introduction of infrastructure earlier than facility-based competition. The result indicates the relationship between the timing of infrastructure builiding and the degree of competitiveness in a product market. Other policy implications are also discussed.

Keywords: Service-based competition, Facility-based competition, Real options, Preemption.

JEL classification: D92; G13; L43; L51

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1 Introduction

This paper addresses how infrastructure investments in network industries, such as telecommunications, electricity, natural gas, and railroads, can be promoted. In particular, we examine the effect of the choice of a competition scheme on a firm's incentive to invest in network infrastructure. To this end, we focus on two specific types of competition schemes: a service-based competition and a facility-based competition. In service-based competition, entrants can enter the market by accessing an incumbent's network facility if and when they desire. On the other hand, facility-based competition requires entrants to construct their own network facilities to enter the market. In this paper, we attempt to clarify the conditions under which one competition scheme would induce a firm to invest more in network infrastructure than the other.

As a matter of fact, promoting investment in network infrastructure is an important concern in a regulator's choice of competition scheme. This is because the adoption of a new type of network infrastructure with innovative technology (e.g., broadband in telecommunications) may lead to price reductions or the provision of better quality products. Furthermore, the construction of a bypass (i.e., an additional network built by a potential entrant) may not only enhance the size of the existing network, but also upgrade it. It may also introduce a positive network externality by reducing congestion, which in turn contributes to welfare enhancement.

There are several studies on the comparison of competition schemes in an open access environment. A closely related study to this paper on the issue of open access policy is Bourreau and Doğan (2005). They focused on an incumbent's incentive for unbundling and the incentive to set an access charge. They showed that an incumbent has an incentive to set too low an access charge, so that an entrant builds its own network too late from a welfare viewpoint.[1] Empirical research by Kaserman and Ulrich (2002) showed the effects of facility-based vs resale entry on competition. According to the results presented in their Table 3, resale entry seemingly has a more drastic effect on competition than facility-based

[1]However, Bourreau and Doğan (2005) did not appear to pay sufficient attention to the dynamic perspective on the policy consequences, even though they stated that they used a dynamic model. In fact, they normalized the discount rate to unity, which implies that a dollar today has the same value as a dollar in the future.

entry, in the sense that resale entry reduces the incumbents' shares in the long-distance telecommunication market more than facility-based entry. Laffont and Tirole (2000) and Woroch (2002) also provide useful discussion in comparing the two types of entry.

Our paper differs from previous work in two main ways. First, most earlier studies on the issue of open access policy did not deal with uncertainty and irreversibility, whereas we examine the firm's incentive for irreversible investment under uncertainty.[2] Uncertainty and irreversibility are decisive for firms in network industries. These two elements come to be commonly recognized among those involved in network industries, including researchers and policy makers. For example, Alleman and Noam (1999) suggested the application of the real options approach to the telecommunications industry. Indeed, the net present value rule, which states that an investment should be undertaken only if its net present value is positive, is inappropriate for firms operating under uncertainty and irreversibility. This is why we employ a real options approach to discuss the above-mentioned issue.[3] We then examine how uncertainty affects the priority of the two competition schemes in terms of a firm's incentive to invest in network infrastructure.

Second, while previous studies assume that firm positions in a market are exogenously given, they are endogenously determined in our model. This means that we study a firm's preemptive incentive in an open access environment.[4] In reality, there are many situations in which the issue of preemption incentive in an open access environment is appropriate.

Figure 1 shows penetration rates of broadband services including fibre-optic networks of G7 countries as of December 2006. One trait which separates Japan from the other countries is high penetration of fibre-optic or LAN based service. In Japan, 30% subscribers of broadband services utilize fibre-optic networks while few subscribers in the other countries use them. This fact reminds us that Japanese Telecom regulatory authority employed some

[2] The effect of uncertainty on retail-price or access-price regulation has been formally examined by Biglaiser and Riordan (2000) and Pindyck (2005), with a focus on the irreversibility of investments. However, neither analyzed an investment game between an incumbent and an entrant, nor allowed an entrant to build an additional network, both of which are included in our paper.

[3] The real options approach applies option concepts to value real assets under uncertainty. It has been an important growth area in investment theory. See Dixit and Pindyck (1994) and Trigeorgis (1996) for a basic treatment of the tools employed. See also Smits and Trigeorgis (2004) for a real options approach to game-theoretic models.

[4] The preemption effect was originally examined by Fudenberg and Tirole (1985) and Katz and Shapiro (1987).

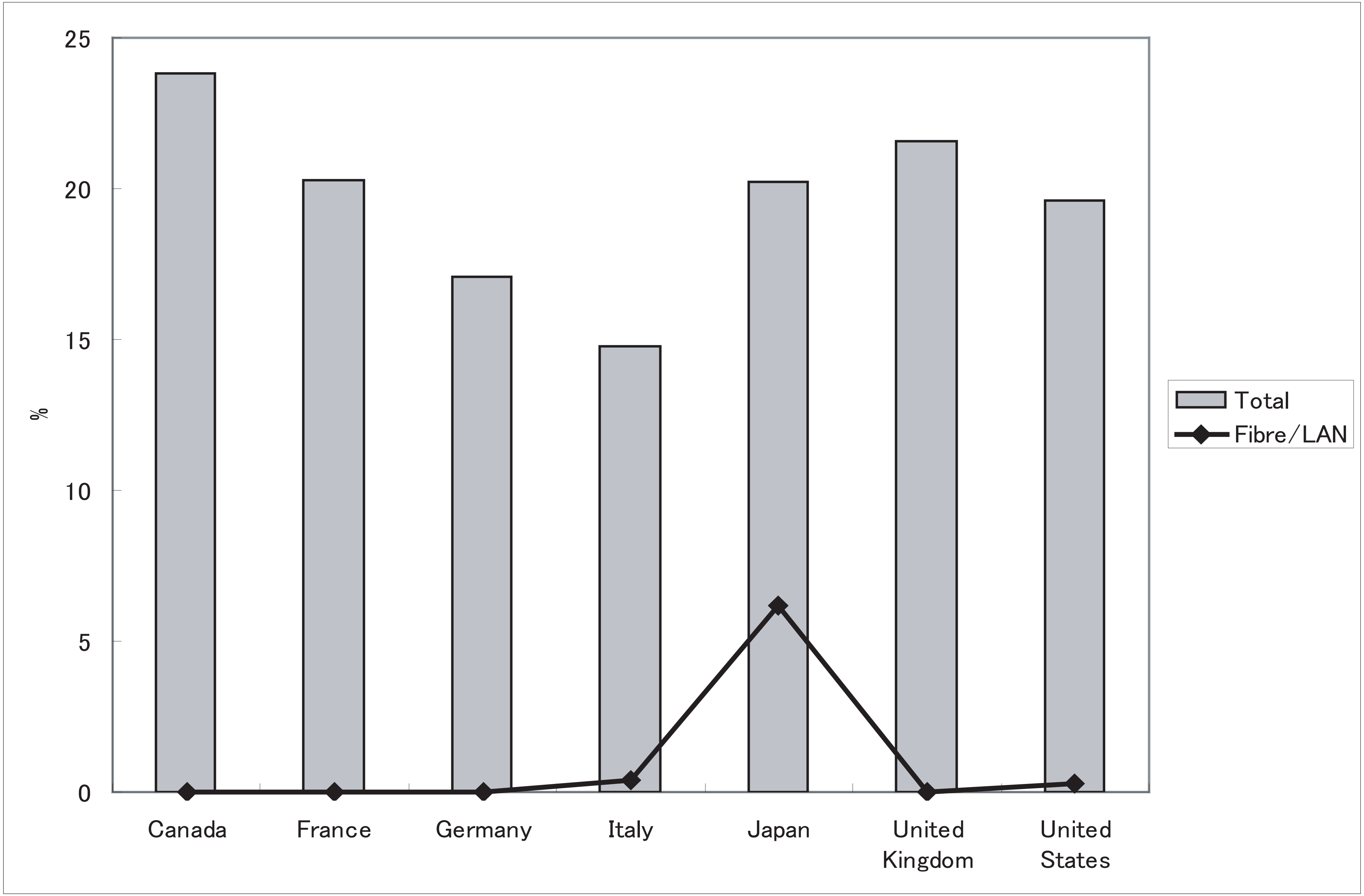


Figure 1: Penetration rates of broardband services

policy that stimulates investments in fibre-optic networks. Indeed, the e-Japan plan was made for the purpose of making 30 million households accessible to broadband networks in 2001. The plan (and the following related plans) explicitly states that broadband networks all over the country should be established through competition among firms. Then, since 2001, several players, including NTT East and West, K-Opticom, Yusen etc., have build their own fibre-optic cable networks to ensure first-mover-advantages. On the other hand, it is argued that the U.S. is not necessarily a well-developed country in the field of fiber-optic networks. As a result, in 2003 the Federal Communications Commission (FCC) adopted new rules regarding the network unbundling obligations of incumbent local phone carriers, with the aim of providing incentives for the carriers to invest in broadband.

Furthermore, Investment in mobile networks to enhance a firm's service coverage is another example that suits the issue of preemption among firms. In mobile network markets, mandated access or mandated roaming is a policy issue.[5] Fully providing several types of infrastructure in developing countries is also appropriate for preemption issue, especially when privatization and unbundling are simultaneously adopted.[6] Needless to say, R&D market with licensing is also a good example that fits the issue of preemption.[7]

In sum, these examples suggest the importance of the study of a firm's preemptive incentive in an open access environment. Here, we should note that, in such an open access environment, there is also a benefit in being a follower, as a follower can avoid the sunk cost of a network by accessing the leader's network with payment of an access charge. This benefit for a follower may deter the introduction of new network infrastructures in a market. Analyzing the effect of allowing access on the preemption incentive in open access environment, we examine the priority of the two competition schemes in terms of the rapidity of the initial construction of network infrastructure.

Using the model developed by Hori and Mizuno (2006), we derive the results on the conditions for one competition scheme which give the firm more incentive to invest in

[5] Interestingly, regulatory agencies took different decisions on this issue among countries. See Hausman (2002) for the discussion.

[6] See Kessides (2004) for the recommendation of privatization and competition in infrastructure industries in developing countries.

