ON ONE INVERSE PROBLEM FOR TRAFFIC FLOW ON THE MULTILANE ROAD

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INTRODUCTION

It is well known, that non-determined models of traffic flows allow random vehicles motion under the influence of accidental impulses. One of the differences between the non-determined and determined (microscopic and macroscopic) models is rooted in the fact that it states to be impossible to predict motion of each independent vehicle in the non-determined models, where vehicle motion supposes only probabilistic description. However, if there are plenty of vehicles, driving "accidentally" and "independently" from each other, a behavior of transport system as a whole can be predicted quite definitely. As a rule, it can be regarded as a subject of primary interest for the traffic flow studies.

While mathematical formulation of the non-deterministic traffic flows process it has been used the continuous flow density model, which could be presented in the form of the following equation ([1-2], [4], [6]):

$$\frac{\partial \rho(x,t)}{\partial t} = a^2(t) \cdot \frac{\partial^2 \rho(x,t)}{\partial x^2}, \; 0 < x < l, \; 0 < t \le T, \tag{1}$$

where $a^2(t) > 0$ is a proportionality coefficient between a flux level and a density gradient:

$$a^2((t)) = -\frac{q(x,t)}{\frac{\partial \rho(x,t)}{\partial t}}, \; \frac{\partial \rho(x,t)}{\partial t} \ne 0, \; 0 < x < l, \; 0 < t \le T,$$

$q(x,t) = \rho(x,t) \cdot \vartheta(x,t)$ is a traffic flux level, $\vartheta(x,t)$ is a traffic flow velocity.

This positive coefficient $a^2(t)$ characterizes the ratio of vehicle flow's absolute velocity through fixed length unit of a road section to the density gradient under the condition that the length of road section is normal to this gradient.

According to [1] for brevity, the coefficient $a^2(t)$ we will call as the sensitivity coefficient of the vehicular traffic in stream. It is obvious that the coefficient $a^2(t)$ is the sensitive characteristic of the curves structure, described by vehicular traffic in flow, and it could be interpreted as a quantity of vehicles passing through the section of unit area at time unit under the condition that the density gradient of traffic flow equals unity.

Since the sensitivity coefficient $a^2(t)$ covers fully the velocity of traffic flow process (this process is defined by non-stationary environment properties, and they can be change as time passes), then this coefficient depends on time and, in general terms, is unknown ([3], [5], [6]). Consequently, a question of its determination under some appropriate initial and boundary conditions from continuous model (1) is emerging. In the present paper the problem of unambiguous determination of the sensitivity coefficient $a^2(t)$ under some additional information is investigated. It is of importance to note that the required additional information must be natural requirements from the point of view of real-life traffic flow process.

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THE DIRECT PROBLEM FOR TRAFFIC FLOW DENSITY

Let's consider the following problem consisting of the equation for traffic flow density

$$\frac{\partial\rho(x,t)}{\partial t}=a^2(t)\cdot\frac{\partial^2\rho(x,t)}{\partial x^2},\ 0<x<l,\ 0<t\le T, \qquad (1)$$

the initial condition

$$\rho(x,t)\big|_{t=0}=h(x),\ 0\le x\le l, \qquad (2)$$

the two boundary conditions of the first type (so-called Dirichlet conditions)

$$\rho(x,t)\big|_{x=0}=0,\ 0\le t\le T, \qquad (3)$$

$$\rho(x,t)\big|_{x=l}=0,\ 0\le t\le T, \qquad (4)$$

the consistency conditions

$$\begin{cases} h(0)=h(l)=0, \\ h''(0)=h''(l)=0 \end{cases} \qquad (5)$$

and finally the requirement

$$a^2(t)\in C[0,T],\ h(x)\in C^4[0,l]. \qquad (6)$$

The direct problem is following: it is necessary to find the traffic flow density $\rho(x,t)$ from the equation (1) and the conditions (2)-(6) under on the assumption that the following conditions are satisfied:

$$\begin{cases} \rho(x,t)\in C\{[0,l]\times[0,T]\}, \\ \rho(x,t)\in C^{2,1}\{(0,l)\times(0,T]\}, \end{cases} \qquad (7)$$

