THE EFFECT OF COMPETITION ON INNOVATION IN THE NETWORK INDUSTRIES

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Abstract

A model of incumbent network operator is developed, and analysed when the incumbent is a monopolist as well as when the incumbent faces an entrant. The objectives of the incumbent are specified in a general manner to allow for revenue, profit, and/or welfare maximisation. The marginal cost of the incumbent is assumed to depend on the investment in new technologies. A strictly convex and decreasing cost function is assumed. The incumbent maximises its objective function with respect to prices and to investment in innovation. The entrant is assumed to maximise profits with respect to prices. The incentives to innovate under monopoly and duopoly are compared. The main results are that the market share of the incumbent under duopoly is a critical determinant of whether monopoly or duopoly creates more incentive to innovate. If the market share is above a certain threshold then monopoly provides more incentives than duopoly. Otherwise, it is duopoly that generates more investment by the incumbent in innovation. As expected from previous literature, the quantity supplied has a direct effect on innovation incentives. However, the results here differ in several respects from previous literature. In particular if regulation has the effect of motivating the incumbent to place greater weight on revenue or profit, relative to consumer welfare, then one can expect to see decreases in investment in innovation. These theoretical developments are examined through some computational experiments, based on a calibrated model of innovation in the postal sector.

Keywords: Innovation, Competition, Postal sector, Network industries, Regulation

JEL Classification: K23, L50, L87, L97, O31

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1. Introduction

Over the past decades, network industries have been going through a process of reform. Most network industries have evolved from being dominated by integrated state-owned monopolies to restructured industries with private sector participation and/or to partially or almost completely liberalized industries. One of the motivations for the reform has been the belief that competition stimulates product innovation (development of new products and services), process innovation (development of new technologies), encourages internal efficiency, and drives prices down.

The literature on the relationship between competition and innovation does not have a clear answer as to whether competition stimulates innovation or not. Increased competition is said to have both positive and negative effects on innovation. The positive effect is a result of the firm's quest to optimize profits through increasing its efficiency and reducing its cost of production. Profitability pushes the development and adoption of more efficient technologies and processes. At the same time, competition decreases the rents of the monopolist and might reduce its market share. Therefore, revenue will also decrease. As a result, firms will have fewer resources to invest, for instance, in research and development. Similarly, they may encounter more difficulties when trying to recover potential investments into new technologies and new processes because of erosion of scale or scope economies resulting from lost market share under competition.

The debate about the influence of the intensity of market competition on technical progress started with Schumpeter (1942) and continued with Arrow (1962). Schumpeter argues that monopoly favours the development of R&D activities because it provides the necessary cash flow to invest in such activities and reduces uncertainty in the market. Twenty years later Arrow investigated the effects of market structure on the firm's incentives to invest in R&D in order to reduce costs. Arrow concluded that under competition the single firm gets more benefits from innovation than under monopoly. The intuition behind this result is that under monopoly, part of the benefits coming from innovation serve only to replace the monopolist's rents earned before innovating, i.e. the monopolist has greater opportunity costs of innovating. Grossman and Helpman (1991) and Romer (1990) support Schumpeter's view that monopoly is a precondition for innovation by arguing that firms innovate because they seek profitable opportunities that arise from monopoly. On the contrary, Nickell (1996) and Boone and Dijk (1998) support the existence of a positive relationship between competition and innovation.

Other authors have elaborated on the relationship between competition and innovation, introducing additional factors like the value of the innovation and the level of fixed and variable costs. Kamien and Schwartz ((1975), (1976)) show that for inventions of small value, the absence of rivalry leads to the most rapid development, while a positive level of rivalry will achieve this for more valuable innovations. Loury (1979) finds that, under certain conditions, the incentives to invest in R&D of individual firms decrease as competition increases. The work developed by Lee and Wilde (1980) reaches rather different conclusions from Loury (1979). The authors conclude that an increase in rivalry increases the equilibrium individual R&D effort. In an attempt to reconcile this conclusion with Loury's earlier work, the authors show that if fixed costs in the R&D technology are larger than the variable costs, then an increase in competition leads to a decrease in the equilibrium level of firm investment in R&D.

Other authors have made a distinction between individual and industry innovation or investment in R&D, and find a positive effect of competition on aggregate innovation and a negative effect of competition on individual innovation (Cellini and Lambertini, 2005, Blundell et al., 1999).