[7] See Baumol (2002) that discusses the similarities and the differences of the issues in infrastructure industries and those in innovation markets. He proposes the efficient componet pricing rule for the determination of an optimal licensing fees.

network infrastructure than the other. Hori and Mizuno (2006) have already analyzed the effect of access charges on firms' incentives for preemption and investment with stochastically growing demand. However, comparison of the competition schemes has not been hitherto attempted. This paper extends the analysis to suggest the optimal choice of competition scheme in order to promote investment in network infrastructure.

We firstly show that service-based competition makes the construction of a bypass by an entrant (or a follower) later than facility-based competition, so long as the entrant accesses an incumbent's (or a leader's) network in service-based competition. We then examine an incumbent's incentive to invest in network infrastructure. In particular, we show that when monopoly rent is large, facility-based competition makes the initial introduction of infrastructure earlier than service-based competition. In addition, we clarify the conditions for the level of access charge at which facility-based competition makes the initial introduction of infrastructure earlier than service-based competition. On the other hand, when not only the monopoly rent but the degree of uncertainty are small, service-based competition makes for the initial introduction of infrastructure earlier than facility-based competition. The result indicates the relationship between the timing of infrastructure building and the degree of competitiveness in a product market. We also discuss other policy implications of the analytical results.

The remainder of the paper is organized as follows. Section 2 presents a model based on Hori and Mizuno (2006). Section 3 describes the equilibria of the two competition schemes. Section 4, which is a main part of the paper, analyzes the conditions that affect the priority of competition schemes in terms of a firm's incentive to invest in network infrastructure. Section 5 discusses some policy implications derived from the analysis. In addition, the section also provides the case in which firms with heterogeneous technology play the investment game as an extension of the model. Some concluding remarks are made in Section 6.

2 The model

The model presented here is based on Hori and Mizuno (2006). That is a simple continuous-time two-firm model of strategic investment under uncertainty.

There are two risk-neutral identical firms with the same production technology, $i = 1, 2$, that plan to enter a market. No firm establishes its facility at the beginning. The firms need two types of facilities to serve consumers in the market: a production facility and a network facility. Investment in these facilities may be undertaken simultaneously or sequentially. A firm builds a production facility at cost I^p, while a network facility (hereafter called a network) is built at cost I^n. These investments are irreversible, hence I^p and I^n are sunk costs.

In service-based competition, however, not all firms must invest in a network. In particular, the firm without a network may utilize the existing network for production by paying a usage access charge, $v\,(> 0)$, which is determined by the regulator and we take it as given in this paper.[8] Let us call the firm that initially invests in both kinds of facilities a leader. The other firm, which may or may not have a network, is called a follower. When a follower uses a leader's existing network, the leader incurs an access cost, c. For simplicity, production costs other than the access costs are assumed to be zero. On the other hand, in facility-based competition, a follower needs to invest not only in a production facility, but also in a network.

The profit flows of the firms are uncertain because the firms face a common exogenous industry shock. We represent the industry shock by Y. Y evolves stochastically according to a geometric Brownian motion given by the following expression.

$$dY_t = \alpha Y_t dt + \sigma Y_t dW, \tag{1}$$

where $\alpha \in \left[\sigma^2/2,\ r\right)$ is the drift parameter measuring the expected growth rate of Y,[9] r is the risk-free interest rate, $\sigma > 0$ is a volatility parameter, and dW is the increment of a standard Wiener process where $dW \sim N\,(0, dt)$.

[8] We restrict our attention in this paper to a one-way access environment.

[9] Note that by means of $\alpha \geq \sigma^2/2$, we implicitly assume that the firm's profit flow is enhanced stochastically.

As we focus on investment or entry timing, we represent each firm's profit flow by a reduced form of $\pi = Y\Pi(N)$, where $\Pi(N)$ is the non-stochastic part of a firm's profit flow at the industry equilibrium with active firms $(N = 0, 1, 2)$. Note that in service-based competition, we need to distinguish two duopolistic market structures: access duopoly, in which the follower has access to the leader's network; and bypass duopoly, in which the follower has its own network, called a bypass. Let $Y\Pi(2)$ represent the profit flow of each firm in the bypass duopoly equilibrium. As the profit flow of a firm in an access duopoly depends on the level of the access charge, we represent the profit flows of a leader and a follower in the access duopoly equilibrium by $Y\Pi^L(2; v)$ and $Y\Pi^F(2; v)$, respectively. In facility-based competition where a follower invests not only in a production facility, but also in a network, the relevant profit flow of each firm in duopoly is $Y\Pi(2)$.

We assume the following relationship among the non-stochastic parts of a firm's reduced profit flow.

Assumptions (i) $\Pi(1) > \Pi(2)$ and $\Pi(1) > \Pi^j(2; v)$ for $j = L, F$, (ii) $\Pi(2) \geq \Pi^F(2; v)$, (iii) $\Pi^L(2; v) \underset{(<)}{\geq} \Pi^F(2; v)$ if $v \underset{(<)}{\geq} c$, (iv) $\frac{\partial \Pi^L(2;v)}{\partial v} > 0$, $\frac{\partial \Pi^F(2;v)}{\partial v} < 0$.

Assumption (i) is natural. In a service-based competition scheme, (iii) and (iv) are also natural. Assumption (ii) comes from the idea that the additional supply of network generally improves the quality of goods or causes a positive externality, which increases consumers' willingness-to-pay.[10] Then (ii) states that these kinds of positive externality are reflected in an increase in each firm's profit flow, which can exceed the profit flow of a follower in access duopoly when the level of access charge is in a reasonable range. In Section 5.2, we show the reduced profit form of $Y\Pi(N)$ and the range of parameters that satisfy the above assumptions in the case of Cournot competition with a linear inverse demand function, by which we discuss a policy implication.

Within each instant $[t, t + dt]$, the timing of the game is the following. In the model, we take a competition scheme and the level of v as given.[11] First, each firm decides whether

[10] For example, the construction of another broadband cable can increase the speed of information flows, so it benefits the population of Internet users.

[11] In section 5, we discuss the policy implications for the choice of a competition scheme and the setting of access charge.

to invest, given the realization of Y. Second, when entering the market, a firm decides the output level in the production stage and the market clears.

We focus on the Markov strategies. That is, the firms' investment and quantity decisions depend only on the current value of Y. The relevant equilibrium concept is a Markov perfect equilibrium.

3 Equilibria in the two competition schemes

Let us examine the equilibria of the game described in Section 2. At first, we characterize the equilibrium in facility-based competition. We then examine the equilibrium in service-based competition.

3.1 Facility-based competition equilibrium

As usual in dynamic game contexts, the game is solved backwards. We start by considering a follower's strategy (i.e., when to enter by building a bypass). A leader's strategy is then examined, given the follower's strategy.[12]

First, a follower's strategy is examined. In a facility-based competition scheme, the follower must invest not only in a production facility, but also in a bypass to serve consumers, so that the total investment cost is $I^p + I^n$. The follower's profit flow is $Y\Pi(2)$.

Given the fact that the other firm already entered the market, the optimal investment policy for a follower is formulated as follows:

$$V_t^F(Y_t) = \max_T E\left\{ e^{-r(T-t)} \left(\int_T^\infty e^{-r\tau} Y_\tau \Pi(2)\, d\tau - (I^p + I^n) \right) \middle| Y_t \right\} \tag{2}$$

where E denotes expectations conditional on Y_t, and T is the future time at which the investment is made.

Equation (2) is regarded as an optimal stopping problem. Given an initial position Y_t, the follower chooses when to stop delaying the investment to maximize its firm value. By

[12] The following procedure is standard in the real options literature concerning strategic investment. See Ch. 9 of Dixit and Pindyck (1994) and Smit and Trigeorgis (2004). See Nielsen (2003) and Weeds (2003) for applications.

solving the problem, we can find the optimal timing when the firm invests in two types of facilities. If the solution is t (i.e., $T = t$), the firm invests immediately.

This optimal stopping problem can be solved as follows (see pp.140-144 of Dixit and Pindyck (1994) for details). We need to distinguish a stopping region and a continuation region. In the stopping region, the firm acts in the market after investing in its facilities. In the continuation region, on the other hand, it does not act and holds the option to invest. First, solving the Bellman equation in the stopping region provides the firm value, $[Y\Pi(2) / (r-\alpha)] - (I^p + I^n)$, which equals the expected value of this project minus the costs of the facilities. Then, we get the firm value in the continuation region by solving the corresponding Bellman equation. The firm value in this region can be thought of as the value of waiting before making the investment. Finally, using the value-matching and the smooth-pasting conditions, we obtain a trigger point. The trigger point is Y at which the firm invests in the two types of facilities.

Using this procedure, we obtain each firm's value function as follows:

$$V_F^B(Y) = \begin{cases} CY^\beta & if \quad Y < Y^B \\ \frac{Y\Pi(2)}{r-\alpha} - (I^p + I^n) & if \quad Y \geq Y^B \end{cases} \tag{3}$$

where $C = \left(Y^F\right)^{-\beta}\left[\frac{Y^F\Pi(2)}{r-\alpha} - (I^p + I^n)\right]$ and $\beta = \frac{1}{2}\left\{1 - \frac{2\alpha}{\sigma^2} + \sqrt{\left(1 - \frac{2\alpha}{\sigma^2}\right)^2 + \frac{8r}{\sigma^2}}\right\}$ (> 1). The trigger point is given by:

$$Y^B = \frac{\beta}{\beta - 1}\frac{r-\alpha}{\Pi(2)}(I^p + I^n) \tag{4}$$

CY^β in (4) is the value of delaying the investment, which corresponds to the value of the call option.

Secondly, we consider a leader's value. If $Y < Y^B$, the leader enjoys the monopoly profit before the follower invests. If $Y \geq Y^B$, it obtains a duopoly profit. Hence, its value function is represented as follows.

$$V_L^B(Y) = \begin{cases} \frac{Y\Pi(1)}{r-\alpha}\left[1-\left(\frac{Y}{Y^B}\right)^{\beta-1}\right] + \left(\frac{Y}{Y^B}\right)^{\beta}\frac{Y^B\Pi(2)}{r-\alpha} - (I^p + I^n) & if \quad Y < Y^B \\ \frac{Y\Pi(2)}{r-\alpha} - (I^p + I^n) & if \quad Y^B \leq Y \end{cases} \tag{5}$$

Finally, we can derive the *facility-based competition equilibrium* from (4) and (5).[13] In the equilibrium, the two firms do not enter the market when $Y \in [0, Y_L)$, where Y_L is the trigger point at which the leader enters the market. Y_L is defined as the smallest value of Y which satisfies $V_L^B(Y) = V_F^B(Y)$. When $Y \in \left[0, Y^B\right)$, the leader enjoys a monopoly profit. When $Y \in \left[Y^B, +\infty\right)$, the follower also serves its customers by building a bypass (i.e., a duopoly). The facility-based competition equilibrium is depicted in Figure 2.