Remark 1. *Strong enough and little bit strange requirement $h(x)\in C^4[0,l]$ in (6) is not the necessary requirement for the formulated problem (1)-(7). The given requirement is caused by that the proof of existence and uniqueness of the solution of (1)-(7) under such requirement essentially becomes simpler. This requirement can be essentially weakened, for example, this condition can be replaced with the requirement $h(x)\in C[0,l],\ \exists h''(0),h''(l)$. All obtained results will hold good also for this case.*

It is not difficult to find the solution of the direct problem (1)-(9) by the variable separation method (see, for example, [15-16]):

$$\rho(x,t)=\frac{2}{l}\cdot\int_0^l G(x,\xi,t)\cdot h(\xi)d\xi,\ 0\le x\le l,\ 0\le t\le T, \qquad (8)$$

where $G(x,\xi,t)\overset{def}{\equiv}\sum_{n=1}^{\infty}\sin\frac{\pi\cdot n\cdot x}{l}\cdot\sin\frac{\pi\cdot n\cdot\xi}{l}\cdot e^{-\frac{\pi^2\cdot n^2}{l^2}\cdot\int_0^t a^2(\tau)d\tau}$ is the Green function of the considered problem.

We must prove that the function (8) is satisfied the conditions (7). For that let's rewrite

$$\rho(x,t)=\sum_{n=1}^{\infty}h_n\cdot\sin\frac{\pi\cdot n\cdot x}{l}\cdot e^{-\frac{\pi^2\cdot n^2}{l^2}\cdot\int_0^t a^2(\tau)d\tau}, \qquad (9)$$

where

$$h_n\overset{def}{\equiv}\frac{2}{l}\cdot\int_0^l h(\xi)\cdot\sin\frac{\pi\cdot n\cdot\xi}{l}d\xi. \qquad (10)$$

From (10) we have got

$$h_n = \frac{2 \cdot l^3}{\pi^4 \cdot n^4} \cdot \int_0^l h^4(\xi) \cdot \sin\frac{\pi \cdot n \cdot \xi}{l} d\xi .$$

Therefore we can write

$$|h_n| \le \frac{const}{n^4} .$$

Then from this inequality and because of continuity of all summands of the infinite series we receive that, firstly,
$\rho(x,t) \in C\{0 \le x \le l, 0 \le t \le T\}$ and secondly, the conditions (2)-(4) are satisfied.
Since the series in the bracketed expression

$$\frac{\partial \rho(x,t)}{\partial t} = \left[\sum_{n=1}^{\infty} h_n \cdot n^2 \cdot \sin\frac{\pi \cdot n \cdot x}{l} \cdot e^{-\frac{\pi^2 \cdot n^2}{l^2} \cdot \int_0^t a^2(\tau) d\tau} \right] \cdot \left(-\frac{\pi^2}{l^2} \cdot a^2(t) \right)$$

is equiconvergent series, and the series in the expression

$$\frac{\partial^2 \rho(x,t)}{\partial x^2} = \left[\sum_{n=1}^{\infty} h_n \cdot n^2 \cdot \sin\frac{\pi \cdot n \cdot x}{l} \cdot e^{-\frac{\pi^2 \cdot n^2}{l^2} \cdot \int_0^t a^2(\tau) d\tau} \right] \cdot \left(-\frac{\pi^2}{l^2} \right)$$

is also equiconvergent series then

$\frac{\partial \rho(x,t)}{\partial t} \in C\{0 < x < l, 0 < t \le T\}$ and

$\frac{\partial^2 \rho(x,t)}{\partial x^2} \in C\{0 < x < l, 0 < t \le T\}$.

In other words, we have proved that
$\rho(x,t) \in C^{2,1}\{0 < x < l, 0 < t \le T\}$, i.e. both conditions in (7) are satisfied, and definite by formula (8) function $\rho(x,t)$ is the solution of the problem (1)-(7).