Between Schumpeter's followers and Arrow's defenders, a third group of authors emerged who have attempted to combine the previous arguments in order to rationalise the "inverted-U" relationship between market concentration and R&D and technological advance found by some authors in the empirical studies. Scherer (1967) observes that the speed of technological research accelerates with rivalry, provided that the number of firms competing is not excessive. Scherer is the first to suggest an inverted-U relationship between competition and innovation.

Later on, Boone ((2000), (2001)) and Aghion et al. (2005) also find a nonlinear relationship between competition and innovation. Aghion et al. (2005) confirm the inverted-U relationship between intensity of competition and R&D incentives.

There is therefore no clear consensus in the literature on whether or not competition has a positive effect on innovation. Moreover, this literature does not take into account the specific context of the network industries where governance structures and regulation play an important role and may further complicate the relationship between competition and innovation.

This paper aims at contributing to the literature by developing an extension of the traditional profit-maximising model to investigate whether investment in new technologies and processes is higher under competition or under monopoly. This extension is intended to encompass contexts, not unusual for many network industries, in which the incumbent has the form of a public enterprise or is a part of a government ministry.

The next section presents the model. For the sake of simplicity a duopoly, where the incumbent and the entrant compete in prices, is assumed. The model accommodates a variety of different objectives for the incumbent, including the maximisation of sales revenue, profit, and/or welfare, subject to the profit being non-negative. The general objective function of the incumbent has the following form: $\alpha_1 R_I + \alpha_2 \prod_I + \alpha_3 W$, where α_1, α_2 and α_3 are weights given to revenue (R_I), profit ($\prod_I$) and welfare (W), respectively, where welfare is measured in the usual fashion as the unweighted sum of producer profits and consumer surplus. The incumbent's marginal cost is a function the amount invested in innovation. We assume this marginal cost function to be convex and decreasing in innovation investment. The entrant maximises its profit with respect to its price. The Nash equilibrium of the ensuring price-innovation game is characterised.

In section 3 the model is calibrated with data for the postal sector. Despite the calibration with sector-specific data the results can, in general, be extrapolated to other network industries.

Section 4 summarises the results of both the theory development and the computational experiments conducted to illustrate the sensitivity of the monopoly and duopoly solutions to relevant parameter changes. Section 5 concludes.

2. Theoretical model

For the purpose of investigating whether the investment in new technologies and processes is higher under competition or under monopoly, in the context of network industries, two stages of competition are assumed. In the first stage, the historical operator or incumbent has a monopoly in the market and in the second stage a new operator (entrant) enters the market and competes on price with the incumbent (duopoly).

<u>Demand side</u>

Consumer preferences are assumed to be quasi-linear with respect to goods and services consumed, and money (*m*), so that:

$$U(q_I, q_E, m) = V(q_I, q_E) + m$$

where the willingness-to-pay function $V(q_I, q_E)$ is assumed to be quadratic over the quantity of goods and services consumed from the incumbent (q_I) and from the entrant (q_E):

$$V(q_I, q_E) = a_I q_I - \frac{b}{2} q_I^2 + a_E q_E - \frac{b}{2} q_E^2 - ebq_I q_E ,$$

where $a > 0$ and $b > 0$ are the parameters that determine the size of the market and the slope of the demand curve, respectively, and m represents money spent on other goods. For the sake of simplicity we assume that the slope of the demand curve (as determined by the parameter *b* in the willingness-to-pay function) is the same for both operators. It is natural to assume that a_E is smaller then a_I since the incumbent has an advantage over the entrant due to switching costs, reputation and simple customer inertia. The parameter e , which varies between zero and one, determines the degree of differentiability of the services offered by the incumbent and by the entrant. If e is close to zero then the services are highly differentiated. As e approaches one, then the services become more homogeneous, being perfect substitutes when $e = 1$.

Only a representative consumer model is considered here; introducing consumer heterogeneity would only add notation, with no additional insights. The (representative) consumer maximises utility with respect to q_I and q_E subject to the following budget constraint, which clearly holds with equality at optimum:

$$p_I q_I + p_E q_E + m \leq M ,$$

where p_I and p_E are the prices of the service supplied by the incumbent and the entrant, respectively. M is the initial wealth endowment of the consumer.