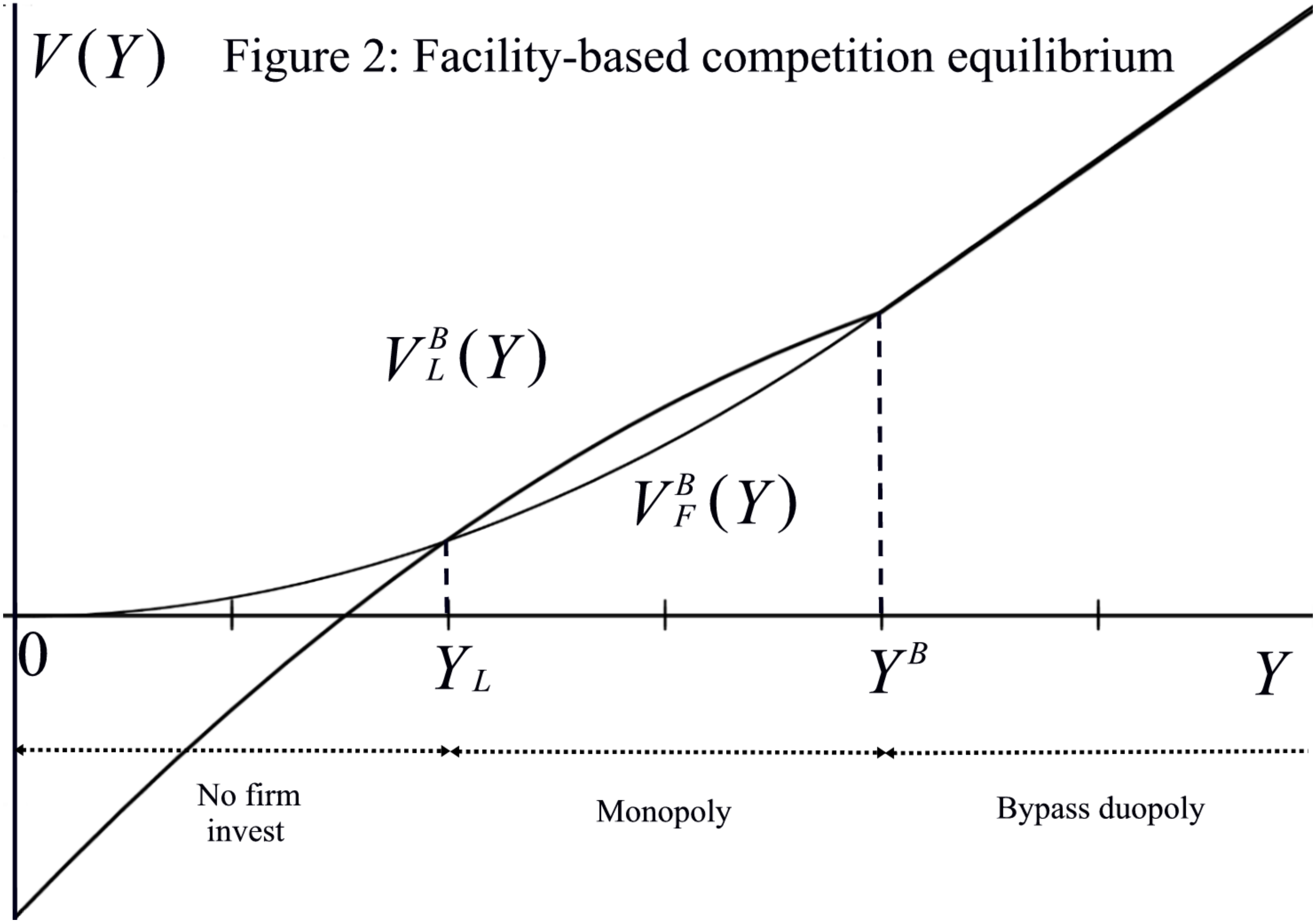
3.2 Service-based competition equilibrium

Next, we derive an equilibrium in service-based competition. When a follower is allowed to access a leader's network, several types of equilibria can occur, depending on the follower's strategy. For example, an equilibrium occurs in which a follower enters the market by building a bypass in spite of the access opportunity. In a stochastically growing demand environment, however, it is natural to examine the situation where a follower enters with access to a leader's network, and in the future the follower will build a bypass by itself. We call the follower's strategy an "access-to-bypass" strategy.

Hori and Mizuno (2006) have already derived the conditions for the existence of *the access-to-bypass equilibrium* in which a leader first enters a market by building a network, a follower takes the access-to-bypass strategy. The access-to-bypass equilibrium has two features: it is a leader–follower equilibrium, and a follower undertakes sequential investment. From this point, we restrict our attention to the access-to-bypass equilibrium. (The details of the derivation of the access-to-bypass equilibrium is in Appendix.)

Let us denote a follower's trigger point above which it enters the market with access

[13] Put more precisely, two equilibria exist in facility-based competition. We shall, however, make use of the term "equilibrium" rather than "equilibria", because the number of equilibrium is unimportant. In fact, there are two facility-based competition equilibria that differ only in the identities of the two firms—firm 1 is the leader in one equilibrium, and firm 2 is the leader in the other equilibrium. The equilibrium outcome is the same for the two equilibria.



$V(Y)$
Figure 2: Facility-based competition equilibrium
$V_L^B(Y)$
$V_F^B(Y)$
0
Y_L
Y^B
Y
No firm invest
Monopoly
Bypass duopoly

by Y^{A*} and the trigger point above which it builds a bypass by Y^{B*}, respectively. In the access-to-bypass equilibrium, Y^{A*} and Y^{B*} are characterized as follows.

$$Y^{A*} = \frac{\beta}{\beta - 1} \frac{r - \alpha}{\Pi^F (2; v)} I^p \tag{6}$$

$$Y^{B*} = \frac{\beta}{\beta - 1} \frac{r - \alpha}{\Delta\Pi (2; v)} I^n, \tag{7}$$

where $\Delta\Pi (2; v) \equiv \Pi (2) - \Pi^F (2; v)$. In fact, Hori and Mizuno (2006) showed that, under $\Delta\Pi (2; v) > 0$ of Assumption (ii), the follower adopts the access-to-bypass strategy (i.e., when it undertakes a sequential investment) if and only if the following condition holds.

$$\Pi (2) \leq \left(1 + \frac{I^n}{I^p}\right) \Pi^F (2; v) \tag{8}$$

When the follower chooses the access-to-bypass strategy, its value function is as follows.

$$V_F^{AB} (Y) = \begin{cases} \left(\frac{Y}{Y^{A*}}\right)^\beta \left\{ \frac{Y^{A*}\Pi^F(2;v)}{r-\alpha} - I^p + \left(\frac{Y^{A*}}{Y^{B*}}\right)^\beta \left[\frac{Y^{B*}\Delta\Pi(2;v)}{r-\alpha} - I^n\right]\right\} & if\ Y < Y^{A*} \\ \frac{Y\Pi^F(2;v)}{r-\alpha} - I^p + \left(\frac{Y}{Y^{B*}}\right)^\beta \left[\frac{Y^{B*}\Delta\Pi(2;v)}{r-\alpha} - I^n\right] & if\ \ Y^{A*} \leq Y < Y^{B*} \\ \frac{Y\Pi(2)}{r-\alpha} - (I^p + I^n) & if\ \ Y^{B*} \leq Y \end{cases} \tag{9}$$

Similarly, we can derive the leader's value function when the follower adopts the access-to-bypass strategy.

$$V_L^{AB} (Y) = \begin{cases} \frac{Y\Pi(1)}{r-\alpha} \left[1 - \left(\frac{Y}{Y^{A*}}\right)^{\beta-1}\right] + \left(\frac{Y}{Y^{A*}}\right)^\beta \left\{ \frac{Y^{A*}\Pi^L(2;v)}{r-\alpha} \left[1 - \left(\frac{Y^{A*}}{Y^{B*}}\right)^{\beta-1}\right] \right. \\ \quad \left. + \left(\frac{Y^{A*}}{Y^{B*}}\right)^{\beta_1} \frac{Y^{B*}\Pi(2)}{r-\alpha} \right\} - (I^p + I^n) \qquad if\ \ Y < Y^{A*} \\ \frac{Y\Pi^L(2;v)}{r-\alpha} \left[1 - \left(\frac{Y}{Y^{B*}}\right)^{\beta-1}\right] + \left(\frac{Y}{Y^{B*}}\right)^\beta \frac{Y^{B*}\Pi(2)}{r-\alpha} - (I^p + I^n) \\ \qquad if\ \ Y^{A*} \leq Y < Y^{B*} \\ \frac{Y\Pi(2)}{r-\alpha} - (I^p + I^n) \qquad if\ \ \ Y^{B*} \leq Y \end{cases} \tag{10}$$

The access-to-bypass equilibrium is drawn in Figure 3. For $Y \in [0, Y_L^*)$, neither of the two firms enter. For $Y \in \left[Y_L^*, Y^{A*}\right)$, only one of the two firms enters the market and