Now we will prove the uniqueness of solution (8). For that it is sufficient to prove that the problem (1)-(6) has only trivial solution under trivial initial condition $h(x) \equiv 0$. Really, let's (1) multiplied by $\rho(x,t)$, then after integration of obtained expression from 0 to l with respect to variable x, and integration from 0 to $t \in (0,T]$ with respect to variable τ, we have got

$$\int_0^l dx \int_0^t \frac{\partial \rho(x,\tau)}{\partial \tau} \cdot \rho(x,\tau) d\tau = \int_0^t a^2(\tau) \int_0^l \frac{\partial \rho(x,\tau)}{\partial x^2} \cdot \rho(x,\tau) dx d\tau .$$

From here

$$\frac{1}{2} \cdot \int_0^l \left[\rho^2(x,\tau) - \rho^2(x,0) \right] dx = \int_0^t a^2(\tau) \cdot \left[\frac{\partial \rho(x,t)}{\partial x} \cdot \rho(x,\tau) \Big|_{x=0}^{x=l} - \int_0^l \left\{ \frac{\partial \rho(x,t)}{\partial x} \right\}^2 dx \right] d\tau ,$$

or in other form

$$\int_0^l \rho^2(x,\tau) dx + 2 \cdot \int_0^t a^2(\tau) \cdot \int_0^l \left\{ \frac{\partial \rho(x,t)}{\partial x} \right\}^2 dx d\tau = 0, \;\; a^2(t) > 0 .$$

It follows that
$\rho(x,t) \equiv 0$ for $\forall x \in [0,l]$ $\forall t \in [0,T]$, and this makes it possible to prove uniqueness of the solution of the direct problem (1)-(7).

STATEMENT OF THE INVERSE PROBLEM FOR DETERMINATION OF THE SENSITIVITY COEFFICIENT

Let's formulate corresponding coefficient inverse problem: it is necessary to determine the coefficient $a^2(t) > 0$, $t \in [0,T]$ from (1)-(6) under the additional information

$$\rho(x,t)\big|_{x=\tilde{x}} = \tilde{\rho}(t),\ 0 \le t \le T\ , \tag{11}$$

where $\tilde{\rho}(t)$ is the known function, and the point $\tilde{x} \in (0,l)$ is given point at which it is preferable to allocate some sensors of distance-measuring equipment for experimental observation and registration of the traffic flow density at all times of observation interval. It is obvious the following two assertions:

1. If we assume $h(x) \equiv 0$, $x \in [0,l]$ then the solution of the problem (1)-(6) is $\rho(x,t) \equiv 0$, $(x,t) \in [0,l] \times [0,T]$. So there are infinitely many different coefficients $a^2(t)$.

2. If we assume $h(x) \ne 0$, $x \in [0,l]$ then the coefficient $a^2(t)$ is not unique. For example let $h(x) = \sin\frac{k \cdot \pi \cdot x}{l}$, where $k \ge 2$.

Then from (9)-(10) we have

$$\rho(x,t) = \sin\frac{k \cdot \pi \cdot x}{l} \cdot e^{-\frac{k^2 \cdot \pi^2}{l^2} \cdot \int_0^t a^2(\tau)d\tau}.$$

For example let's examine for the point $\tilde{x} = \frac{l}{k} \in (0,l)$, $k \ge 2$.

Then $\rho(x,t) = e^{-\frac{k^2 \cdot \pi^2}{l^2} \cdot \int_0^t a^2(\tau)d\tau} \cdot \sin\left(\frac{k \cdot \pi}{l} \cdot \frac{l}{k}\right) \equiv 0$.

From here we receive that $\rho(x,t) \equiv 0$ independently of any choice $a^2(t)$, i.e. the sensitivity coefficient $a^2(t)$ can be chosen arbitrary.

Thus, if we have the additional condition (4'), at that $\tilde{\rho}(0) = h(\tilde{x})$, then both in case of $h(x) \equiv 0$, and in case of $h(x) \ne 0$ the determination of the coefficient $a^2(t)$ from the formulated inverse problem (1)-(6), (11) is non-unique.

Therefore we must investigate the following important question: what requirements are necessary for unique determination of the sensitivity coefficient $a^2(t)$. In the next section such sufficient conditions for unique determination of the sensitivity coefficient $a^2(t)$ are expounded.