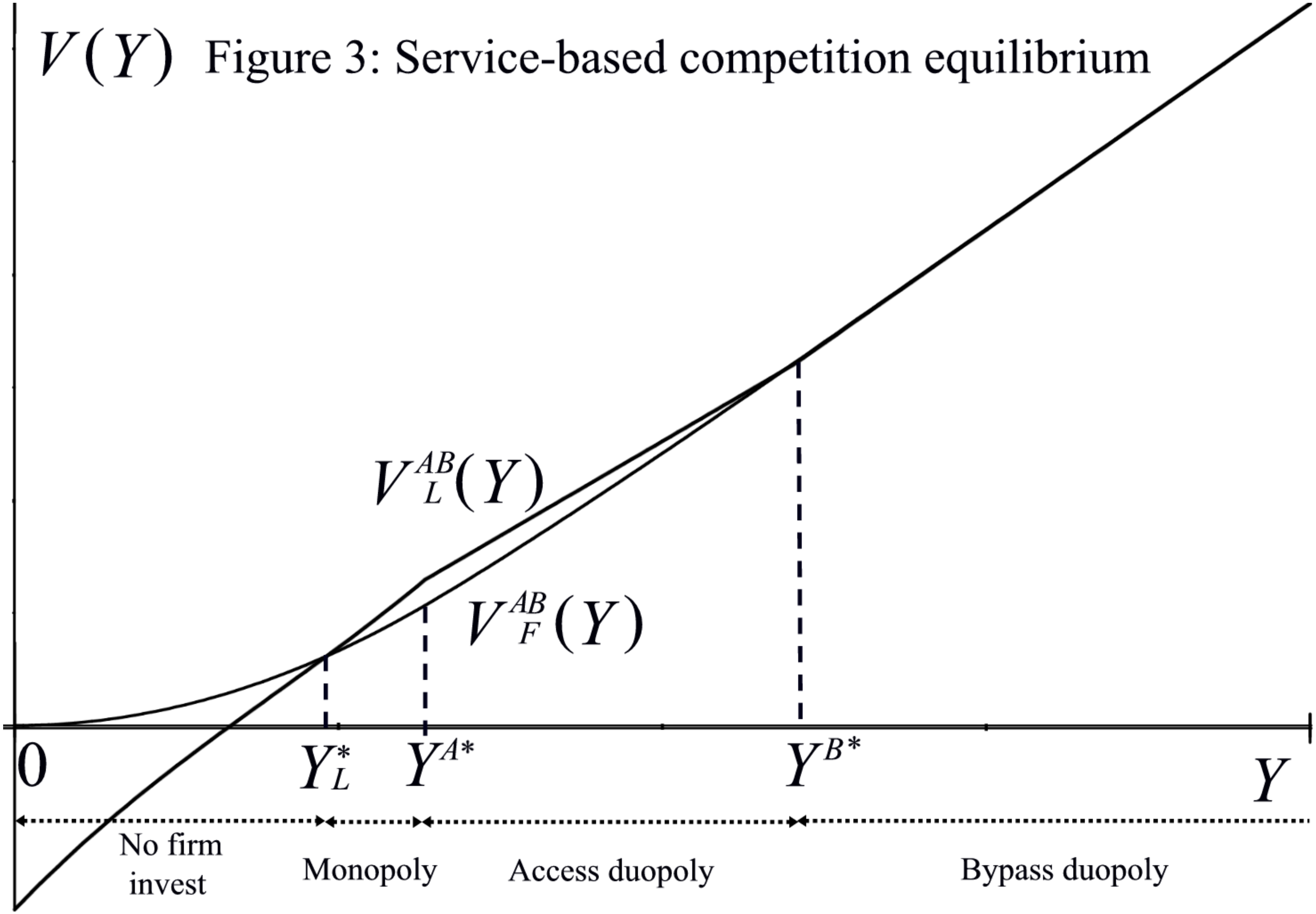


Figure 3: Service-based competition equilibrium

earns a monopoly profit. For $Y \in [Y^{A*}, Y^{B*})$, the other firm as a follower has access to a leader's network (access duopoly). For $Y \in [Y^{B*}, +\infty)$, the follower builds its own network (bypass duopoly).

4 The comparison of two competition schemes

We are now in a position to examine the priority of competition schemes in terms of firms' investment decisions. The next section discusses its policy implications.

4.1 A follower's investment decision

We can confirm that, in service-based competition, a follower enters the market earlier and builds a bypass later than in facility-based competition.

Proposition 1 *In the access-to-bypass equilibrium, (i) a follower enters the market earlier than a follower in the facility-based competition equilibrium, and (ii) a follower builds a bypass later than a follower in the facility-based competition equilibrium. That is,* $Y^{A*} < Y^{B} < Y^{B*}$.

Proof. See Appendix. ■

The first inequality in Proposition 1 shows that a follower's entry in the service-based competition can be earlier than that in the facility-based competition. We observe that it holds as long as the access charge is in the range where the follower adopts the access-to-bypass strategy. The second inequality in Proposition 1 states that a follower in the service-based competition builds a bypass later than one in the facility-based competition.

The second result is explained by the replacement effect: The follower that already obtains a profit through access to the leader's network has less incentive to build a bypass than the potential follower that now intends to enter the market. The replacement effect also appears in Lemma 6 of Bourreau and Doğan (2005). They stated that when there is unbundling, a follower builds its bypass later than when there is no unbundling. While there are a few differences in the setting of the model between the two studies, the replacement effect is robust.

From Proposition 1, we can immediately verify the effect of uncertainty on a follower's investment decisions in the two competition schemes.

Proposition 2 *As the degree of uncertainty increases, the difference of a follower's investment timing gets larger between the two competition schemes.*

Proof. As is already known, β gets small as σ gets large.[14] Then, from the forms of Y^{A*}, Y^{B}, and Y^{B*} and the fact that $Y^{A*} < Y^{B} < Y^{B*}$, the claim in the proposition automatically holds. ■

Proposition 2 implies that as uncertainty increases, the difference of a follower's entry timing gets larger, which in turn causes a significant impact on social welfare. This point will be discussed in the next section.

4.2 A leader's investment decision

We next compare the entry timing of a leader between the two competition schemes. The following proposition states the necessary and sufficient condition for a leader in the access-to-bypass equilibrium to enter earlier than that in the facility-based competition equilibrium.

Proposition 3 *A leader enters earlier (later) in the facility-based competition equilibrium than in the access-to-bypass equilibrium (i.e., $Y_L^* > (<) \, Y_L$) if and only if*

$$Q^{AB}(Y_L) < (>) \, 0$$

where $Q^{AB}(Y) \equiv V_L^{AB}(Y) - V_F^{AB}(Y)$ is the difference in firm value between the leader and the follower in the access-to-bypass equilibrium.

Proof. Y_L^* is the smallest value of Y which satisfies $Q^{AB}(Y) = 0$. Therefore, $Q^{AB}(Y_L)$ is negative (positive) if and only if $Y_L^* > (<) \, Y_L$. ■

However, substituting (9) and (10) directly into $Q^{AB}(Y_L)$ yields too complicated an expression to obtain an intuitive interpretation. Hence, we instead attempt to seek a

[14] See p. 144 of Dixit and Pindyck (1994).

sufficient condition for one competition scheme to make a leader's entry earlier than in the other. In fact, we show that, three elements (i.e., a monopoly rent, the level of access charge, and the degree of uncertainty) are key factors to determine the priority of competition schemes in terms of a leader's investment timing.

First, we show that, when a monopoly rent is large, facility-based competition induces a leader to invest earlier than in service-based competition.

Proposition 4 *If* $\Pi(1)$ *is sufficiently large, a leader in the facility-based competition equilibrium invests in both production and network facilities earlier than the one in the access-to-bypass equilibrium (i.e.,* $Y_L < Y_L^*$*).*

Proof. See Appendix. ■

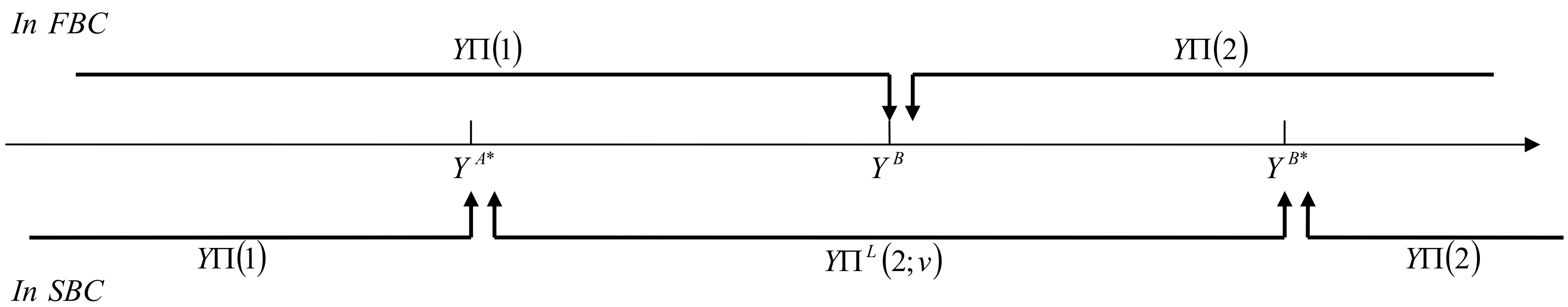
Note that $\Pi(1)$ does not affect the follower′s decision on access to the leader's network and the construction of a bypass. Hence, when $\Pi(1)$ is sufficiently large, service-based competition decelerates introduction of new infrastructure in a region which is not yet covered, without any influence on competition in the product market.

The intuitive reasoning of Proposition 4 is simple. See Figure 4. According to Proposition 1, $Y^{A*} < Y^B$, which means a leader in the facility-based competition equilibrium enjoys monopoly rent in a longer period than that in the access-to-bypass equilibrium. Hence, the preemption incentive of a leader in the facility-based competition equilibrium is larger than that in the access-to-bypass equilibrium.

We next examine the effect of a change in the level of access charge on investment decisions in the two competition schemes. Indeed, concerning the follower′s investment decision, Proposition 1 already discusses its effect: when the level of access charge is in a range where the access-to-bypass equilibrium exists, a follower's entry in service-based competition is earlier than in facility-based competition, with a bypass being built later.[15] What then is its effect on a leader's investment timing?

Since the effect of the access charge on $Q^{AB}(Y_L)$ works nonlinearly, it is difficult to analytically derive a necessary and sufficient condition for one competition scheme to make

[15] If the access charge is above the range, the follower builds a bypass, so that the follower's investment timings are the same in the two competition schemes.



Note: *FBC* and *SBC* stand for a facility-based competition and a service-based competition, respectively.

Figure 4: A Leader's Profit in the Two Competition Schemes

a leader's entry earlier than the other. Hence, we again seek a sufficient condition for one competition scheme to do so. The next proposition provides a sufficient condition for a leader in facility-based competition to invest earlier than one in service-based competition.

Proposition 5 *(i) A leader in the facility-based competition equilibrium invests in both kinds of facilities earlier than the one in the access-to-bypass equilibrium (i.e., $Y_L < Y_L^*$) when the level of access charge satisfies the following condition.*

$$\frac{\Pi(1) - \Pi^L(2; v)}{\Pi^F(2; v)} \geq \frac{\Pi(1) - \Pi(2)}{\Pi(2)} \tag{11}$$

(ii) The timing of the leader's investment in the access-to-bypass equilibrium gets later as the access charge gets small, as long as (11) holds.

Proof. See Appendix. ■

The intuitive meaning of the condition (11) is as follows. The numerator of the left-hand side represents the profit or loss a leader incurs when the market structure changes from monopoly to access duopoly, while the denominator represents the profit a follower obtains in an access duopoly. Hence, the left-hand side is interpreted as the relative cost a leader incurs from structural change in the market in the service-based competition scheme. The right-hand side has exactly the same meaning as in the case of facility-based competition. Therefore, condition (11) states that when the relative cost a leader incurs from the structural change of the market in facility-based competition scheme is less than that in service-based competition, the preemption effect is larger in the facility-based competition scheme.

Notice that condition (11) also implies the range of monopoly rent $\Pi(1)$ that induces a leader in the facility-based competition to invest in both kinds of facilities earlier than the one in the access-to-bypass equilibrium. In fact, rearranging (11) gives us the following.

$$\Pi(1) \geq \frac{\Pi(2)\left(\Pi^L(2; v) - \Pi^F(2; v)\right)}{\Pi(2) - \Pi^F(2; v)} \tag{12}$$

The difference between (12) and Proposition 4 is that (12) refers to the relationship between the monopoly rent and the level of access charge or that of the network externality, whereas Proposition 4 mentions only the degree of monopoly rent. The intuitive meaning of (12) is the same as that of Proposition 4.

So far, we provide only sufficient conditions for facility-based competition to cause a leader to invest earlier than in service-based competition. The next proposition, which highlights not only the monopoly rent but the degree of uncertainty, gives a sufficient condition for service-based competition to cause a leader to invest earlier than facility-based competition.

Proposition 6 *If the monopoly rent is small such that* $\Pi(1) \leq \frac{\Pi^F(2;v)(I^p+I^n)}{I^p}$, *a leader in the access-to-bypass equilibrium invests earlier than a leader in the facility-based competition equilibrium (i.e.,* $Y_L^* < Y_L$*) as* σ *approaches to zero.*

.

Proof. See Appendix. ■

Proposition 6 states that, when not only the monopoly rent but also uncertainty are small, the preemption effect in the access-to-bypass equilibrium is stronger than that in the facility-based competition equilibrium. This is explained as follows.

Remember that $Y^{A*} < Y^B$. (See Figure 4 again.) This implies that a leader in the facility-based competition equilibrium can earn a monopoly rent for a longer period than the one in the access-to-bypass equilibrium. According to Proposition 2, a decrease in σ makes Y^{A*} closer to Y^B. That is, the difference in the period in which the leader can earn the larger rent in facility-based competition equilibrium than in the access-to-bypass equilibrium (i.e., $\Pi(1) > \Pi^L(2;v)$) becomes shorter, as uncertainty is reduced.[16]

Moreover, for $Y \in [Y^B, Y^{B*}]$, the leader in the access-to-bypass equilibrium can obtain the higher profit than that in the facility-based competition equilibrium (i.e.,

[16] However, note that the effect of uncertainty on a leader's entry timing in a *given* competition scheme in our model is still ambiguous. This ambiguity stems from a conflict between the preemption effect and the real options effect. Since a follower enters later when uncertainty is higher, the incentive to become a leader becomes larger. This is because a leader can enjoy a monopoly rent for a longer period. Hence, the preemption motive is a countervailing force to the real-option effect. This point is also illustrated by Mason and Weeds (2003). They demonstrate that if monopoly profits are large, an increase in uncertainty hastens the leader's entry.

$\Pi^L(2; v) > \Pi(2))$, when the access-to-bypass equilibrium exists. Then, the access profit can compensate the loss caused by a shorter period of monopoly rent in the access-to-bypass equilibrium, when the monopoly rent is small. Therefore, a leader in the access-to-bypass equilibrium has a larger incentive to enter than a leader in the facility-based competition equilibrium, when both the monopoly rent and uncertainty are small.

5 Discussion

5.1 An investment game of firms with heterogeneous technology

So far we have assumed that there are two identical firms with the same production technology. In a real business environment, however, different firms may have different technologies. Hence, we extend our model to deal with the case where there are two firms with heterogeneous technology. In this section, we derive an access-to-bypass equilibrium where the cost of the network facility, I^n, is different between the two firms.

Let I_1^n and I_2^n denote the respective costs for firms 1 and 2. Without loss of generality, we assume that I_1^n is smaller than I_2^n. Furthermore, I_1^n is specified as $I^n - \epsilon$, while I_2^n is specified as $I^n + \epsilon$ where $\epsilon > 0$. Under these assumptions, we can show the following proposition.

Proposition 7 *Assume that there are two identical firms, except for the cost of the network facility, and the investment game begins with a sufficiently low level of Y. In the access-to-bypass equilibrium, (i) a firm with a lower cost of the network facility does not construct a bypass later than a firm with the higher cost; (ii) the difference in the cost of the network facility does not matter for the timing of access by the entrant; and (iii) the firm with the lower cost enters the market earlier than one with a higher cost.*

Proof. See Appendix ■

If the investment game begins with a sufficiently low level of Y, firm 1 becomes a leader in service-based competition, while firm 2 becomes a follower. This suggests that firms with advanced technology or lower costs crowd out other firms and enter the market as leaders. Moreover, the larger the ϵ, implying a wider discrepancy in technology, the longer the expected period of monopoly and access duopoly.

The results of (i) and (iii) of Proposition 7 can apply to the facility-based competition equilibrium. Therefore, the heterogeneous technology would only identify the incumbent and the entrants, but not alter the implications obtained from Propositions 1 to 6.

5.2 Policy implications

5.2.1 The relationship with product market competition

According to Proposition 5, the two elements concerning the product market competition (i.e., the level of access charge and the magnitude of a monopoly rent) are key factors to determine a high priority of facility-based competition in terms of a leader's investment timing. However, it seems to be complicated to derive the explicit representation of the range of access charge that satisfies condition (11), since the profit flows depend not only on access charge but also on the characterization of the demand function, the competition mode in duopoly, the degree of positive externality, etc. Therefore, we specify the product market competition in order to derive explicitly some policy implications of the proposition.

Consider the case of Cournot competition with a linear inverse demand $p_t^j = a - b_t\left(q^j + \gamma q^k\right)$ $(j,k = L, F)$ where $\gamma\,(\in[0,1])$ represents a degree of product differentiation and b_t is a stochastic variable that relates to the industry shock. Let us suppose that, when the market is a bypass duopoly, the inverse demand becomes $p_t^j = (a+\theta) - b_t\left(q^j + \gamma q^k\right)$, where $\theta\,(>0)$ is a parameter of positive externality generated from network expansion. Defining $A \equiv a - c$ and $X \equiv v - c$, we can derive the reduced profits as follows.

$$
\begin{aligned}
Y_t &= 1/b_t,\ \Pi(1) = \frac{A^2}{4},\ \Pi(2) = \frac{(A+\theta)^2}{(2+\gamma)^2}, \\
\Pi^L(2;X) &= \frac{((2-\gamma)A+\gamma X)^2 + (2+\gamma)(2-\gamma)X((2-\gamma)A-2X)}{(2+\gamma)^2(2-\gamma)^2}, \\
\text{and } \Pi^F(2;X) &= \frac{((2-\gamma)A-2X)^2}{(2+\gamma)^2(2-\gamma)^2}.
\end{aligned}
$$

Then, the assumption that $\Pi(1) > \Pi(2)$ of (i) is satisfied,[17] when

$$\gamma > \frac{2(A+\theta)}{A} - 2. \tag{13}$$

Assumption (ii) is satisfied, when

$$X > -\left(\frac{2-\gamma}{2}\right)\theta. \tag{14}$$

Assumptions (iii) and (iv) are also satisfied in this case.[18]

Then, it can be shown that, when the level of access charge is low (i.e., near to access cost), the facility-based competition scheme induces a leader to invest in infrastructure earlier than the service-based competition, and, as the degree of product differentiation becomes small, the range of v that yields this result becomes large. We report the finding as a proposition.

Proposition 8 *Consider the case of Cournot competition with a linear inverse demand $p_t^j = a - b_t\left(q^j + \gamma q^k\right)$ in the product market. Then, when the level of access charge v is low (i.e., near to access cost c), the facility-based competition scheme induces a leader to invest in infrastructure earlier than the service-based competition. In addition, given a level of access charge, a leader in the facility-based competition scheme comes to invest in infrastructure earlier than the one in the service-based competition scheme as γ gets large.*

Proof. See Appendix. ■

In particular, we ensure that, when $\gamma = 1$ (i.e. the goods are perfectly substitute), (11) holds for any X (> 0). That is, given any access charge that is larger than access cost, when the goods are perfectly substitute, the facility-based competition scheme induces a leader to invest in infrastructure earlier than the service-based competition scheme.

[17] We can ensure that the second part of (i) is automatically satisfied when (ii) and the sufficient condition for the existence of access-to-bypass equilibrium are held.

[18] It is easy to verify that $\Pi^L(2; X, \gamma)$ and $\Pi^F(2; X, \gamma)$ are decreasing functions of γ, as long as A is sufficiently large relative to X.

Since the degree of product differentiation is interpreted as a degree of competitiveness in a product market, this finding seems to be intuitively appealing. That is, when product market competition is severe (i.e., when γ is large), the monopoly rent is relatively large when compared with the duopoly rent. In that case, the facility-based competition scheme induces a leader to invest in infrastructure earlier than the service-based competition scheme.

5.2.2 The choice of competition scheme and social welfare

In the above analysis, we took a competition scheme and a level of access charge as given, and examined the relationship between the choice of a competition scheme and the investment decisions of both the leader and the follower. In fact, it is hard to determine which of the two competition schemes is better from a welfare viewpoint. This point is explained as follows.