SUFFICIENT CONDITIONS FOR UNIQUE DETERMINATION OF THE SENSITIVITY COEFFICIENT

Theorem 1. *Let us suppose that the following two conditions are occurred in the inverse problem (1)-(7), (11):*

A. $\forall x \in (0,l)$ $h''(x) > 0$ *or* $h''(x) < 0$, *(12)*

B. $0 < a^2(t) \in C^1[0,T]$. *(13)*

Then the coefficient $a^2(t)$ *can be uniquely determined from (1)-(7), (11)-(13).*

Proof of Theorem.

We will make a inverse proposition: let there are $a_1^2(t)$, $a_2^2(t)$, $a_1^2(t) \neq a_2^2(t)$ and let the following conditions have been satisfied:

$$\frac{\partial \rho_1(x,t)}{\partial t} = a_1^2(t) \cdot \frac{\partial^2 \rho_1(x,t)}{\partial x^2},\ 0 < x < l,\ 0 < t \le T,$$

$$\frac{\partial \rho_2(x,t)}{\partial t} = a_2^2(t) \cdot \frac{\partial^2 \rho_2(x,t)}{\partial x^2},\ 0 < x < l,\ 0 < t \le T,$$

$$\rho_1(x,t)\big|_{t=0} = h(x) = \rho_2(x,t)\big|_{t=0},\ 0 < x < l,$$

$$\begin{cases} \rho_1(x,t)\big|_{x=0} = 0 = \rho_2(x,t)\big|_{x=0},\ 0 < t \le T, \\ \rho_1(x,t)\big|_{x=l} = 0 = \rho_2(x,t)\big|_{x=l},\ 0 < t \le T, \end{cases} \tag{14}$$

$$h(0) = h(l) = 0,$$

$$h''(0) = h''(l) = 0.$$

$$a_1^2(t),\ a_2^2(t) \in C^1[0,T],\ a_i^2(t) > 0\ \ (i = 1,2),$$

$$\rho_1(x,t)\big|_{x=\tilde{x}} = \tilde{\rho}(t) = \rho_2(x,t)\big|_{x=\tilde{x}},\ \tilde{x} \in (0,l),$$

$$|h''(x)| > 0 \ \ \forall x \in (0,l).$$

As it has been told in the Remark 1, in the problem (14) (also in the initial direct problem (1)-(7)) we assume $h(x) \in C^4[0,l]$. Once again we remind that instead of this condition we can assume more weak condition. All obtained results are also valid under more weak condition.
Since

$$\rho_i(x,t) = \sum_{n=1}^{\infty} h_n \cdot \sin\frac{\pi \cdot n \cdot x}{l} \cdot e^{-\frac{\pi^2 \cdot n^2}{l^2} \cdot \int_0^t a_i^2(\tau) d\tau},\ i = 1,2,\ 0 \le x \le l,\ 0 \le t \le T, \tag{15}$$

where $h_n \equiv \frac{2}{l} \cdot \int_0^l h(\xi) \cdot \sin\frac{\pi \cdot n \cdot \xi}{l} d\xi$,

then the additional information (11) means that

$$\begin{cases} \sum_{n=1}^{\infty} h_n \cdot \sin\frac{\pi \cdot n \cdot \tilde{x}}{l} \cdot e^{-\frac{\pi^2 \cdot n^2}{l^2} \cdot \int_0^t a_1^2(\tau) d\tau} = \tilde{\rho}(t),\ \ 0 \le t \le T, \\ \sum_{n=1}^{\infty} h_n \cdot \sin\frac{\pi \cdot n \cdot \tilde{x}}{l} \cdot e^{-\frac{\pi^2 \cdot n^2}{l^2} \cdot \int_0^t a_2^2(\tau) d\tau} = \tilde{\rho}(t),\ \ 0 \le t \le T. \end{cases}$$