Denoting the social surplus flow in the monopoly and that in the (bypass) duopoly by $SS(1)$ and $SS(2)$ respectively, we can derive the socially optimal investment timings in our stochastic environment as follows.

$$Y_L^{**} = \frac{\beta}{\beta - 1} \frac{r - \alpha}{SS(1)} \left(I^p + I^n \right), \text{ and } Y^{B**} = \frac{\beta}{\beta - 1} \frac{r - \alpha}{\Delta SS} \left(I^p + I^n \right), \tag{15}$$

where $\Delta SS \equiv SS(2) - SS(1)$. Then, it is easy to verify that the comparison of these social optimal timings with the investment timings in the equilibrium of each competition scheme is hard to be compared. The bottom line is that the comparison depends on various factors such as the shape of the inverse demand function, the investment costs of production and network facilities, the preemption incentive, and the parameters of (1) that define the geometric Brownian motion. This reasoning also works when we try to compare the two competition schemes from welfare viewpoint.

Nevertheless, we can obtain some policy implications for the choice of a competition scheme. According to Proposition 1, a follower under service-based competition are more proactive in entering a market than those under facility-based competition, since they need not incur the large sunk costs of building their own network. Hence, service-based competition contributes to an early realization of a competitive environment in the product market. This, in turn, enhances social welfare.[19] However, service-based competition simultaneously entails the late construction of a bypass. This implies the later introduction of a positive externality generated by the prevalence of infrastructure. Since consumers also obtain benefit from the positive externality, service-based competition may be harmful in this respect. In sum, service-based competition includes benefits and costs from a welfare perspective when focusing on the follower's entry. Notice also that the magnitude of the benefits and costs gets larger, as uncertainty increases (Proposition 2).

On the other hand, according to Propositions 4, 5, and 6, service-based competition may deter the initial construction of network infrastructure, which depends on the environment surrounding firms. Therefore, we cannot confirm whether service-based competition enhances social welfare in regard to a leader's entry.

In reality, the negative effect of service-based competition on social welfare appears to be recognized, particularly in telecommunications. For example, it is argued that the U.S. is not necessarily a well-developed country in the field of broadband networks such as fiber-optics. As a result, in 2003 the Federal Communications Commission (FCC) adopted new rules regarding the network unbundling obligations of incumbent local phone carriers, with the aim of providing incentives for the carriers to invest in broadband.

6 Concluding Remarks

This paper compared the two specific types of competition schemes (i.e., a service-based competition and a facility-based competition) by focusing on a firm's incentive to invest in network infrastructure. To address the issue, we employed a real options approach, since the uncertainty and irreversibility of investment are influential factors when considering investment problems.

The analysis showed that service-based competition makes the construction of a bypass by an entrant or a follower later than facility-based competition, so long as an entrant

[19] One may speculate if a *follower*'s entry timing can be too early from a welfare perspective. Since a firm's incentive to enter a market depends only on the profit motive, the possibility for a follower's entry timing to be too early appears to be low.

accesses an incumbent's network in service-based competition. We then examined the leader's (i.e., the first entrant's) incentive to invest in network infrastructure. In particular, we showed that when monopoly rent is large, facility-based competition makes the introduction of a new type of infrastructure earlier than service-based competition. In addition, we clarified the conditions about the level of access charge at which facility-based competition makes the introduction of a new type of infrastructure earlier than service-based competition. On the other hand, when not only the monopoly rent but the degree of uncertainty are small, service-based competition makes the introduction of a new type of infrastructure earlier than facility-based competition. These results indicated the relationship between the timing of infrastructure building and the degree of competitiveness in product markets. We also discussed other policy implications of the analytical results.

These findings seem to explain recently occurring phenomena in network industries. For example, it is well known that the introduction of cable broadband networks in U.S. telecommunications was deterred by the 1996 Telecommunications Act. According to our analysis, this can be explained by low (regulated) access charges, large monopoly rents and the greater degree of uncertainty.

Appendix

Summary of the access-to-bypass equilibrium

Let us review the derivation procedure of the access-to-bypass equilibrium in Hori and Mizuno (2006).

As in the game of facility-based competition, we first characterize a follower's strategy. Let us represent the trigger point above which the follower enters the market with access by Y^{A*}. Similarly, we represent the trigger point above which the follower builds a bypass by Y^{B*}. It is apparent that when $(0 <)\, Y^{A*} \leq Y^{B*}\, (< +\infty)$, the follower adopts the access-to-bypass strategy, while when $Y^{B*} < Y^{A*}$, the follower adopts the bypass strategy. Indeed, Y^{A*} and Y^{B*} are derived by defining the option value of transition from the access project to the bypass project and using the standard technique in the real options approach.

Given the firm values of a leader and a follower, i.e., $V_L^{AB}(Y)$ and $V_F^{AB}(Y)$, we now derive the access-to-bypass equilibrium. Each firm i $(=1,2)$ has the following equilibrium strategy. It invests in the production facility at Y^{A*} and the network at Y^{B*} if a rival has invested. If the rival has not invested, it invests in both facilities at Y where $Y < Y^{A*}$ and $V_L^{AB}(Y) > V_F^{AB}(Y)$. Irrespective of the rival's decision, it never invests at Y where $V_L^{AB}(Y) < V_F^{AB}(Y)$. The leader's trigger point Y_L^* where the leader immediately invests is the smallest value of Y that satisfies $V_L^{AB}(Y) = V_F^{AB}(Y)$.

The following proposition (Proposition 1 of Hori and Mizuno (2006)) gives the conditions for the existence of the access-to-bypass equilibrium.

Proposition There exist two access-to-bypass equilibria differing only in the identities of the two firms. A unique leader's trigger point $Y_L^* \in \left(0, Y^{B*}\right)$ in the equilibria is characterized by:

$$
\begin{aligned}
V_L^{AB}(Y) &< V_F^{AB}(Y) \quad if \quad Y < Y_L^* \\
V_L^{AB}(Y) &= V_F^{AB}(Y) \quad if \quad Y = Y_L^* \\
V_L^{AB}(Y) &> V_F^{AB}(Y) \quad if \quad Y \in \left(Y_L^*, Y^{B*}\right) \\
V_L^{AB}(Y) &= V_F^{AB}(Y) \quad if \quad Y \geq Y^{B*},
\end{aligned}
$$

under the conditions that

$$\Pi^L(2; v) \geq \Pi(2), \tag{16}$$

$$\Pi(2) \leq \left(1 + \frac{I^n}{I^p}\right) \Pi^F(2; v), \tag{17}$$

and

$$\chi(v, I^n, I^p) > \frac{I^n}{\Pi^{LF}(2; v) / (r - \alpha)}, \tag{18}$$

where $\Pi^{LF}(2; v) \equiv \Pi^L(2; v) - \Pi^F(2; v)$ and $\chi(v, I^n, I^p) \equiv \frac{1-\left(Y^{A*}/Y^{B*}\right)^{\beta-1}}{1-\left(Y^{A*}/Y^{B*}\right)^{\beta}} Y^{A*}$.

Condition (17) defines the upper bound of v for the access-to-bypass equilibrium to exist. On the other hand, condition (18) defines a lower bound of v. Indeed, condition (18) implies $Y^{A*} > I^n(r-\alpha)/\Pi^{LF}(2; v)$, which is equivalent to $\Pi^{LF}(2; v)/\Pi^F(2; v) >$

$[(\beta-1)/(\beta I^p)]\, I^n$. Notice that $\Pi^{LF}(2;v)/\Pi^F(2;v)$ is an increasing function of v. Hence, if v is too small, condition (18) is violated. That is, v cannot be too small compared with I^n. This is because if v is too small, this gives the two firms an incentive to be a follower rather than a leader, which implies an equilibrium cannot exist. It is shown that v which satisfies condition (18) does not violate condition (16), so long as condition (17) holds. Note also that the monopoly profit $\Pi(1)$ is not included in any condition in the above proposition. That is, the access-to-bypass equilibrium exists for any level of monopoly profit that satisfies Assumption (i).

Proof of Proposition 1

(i) In the access-to-bypass equilibrium, the relationship that $Y^{A*} < Y^{B*}$ holds. We show that $Y^{A*} < Y^{B}$ if and only if $Y^{A*} < Y^{B*}$. This is because:

$$\begin{aligned}
Y^{A*} &\equiv \frac{\beta}{\beta-1}\frac{r-\alpha}{\Pi^F(2;v)}I^p < \frac{\beta}{\beta-1}\frac{r-\alpha}{\Delta\Pi(2;v)}I^n \equiv Y^{B*} \\
&\Leftrightarrow \left(\Pi(2)-\Pi^F(2;v)\right)I^p < \Pi^F(2;v)\,I^n \\
&\Leftrightarrow \Pi(2)\,I^p < \Pi^F(2;v)\left(I^p+I^n\right) \\
&\Leftrightarrow Y^{A^*} \equiv \frac{\beta}{\beta-1}\frac{r-\alpha}{\Pi^F(2;v)}I^p < \frac{\beta}{\beta-1}\frac{r-\alpha}{\Pi(2)}\left(I^p+I^n\right) \equiv Y^B.
\end{aligned}$$

(ii) Similarly, it is easy to show that $Y^B < Y^{B*}$ if and only if $Y^{A*} < Y^{B*}$. In fact,

$$\begin{aligned}
Y^{A*} &\equiv \frac{\beta}{\beta-1}\frac{r-\alpha}{\Pi^F(2;v)}I^p < \frac{\beta}{\beta-1}\frac{r-\alpha}{\Delta\Pi(2;v)}I^n \equiv Y^{B*} \\
&\Leftrightarrow \left(\Pi(2)-\Pi^F(2;v)\right)I^p < \Pi^F(2;v)\,I^n \\
&\Leftrightarrow \left(\Pi(2)-\Pi^F(2;v)\right)\left(I^p+I^n\right) < \Pi(2)\,I^n \\
&\Leftrightarrow Y^B \equiv \frac{\beta}{\beta-1}\frac{r-\alpha}{\Pi(2)}\left(I^p+I^n\right) < \frac{\beta}{\beta-1}\frac{r-\alpha}{\Delta\Pi(2;v)}I^n \equiv Y^{B*}.
\end{aligned}$$

Therefore, we have $Y^B < Y^{B*}$. ■

Proof of Proposition 4

Substituting (9) and (10) for $\forall Y \in \left(0, Y^{A*}\right)$ into $Q^{AB}(Y) \equiv V_L^{AB}(Y) - V_F^{AB}(Y)$ and rearranging it, we have:

$$\begin{aligned} Q^{AB}(Y) &= \frac{Y\Pi(1)}{r-\alpha} - (I^p + I^n) \\ &\quad + \left(\frac{Y}{Y^{A*}}\right)^{\beta} \left\{ I^p + \frac{Y^{A*}\Pi^{LF}(2; v)}{r-\alpha} - \frac{Y^{A*}\Pi(1)}{r-\alpha} \right\} \\ &\quad + \left(\frac{Y}{Y^{B*}}\right)^{\beta} \left\{ I^n - \frac{Y^{B*}\Pi^{LF}(2; v)}{r-\alpha} \right\}. \end{aligned} \tag{19}$$

To prove the proposition, we check the sign of $Q^{AB}(Y_L)$ where Y_L is the trigger point of the leader under facility-based competition equilibrium.

Note that Y_L is characterized by $V_L^B(Y_L) = V_F^B(Y_L)$, so that we have:

$$\begin{aligned} &\frac{Y_L\Pi(1)}{r-\alpha}\left[1 - \left(\frac{Y_L}{Y^B}\right)^{\beta-1}\right] + \left(\frac{Y_L}{Y^B}\right)^{\beta} \frac{Y^B\Pi(2)}{r-\alpha} - (I^p + I^n) \\ = &\left(\frac{Y_L}{Y^B}\right)^{\beta_1} \left[\frac{Y^B\Pi(2)}{r-\alpha} - (I^p + I^n)\right], \end{aligned}$$

or

$$\frac{Y_L\Pi(1)}{r-\alpha} - (I^p + I^n) = \left(\frac{Y_L}{Y^B}\right)^{\beta} \left[\frac{Y^B\Pi(1)}{r-\alpha} - (I^p + I^n)\right]. \tag{20}$$

Substituting (20) into $Q^{AB}(Y_L)$ gives:

$$Q^{AB}(Y_L) = (Y_L)^{\beta} \chi(\mathbf{x}),$$

where

$$\begin{aligned} \chi(\mathbf{x}) &\equiv \left(Y^B\right)^{-\beta} \left[\frac{Y^B\Pi(1)}{r-\alpha} - (I^p + I^n)\right] \\ &\quad + \left(Y^{A*}\right)^{-\beta} \left[I^p + \frac{Y^{A*}\Pi^{LF}(2; v)}{r-\alpha} - \frac{Y^{A*}\Pi(1)}{r-\alpha}\right] \\ &\quad + \left(Y^{B*}\right)^{-\beta} \left[I^n - \frac{Y^{B*}\Pi^{LF}(2; v)}{r-\alpha}\right], \end{aligned} \tag{21}$$

and $x \equiv (v, \Pi(1), \Pi(2), I^p, I^n)$.

Hence, $Q^{AB}(Y_L) < 0$ if $\chi(\mathbf{x}) < 0$. Note that the terms in the bracket of the third term of (21) are negative under the condition of (8). Hence, $\chi(\mathbf{x}) < 0$ if:

$$\left(Y^B\right)^{-\beta}\left[\frac{Y^B\Pi(1)}{r-\alpha} - (I^p + I^n)\right] + \left(Y^{A*}\right)^{-\beta}\left[I^p + \frac{Y^{A*}\Pi^{LF}(2;v)}{r-\alpha} - \frac{Y^{A*}\Pi(1)}{r-\alpha}\right] < 0, \quad (22)$$

or

$$\begin{aligned}&\left(Y^B\right)^{-\beta}(I^p + I^n)\left[\frac{\beta}{\beta-1}\frac{\Pi(1)}{\Pi(2)} - 1\right] \\ &+ \left(Y^{A*}\right)^{-\beta} I^p\left[1 + \frac{\beta}{\beta-1}\frac{\Pi^{LF}(2;v)}{\Pi^F(2;v)} - \frac{\beta}{\beta-1}\frac{\Pi(1)}{\Pi^F(2;v)}\right] < 0. \end{aligned} \quad (23)$$

We will show (23) holds if $\Pi(1)$ is sufficiently large.

Define $\Gamma[\Pi(1)] \equiv [m\Pi(1) - l] / [j\Pi(1) - k]$ where $m \equiv \frac{\beta}{\beta-1}\frac{I^p}{\Pi^F(2;v)}$, $l \equiv I^p + \frac{\beta}{\beta-1}\frac{\Pi^{LF}(2;v)}{\Pi^F(2;v)}$, $j \equiv \frac{\beta}{\beta-1}\frac{I^p+I^n}{\Pi(2)}$, and $k \equiv (I^p + I^n)$. Note that (8) ensures $j \geq m$ as long as the access-to-bypass equilibrium exists. In addition, $l > k$ holds if (8) holds. Hence:

$$\frac{d\Gamma[\Pi(1)]}{d\Pi(1)} = \frac{jl - km}{[j\Pi(1) - k]^2} > 0. \quad (24)$$

Equation (24) shows that $\Gamma[\Pi(1)]$ is an increasing function of $\Pi(1)$ and that $\Gamma[\Pi(1)]$ monotonically converges to m/j as $\Pi(1)$ goes to infinity.

Let us compare $\left(Y^{A*}/Y^B\right)^\beta$ with $\Gamma[\Pi(1)]$. Since $Y^{A*}/Y^B = m/j < 1$ and $\beta > 1$, there exists a threshold of $\Pi(1)$ above which $(m/j)^\beta < \Gamma[\Pi(1)]$ or:

$$\left(\frac{Y^{A*}}{Y^B}\right)^\beta < \frac{m\Pi(1) - l}{j\Pi(1) - k}. \quad (25)$$

Since $\Pi(1) > \Pi(2)$, we have $j\Pi(1) - k > 0$. Hence, rearranging (25) gives (23). That is, if $\Pi(1)$ is sufficiently large, a leader enters earlier in the facility-based competition equilibrium than in the access-to-bypass equilibrium. ■

Proof of Proposition 5

Let us rewrite $Q^{AB}(Y_L)$ again.

$$\begin{aligned} Q^{AB}(Y_L) &= \frac{Y_L\Pi(1)}{r-\alpha} - (I^p + I^n) \\ &+ \left(\frac{Y_L}{Y^{A*}}\right)^{\beta}\left\{I^p + \frac{Y^{A*}\Pi^{LF}(2;v)}{r-\alpha} - \frac{Y^{A*}\Pi(1)}{r-\alpha}\right\} \\ &+ \left(\frac{Y_L}{Y^{B*}}\right)^{\beta}\left\{I^n - \frac{Y^{B*}\Pi^{LF}(2;v)}{r-\alpha}\right\}. \end{aligned} \tag{26}$$

Using $I^p = (\beta/(\beta-1))\left(\Pi^F(2;v)/(r-\alpha)\right)Y^{A*}$ and $I^n = (\beta/(\beta-1)) \times (\Delta\Pi(2;v)/(r-\alpha))Y^{B*}$, $Q^{AB}(Y_L)$ is rewritten as follows.

$$Q^{AB}(Y_L) = \frac{Y_L\Pi(1)}{r-\alpha} - (I^p + I^n) - (Y_L)^{\beta}[B_F + B_L],$$

where $B_F \equiv \frac{1}{\beta(r-\alpha)}\left[\Pi^F(2;v)\left(Y^{A*}\right)^{1-\beta} + \Delta\Pi(2;v)\left(Y^{B*}\right)^{1-\beta}\right]$,
and $B_L \equiv \frac{1}{(r-\alpha)}\left[\left(\Pi(1) - \Pi^L(2;v)\right)\left(Y^{A*}\right)^{1-\beta} + \left(\Pi^L(2;v) - \Pi(2)\right)\left(Y^{B*}\right)^{1-\beta}\right]$. Differentiating $Q^{AB}(Y_L)$ with respect to v gives

$$\frac{\partial Q^{AB}(Y_L)}{\partial v} = -(Y_L)^{\beta}\frac{\partial[B_F + B_L]}{\partial v}.$$

To prove (i) and (ii) of the proposition, we firstly show that, under condition (11), $Q^{AB}(Y_L)$ is a monotone increasing function of v. Then we show that, at the upper limit of v that guarantees the existence of the access-to-bypass equilibrium, $Q^{AB}(Y_L) = 0$.

In fact, we have:

$$\frac{\partial B_F}{\partial v} = \frac{\beta}{\beta-1}\frac{\partial\Pi^F(2;v)}{\partial v}\left[\left(Y^{A*}\right)^{-\beta}\frac{I^p}{\Pi^F(2;v)} - \left(Y^{B*}\right)^{-\beta}\frac{I^n}{\Delta\Pi(2;v)}\right]. \tag{27}$$

$$\begin{aligned} \frac{\partial B_L}{\partial v} &= \frac{-1}{r-\alpha}\frac{\partial\Pi^L(2;v)}{\partial v}\left[\left(Y^{A*}\right)^{1-\beta} - \left(Y^{B*}\right)^{1-\beta}\right] + \beta\frac{\partial\Pi^F(2;v)}{\partial v} \times \\ &\left[\left(Y^{A*}\right)^{-\beta}\frac{\left(\Pi(1) - \Pi^L(2;v)\right)I^p}{\left(\Pi^F(2;v)\right)^2} - \left(Y^{B*}\right)^{-\beta}\frac{\left(\Pi^L(2;v) - \Pi(2)\right)I^n}{\left(\Delta\Pi(2;v)\right)^2}\right] \end{aligned} \tag{28}$$

Since $Y^{A*}/Y^{B*} = \Delta\Pi(2;v)\, I^p/\Pi^F(2;v)\, I^n\ (<1)$, both the square bracket of (27) and the one in the first term of (28) are positive. Hence, a sufficient condition for $\partial[B_F + B_L]/\partial v < 0$ is:

$$\left(Y^{A*}\right)^{-\beta}\frac{\left(\Pi(1)-\Pi^L(2;v)\right)I^p}{\left(\Pi^F(2;v)\right)^2} \geq \left(Y^{B*}\right)^{-\beta}\frac{\left(\Pi^L(2;v)-\Pi(2)\right)I^n}{\left(\Delta\Pi(2;v)\right)^2}. \tag{29}$$

Since $\left(Y^{A*}\right)^{-\beta}\frac{I^p}{\Pi^F(2;v)} > \left(Y^{B*}\right)^{-\beta}\frac{I^n}{\Delta\Pi(2;v)}$, the sufficient condition for $\partial[B_F + B_L]/\partial v < 0$ or $\partial Q^{AB}(Y_L)/\partial v > 0$ is

$$\frac{\Pi(1)-\Pi^L(2;v)}{\Pi^F(2;v)} \geq \frac{\Pi^L(2;v)-\Pi(2)}{\Delta\Pi(2;v)} \tag{30}$$

Rearranging it, we have

$$\frac{\Pi(1)-\Pi^L(2;v)}{\Pi^F(2;v)} \geq \frac{\Pi(1)-\Pi(2)}{\Pi(2)},$$

which is the condition of (11).