After subtracting the second equality from the first equality, we have got

$$\sum_{n=1}^{\infty} h_n \cdot \sin\frac{\pi \cdot n \cdot \tilde{x}}{l} \cdot \left[e^{-\frac{\pi^2 \cdot n^2}{l^2} \cdot \int_0^t a_1^2(\tau) d\tau} - e^{-\frac{\pi^2 \cdot n^2}{l^2} \cdot \int_0^t a_2^2(\tau) d\tau} \right] = 0,\ \ \forall t \in [0,T]. \tag{16}$$

Obviously that

$$\int_0^1 e^{-\frac{\pi^2 \cdot n^2}{l^2} \cdot \int_0^t a_2^2(\tau) d\tau} d\xi - \xi \cdot \left[\frac{\pi^2 \cdot n^2}{l^2} \cdot \int_0^t a_1^2(\tau) d\tau - \frac{\pi^2 \cdot n^2}{l^2} \cdot \int_0^t a_2^2(\tau) d\tau \right] =$$

$$= \frac{e^{-\frac{\pi^2 \cdot n^2}{l^2}\cdot\int_0^t a_2^2(\tau)d\tau}}{\frac{\pi^2 \cdot n^2}{l^2}\cdot\int_0^t a_1^2(\tau)d\tau - \frac{\pi^2 \cdot n^2}{l^2}\cdot\int_0^t a_2^2(\tau)d\tau}\cdot e^{-\xi\cdot\left[\frac{\pi^2 \cdot n^2}{l^2}\cdot\int_0^t a_1^2(\tau)d\tau - \frac{\pi^2 \cdot n^2}{l^2}\cdot\int_0^t a_2^2(\tau)d\tau\right]}\Bigg|_{\xi=0}^{\xi=1} =$$

$$= \frac{e^{-\frac{\pi^2 \cdot n^2}{l^2}\cdot\int_0^t a_2^2(\tau)d\tau}\cdot\left[e^{-\frac{\pi^2 \cdot n^2}{l^2}\cdot\int_0^t a_1^2(\tau)d\tau + \frac{\pi^2 \cdot n^2}{l^2}\cdot\int_0^t a_2^2(\tau)d\tau} - 1\right]}{\frac{\pi^2 \cdot n^2}{l^2}\cdot\left[\int_0^t a_1^2(\tau)d\tau - \int_0^t a_2^2(\tau)d\tau\right]}.$$

From here

$$e^{-\frac{\pi^2 \cdot n^2}{l^2}\cdot\int_0^t a_1^2(\tau)d\tau} - e^{-\frac{\pi^2 \cdot n^2}{l^2}\cdot\int_0^t a_2^2(\tau)d\tau} =$$

$$= \frac{\pi^2}{l^2}\cdot\left[n^2\cdot\int_0^t a_1^2(\tau)d\tau - n^2\cdot\int_0^t a_2^2(\tau)d\tau\right]\cdot\int_0^1 e^{-\frac{\pi^2 \cdot n^2}{l^2}\cdot\int_0^t a_2^2(\tau)d\tau - \xi\cdot\left[\frac{\pi^2 \cdot n^2}{l^2}\cdot\int_0^t a_1^2(\tau)d\tau - \frac{\pi^2 \cdot n^2}{l^2}\cdot\int_0^t a_2^2(\tau)d\tau\right]}d\xi. \tag{17}$$

Now taking into account (13) in (12) we receive

$$\frac{\pi^2}{l^2}\cdot\left[\int_0^t a_1^2(\tau)d\tau - \int_0^t a_2^2(\tau)d\tau\right]\cdot\sum_{n=1}^{\infty} h_n \cdot n^2 \cdot \sin\frac{\pi \cdot n \cdot \tilde{x}}{l}\times$$

$$\times\int_0^1 e^{-\frac{\pi^2 \cdot n^2}{l^2}\cdot\int_0^t a_2^2(\tau)d\tau - \xi\cdot\left[\frac{\pi^2 \cdot n^2}{l^2}\cdot\int_0^t a_1^2(\tau)d\tau - \frac{\pi^2 \cdot n^2}{l^2}\cdot\int_0^t a_2^2(\tau)d\tau\right]}d\xi = 0, \quad \forall t \in [0,T]. \tag{18}$$