Next, let us examine the upper limit, $\overline{v}$, of the access charge that guarantees the existence of the access-to-bypass equilibrium. It is apparent that (8) defines $\overline{v}$. That is, $\Pi(2) = \left(1+\frac{I^p}{I^n}\right)\Pi^F(2;\overline{v})$. It is then easy to check that $Q^{AB}(Y_L) = 0$ at $\overline{v}$.

Hence, for $v \leq \overline{v}$, $Q^{AB}(Y_L) < 0$. This means that, under condition (11), a leader in the facility-based competition equilibrium enters earlier than the one in the access-to-bypass equilibrium. ■

Proof of Proposition 6

Let us rewrite $Q^{AB}(Y_L)$ as follows.

$$Q^{AB}(Y_L) = \left(\frac{Y^L}{Y^{A*}}\right)^{\beta}\widetilde{\chi}(\mathbf{x}),$$

where $\widetilde{\chi}(\mathbf{x}) \equiv \left(Y^{A*}\right)^{\beta}\chi(\mathbf{x})$ and $\chi(\mathbf{x})$ is defined in the proof of Proposition 3.

Using $Q^{AB}\left(Y^{A*}\right) = \frac{Y^{A*}\Pi^{LF}(2;v)}{r-\alpha} - I^n + \left(\frac{Y^{A*}}{Y^{B*}}\right)^{\beta} I^n\left[1-\frac{\beta}{\beta-1}\frac{\Pi^{LF}(2;v)}{\Delta\Pi(2;v)}\right]$, $\widetilde{\chi}(\mathbf{x})$ is rewritten as:

$$\begin{aligned}\widetilde{\chi}(\mathbf{x}) &= \left(\frac{Y^{A*}}{Y^B}\right)^{\beta}(I^p+I^n)\left[\frac{\beta}{\beta-1}\frac{\Pi(1)}{\Pi(2)}-1\right] \\ &\quad +I^p\left[1-\frac{\beta}{\beta-1}\frac{\Pi(1)}{\Pi^F(2;v)}\right]+I^n+Q^{AB}\left(Y^{A*}\right) \\ &= (I^p+I^n)\left[\left(1-N^{\beta}\right)-\frac{\beta}{\beta-1}\frac{\Pi(1)}{\Pi(2)}N\left(1-N^{\beta-1}\right)\right]+Q^{AB}\left(Y^{A*}\right), \quad (31)\end{aligned}$$

where $N \equiv \frac{\Pi(2)I^p}{\Pi^F(2;v)(I^p+I^n)}\left(=\frac{Y^{A*}}{Y^B}\right) < 1$. Let us denote the bracket part of the first term of (31) by $\Omega(N,\beta)$:

$$\Omega(N,\beta) \equiv \left(1-N^{\beta}\right)-\frac{\beta}{\beta-1}\frac{\Pi(1)}{\Pi(2)}N\left(1-N^{\beta-1}\right).$$

Since $Q^{AB}\left(Y^{A*}\right) > 0$, a sufficient condition for $\widetilde{\chi}(\mathbf{x}) > 0$ is $\Omega(N,\beta) > 0$. Then, as $\beta \to \infty$, $\Omega(N,\beta) \to 1-\frac{\Pi^F(2;v)(I^p+I^n)}{\Pi(1)I^p}$.[20] Therefore, when $\Pi(1) \leq \frac{\Pi^F(2;v)(I^p+I^n)}{I^p}$, $\Omega(N,\beta) > 0$ as $\beta \to \infty$ or $\sigma \to 0$. ■

Proof of Proposition 7

(i) A trigger point at which firm 1 builds a bypass is represented by Y_1^{B*} and the one for firm 2 is represented by Y_2^{B*}. It is easy to verify that $Y_1^{B*} < Y_2^{B*}$ because $I_1^n < I_2^n$.

(ii) A trigger point at which an entrant accesses to the incumbent's network facility for both firms remains as Y^{A*}. This is obvious, since Y^{A*} does not depend on the cost of the network facility.

(iii) We show that $Y_{L1}^* < Y_{L2}^*$ where Y_{L1}^* is a trigger point at which firm 1 enters the market as a leader and Y_{L2}^* is the one for firm 2. Firm 1 is assumed to enter the market as a leader at which $Y = Y_{L1}^*$ is defined by $V_F^{AB}(Y_{L1}^*) = V_L^{AB}(Y_{L1}^*)$, or:

$$B_F(Y_{L1}^*)^{\beta} = \frac{\Pi(1)}{r-\alpha}Y_{L1}^* - B_L(Y_{L1}^*)^{\beta} - (I^p+I^n-\varepsilon), \quad (32)$$

where $B_F \equiv \frac{\Pi^F(2;v)}{\beta(r-\alpha)}\left(Y^{A*}\right)^{1-\beta}+\frac{\Delta\Pi(2;v)}{\beta(r-\alpha)}\left(Y_2^{B*}\right)^{1-\beta}$ and $B_L \equiv \frac{\Pi(1)-\Pi^F(2;v)}{r-\alpha}\left(Y^{A*}\right)^{1-\beta} -$

[20] Notice that, as $\sigma \to 0$, $\beta \to \frac{r}{\alpha}$ for $\alpha > 0$. (See p.144 of Dixit and Pindyck.) Hence, the claim in this sentence holds when α is sufficiently small when compared with r.

$\frac{\Delta\Pi(2;v)}{r-\alpha}\left(Y_2^{B*}\right)^{\beta}$. Differentiating it, we have:

$$CdY_{L1}^{*}+\left[\frac{\partial\left(B_F+B_L\right)}{\partial\epsilon}\left(Y_{L1}^{*}\right)^{\beta}-1\right]d\epsilon=0, \tag{33}$$

where $C\equiv V_F^{AB\prime}\left(Y_{L1}^{*}\right)-V_L^{AB\prime}\left(Y_{L1}^{*}\right)<0$. As $\partial Y_2^{B*}/\partial\epsilon>0$, we have

$$\begin{aligned}\frac{\partial\left(B_F+B_L\right)}{\partial\epsilon} &= \frac{\partial\left(B_F+B_L\right)}{\partial Y_2^{B*}}\frac{\partial Y_2^{B*}}{\partial\epsilon}\\ &= \left[\frac{(1-\beta)\,\Delta\Pi\,(2;v)}{\beta\,(r-\alpha)}\left(Y_2^{B*}\right)^{-\beta}-\frac{\beta\Delta\Pi\,(2;v)}{r-\alpha}\left(Y_2^{B*}\right)^{\beta-1}\right]\frac{\partial Y_2^{B*}}{\partial\epsilon}<0.\end{aligned}$$

Since $\partial Y_{L1}^{*}/\partial\epsilon<0$, we can show that $Y_{L1}^{*}<Y_L^{*}$.

On the other hand, we examine the case where firm 2 enters the market as a leader. Through similar calculations, it can be easily shown that $Y_L^{*}<Y_{L2}^{*}$. Therefore, $Y_{L1}^{*}<Y_{L2}^{*}$. ■

6.1 Proof of Proposition 8

In the specification of Cournot competition with a linear inverse demand function stated here, the condition (11) is expressed as follows.

$$4D\left(\gamma\right)X^2-4E\left(\gamma\right)X+F\left(\gamma\right)\geq 0, \tag{34}$$

where

$$\begin{aligned}D\left(\gamma\right) &\equiv 3\left(2-\gamma\right)\left(A+\theta\right)^2-\left(2+\gamma\right)A^2,\\ E\left(\gamma\right) &\equiv \left(2-\gamma\right)A\left[\left(4-\gamma\right)\left(A+\theta\right)^2-\left(2+\gamma\right)A^2\right],\\ \text{and } F\left(\gamma\right) &\equiv \left(2+\gamma\right)\left(2-\gamma\right)^2A^2\left[\left(A+\theta\right)^2-A^2\right].\end{aligned}$$

The left-hand side of (34) is a quadratic function of X with $D\left(\gamma\right)>0$, $E\left(\gamma\right)>0$, and $F\left(\gamma\right)>0$ for any $\gamma\in[0,1]$. Then, a tedious calculation shows that $\left(E\left(\gamma\right)\right)^2-4D\left(\gamma\right)F\left(\gamma\right)\geq 0$ for any $\gamma\in[0,1]$, so that there exists at least one solution for $4D\left(\gamma\right)X^2-4E\left(\gamma\right)X+F\left(\gamma\right)=0$.

Let us denote the small solution by $X^*(\gamma)$. Then, we have

$$X^*(\gamma) = \frac{(2-\gamma)(2+\gamma)A\left((A+\theta)^2 - A^2\right)}{2\left(3(2-\gamma)(A+\theta)^2 - (2+\gamma)A^2\right)}.$$

Since $D(\gamma) > 0$, $E(\gamma) > 0$, and $F(\gamma) > 0$ for any $\gamma \in [0,1]$, (11) holds in the range of $[0, X^*(\gamma)]$. This means that, when the level of access charge is low, (i.e., near to access cost c), the facility-based competition scheme induces a leader to invest in infrastructure earlier than the service-based competition.

In addition, differentiating $X^*(\gamma)$ with respect to γ gives

$$X^{*\prime}(\gamma) = \frac{A\left((A+\theta)^2 - A^2\right)\left[3(\gamma-2)^2(A+\theta)^2 + (\gamma-2)^2 A^2\right]}{2\left(3(2-\gamma)(A+\theta)^2 - (2+\gamma)A^2\right)^2} > 0.$$

That is, $X^*(\gamma)$ is an increasing function of γ. This means that the range of $X \equiv v - c$ that satisfies (11) (or (12)) gets large as γ gets large.[21] ■

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[21] Of course, Assumption (i) and the condition that $\Pi^L(2; v) > \Pi(2)$ (i.e., a sufficient condition for the existence of access-to-bypass equilibrium) restricts the range of $X^*(\gamma)$. This statement holds when we pay attention to these restrictions.

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