The series in this formula consist of the continuous functions. Besides this series is the equiconvergent series. Consequently, this series give us a continuous function, moreover at the point $t = 0$ we have

$$\sum_{n=1}^{\infty} h_n \cdot n^2 \cdot \sin\frac{\pi \cdot n \cdot \tilde{x}}{l}\cdot 1 = -h''(\tilde{x}) < 0 .$$

The following well-known Theorem is necessary for further evidence ([14]):

Theorem 2. *If the function $V(x,t)$ satisfies the equation*

$$\frac{\partial V(x,t)}{\partial t} = a^2(t)\cdot\frac{\partial^2 V(x,t)}{\partial x^2} + \frac{\left[a^2(t)\right]'}{a^2(t)}\cdot V(x,t),\ a^2(t) > 0,\ 0 < x < l,\ 0 < t \le T ,$$

if $V(x,t) \ge 0$ in the closed rectangle $(0 \le x \le l)\times(0 \le t \le T)$, and if there is such point (x_0, t_0), $0 < x_0 < l,\ 0 < t_0 \le T$ that $V(x_0,t_0) = 0$, then $V(x,t) \equiv 0$ for all $x \in [0,l]$ and all $t \in [0,t_0]$.

Using the Theorem 2 we can assert that the solution of the problem

$$\begin{cases} \frac{\partial V(x,t)}{\partial t} = a^2(t)\cdot\frac{\partial^2 V(x,t)}{\partial x^2} + \frac{\left[a^2(t)\right]'}{a^2(t)}\cdot V(x,t),\ a^2(t) > 0,\ 0 < x < l,\ 0 < t \le T, \\ V(0,t) = V(l,t) = 0,\ 0 \le t \le T, \\ V(x,0) = a^2(0)\cdot h''(x),\ 0 \le x \le l \end{cases}$$

is strictly positive function, i.e. $V(x,t)>0$ for all $x\in[0,l]$ and all $t\in[0,T]$.
Really, if this is not the case then there is such point (x_0,t_0), $0<x_0<l,\ 0<t_0\le T$ that $V(x_0,t_0)=0$. Consequently, $V(x,t)=0$ for all $0<x<l,\ 0<t\le t_0$. But in this case $V(x,0)=a^2(0)\cdot h''(x)=0$ for all $0\le x\le l$. Obtained result conflicts with the conditions of the Theorem 2. Consequently, $V(x,t)>0$ for all $x\in[0,l]$ and all $t\in[0,T]$.
It is elementary to prove that for arbitrary $\tilde{t}\ge 0$ the function-series

$$\sum_{n=1}^{\infty} h_n\cdot n^2\cdot\sin\frac{\pi\cdot n\cdot\tilde{x}}{l}\cdot\int_0^1 e^{-\frac{\pi^2\cdot n^2}{l^2}\cdot\int_0^t a_2^2(\tau)d\tau-\xi\cdot\left[\frac{\pi^2\cdot n^2}{l^2}\cdot\int_0^t a_1^2(\tau)d\tau-\frac{\pi^2\cdot n^2}{l^2}\cdot\int_0^t a_2^2(\tau)d\tau\right]}d\xi$$

is a nonvanishing series. Consequently, at each point $t\in\left[0,\tilde{t}\right)$ we have

$$\sum_{n=1}^{\infty} h_n\cdot n^2\cdot\sin\frac{\pi\cdot n\cdot\tilde{x}}{l}\cdot\int_0^1 e^{-\frac{\pi^2\cdot n^2}{l^2}\cdot\int_0^t a_2^2(\tau)d\tau-\xi\cdot\left[\frac{\pi^2\cdot n^2}{l^2}\cdot\int_0^t a_1^2(\tau)d\tau-\frac{\pi^2\cdot n^2}{l^2}\cdot\int_0^t a_2^2(\tau)d\tau\right]}d\xi<0.$$

Thus, we have proved that

$$\frac{\partial^2\rho(x,t)}{\partial x^2}=-\sum_{n=1}^{\infty} h_n\cdot n^2\cdot\sin\frac{\pi\cdot n\cdot\tilde{x}}{l}\cdot\int_0^1 e^{-\frac{\pi^2\cdot n^2}{l^2}\cdot\int_0^t a_2^2(\tau)d\tau-\xi\cdot\left[\frac{\pi^2\cdot n^2}{l^2}\cdot\int_0^t a_1^2(\tau)d\tau-\frac{\pi^2\cdot n^2}{l^2}\cdot\int_0^t a_2^2(\tau)d\tau\right]}d\xi>0.$$

On the other hand, from (14) follows that for each $t\in\left[0,\tilde{t}\right]$ the equality $\int_0^t a_1^2(\tau)d\tau=\int_0^t a_2^2(\tau)d\tau$ is valid. We will consider this function-series at the point $\tilde{t}$:

$$\sum_{n=1}^{\infty} h_n\cdot n^2\cdot\sin\frac{\pi\cdot n\cdot\tilde{x}}{l}\cdot e^{-n^2\cdot\int_0^{\tilde{t}} a_2^2(\tau)d\tau}=-\frac{\partial^2\rho_2\left(\tilde{x},\tilde{t}\right)}{\partial x^2},$$

Here without losing generality we assume the point $\tilde{t}$ is the minimal zero of this function-series. Since $\frac{\partial^2\rho_2(x,t)}{\partial x^2}>0$, $\forall 0\le x\le l$, $0\le t\le T$ then this function-series is not equal to zero at the point $\tilde{t}$. Consequently, our function is nowhere equal to zero in the closed segment $[0,T]$. Then

$$\int_0^t a_1^2(\tau)d\tau-\int_0^t a_2^2(\tau)d\tau=0 \text{ for } \forall t\in[0,T].$$

It means $a_1^2(t)=a_2^2(t),\ t\in[0,T]$.
The Theorem 1 is proved.
From the formula (8) follows that above formulated coefficient inverse problem presents the problem for solving of the following nonlinear operator equation for unique determination of the sensitivity coefficient $a^2(t)$:

$$Aa^2(t)=\tilde{\rho}(t), \tag{19}$$

where $$Aa^2(t)\overset{def}{\equiv}\sum_{n=1}^{\infty} h_n\cdot\sin\frac{\pi\cdot n\cdot\tilde{x}}{l}\cdot e^{-\frac{\pi^2\cdot n^2}{l^2}\cdot\int_0^t a^2(\tau)d\tau}. \tag{20}$$

The equation (19) with nonlinear operator (20) is ill-posed problem if we will suppose $A:C[0,T]\to C[0,T]$. In other words, the function $\tilde{\rho}(t)$ in the additional information (11)

can be given approximately in the uniform metric. This passing remark is favorable circumstance in case of traffic flow, when we have only discrete data as a result of work of some sensors for experimental observation and registration of the traffic flow density.
In view of the aforesaid we must take the following circumstances into account:

1) It is quite possible that there is no solution $a^2(t) \in C[0,T]$ if $\tilde{\rho}(t) \in C[0,T]$, but $\tilde{\rho}(t) \notin C^1[0,T]$, i.e. the condition $\tilde{\rho}(t) \in C^1[0,T]$ is the necessary condition for existence of solution;
2) The found solution is instable. Really, if we will assume that $\overline{a}_m^2(t) = a^2(t) + C \cdot \cos(m \cdot t)$, $a^2(t) \in C[0,T]$, $0 < C < \min_{t \in [0,T]} a^2(t)$ then we have got $\overline{a}_m^2(t) \in C[0,T]$ and $\overline{a}_m^2(t) > 0$ for $\forall t \in [0,T]$. Consequently, $\left\| \overline{a}_m^2(t) - a^2(t) \right\| = C$, but $\left\| A\overline{a}_m^2(t) - Aa^2(t) \right\| \xrightarrow{m \to \infty} 0$.

Closely similar to our problem (19)-(20) have been investigated by different authors. Because of an ill-posedness of such problems special methods and approaches are necessary for their solving. In the fundamental monograph [11] have been introduced and developed such basic concepts as "regularizing operator", "regularizing algorithm" and "regularized solution". In the present work we do not provide some effective regularizing algorithms for the solution of the nonlinear ill-posed problem (19)-(20), and only we will refer to works [7-13].

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