Solving the consumer's problem, the following demand functions are obtained:

$$q_I(p_I, p_E) = \frac{1}{b(1-e^2)}\left(a_I - ea_E - p_I + ep_E\right) \qquad (1)$$

$$q_E(p_I, p_E) = \frac{1}{b(1-e^2)}\left(a_E - ea_I - p_E + ep_I\right) \qquad (2)$$

A viable outcome in terms of non-negative quantities exists under the following condition:

$$e < \frac{a_E - p_E}{a_I - p_I} \quad \text{or} \quad e < \frac{q_E + eq_I}{q_I + eq_E}$$

Adapting the utility function and the budget constraint of the consumer to a monopoly situation, i.e. setting q_E equal to zero, the demand function in monopoly becomes:

$$q_I(p_I) = \frac{1}{b}\left(a_I - p_I\right)$$

<u>Supply side</u>

On the supply side, in both stages of competition the marginal cost of the incumbent (c_I) is assumed to depend on the investment in new technologies (k). The following marginal cost function, which is strictly convex and decreasing, is assumed:

$$c_I(k) = c_{I0} e^{-\gamma k}$$

where c_{I0} is the initial marginal cost of the incumbent and γ establishes the relationship between the investment in innovation or new technologies and the reduction in the marginal cost. The higher the value of γ the higher the investment needed to attain a certain percentage of cost reduction. This equation accommodates the assumption that if initial cost is high (perhaps due to internal inefficiency), then a smaller level of investment is needed to obtain a certain reduction in the marginal cost as compared to a situation where the initial cost is low.

The objective is to analyse how the incentives to innovate change under monopoly as compared to a competitive environment. Therefore, only the case where the incumbent has the choice to invest in innovation in order to reduce its marginal cost is considered.

In duopoly, the equilibrium is given by the intersection of the reaction functions of the incumbent and entrant. The entrant maximizes his profit

$$\Pi_E(p_I, p_E) = (p_E - c_E) q_E(p_I, p_E) - F_E$$

with respect to p_E, where c_E is the marginal cost of the entrant and F_E represents his fixed costs. The reaction function of the entrant has the following form:

$$p_E(p_I) = \frac{a_E - e a_I + e p_I + c_E}{2} \qquad (3)$$

Many incumbents are public enterprises or have other forms of ownership and governance than profit-maximizing private firms. To capture some of the richness inherent in these alternative ownership and governance structures, as well as the potential of regulation on these firms, various objectives are posited for the incumbent in the analysis that follows. In particular, the incumbent modelled here can maximize sales revenue, profit, and/or welfare, subject to a breakeven constraint on profit. For example, the sales revenue maximization objective might be relevant for a traditional public bureaucracy whose management is concerned primarily with maximizing the size of the organization (e.g., Niskanen, 1971), while a welfare maximization objective might be relevant for a public enterprise that is explicitly regulated to achieve efficiency (in pricing). The point of this rather general analysis is to consider the impact on investment strategies of alternative objectives that might credibly be advanced as representing the objectives of incumbents[1].

The objective function of the historical operator is assumed to have the following form: $\alpha_1 R_I + \alpha_2 \prod_I + \alpha_3 W$, where α_1, α_2 and α_3 are non-negative weights, which without loss of generality are assumed to sum one, attributed to the size of the firm as measured by revenue (R_I), profit ($\prod_I$) and welfare (W), respectively. The revenue, profit and welfare of the incumbent are, respectively, given by:

$$R_I(p_I, p_E) = p_I q_I(p_I, p_E)$$

$$\prod_I(p_I, p_E, k) = \left(p_I - c_I(k)\right) q_I(p_I, p_E) - k - F_I$$

$$W(p_I, p_E, k) = V\left(q_I(p_I, p_E), q_E(p_I, p_E)\right) - c_I(k) q_I(p_I, p_E) - c_E q_E(p_I, p_E) - k - F_I - F_E$$

Therefore, the Lagrangian for the breakeven-constrained incumbent can be written as:

$$\begin{aligned} L\left(p_I, k, \lambda\right) = & \left(\alpha_1 + \alpha_2 + \lambda\right) p_I q_I(p_I, p_E) - \left(\alpha_2 + \alpha_3 + \lambda\right) c_I(k) q_I(p_I, p_E) - \left(\alpha_2 + \alpha_3 + \lambda\right) k \\ & - \left(\alpha_2 + \lambda\right) F_I + \alpha_3 \left[V\left(q_I(p_I, p_E), q_E(p_I, p_E)\right) - c_E q_E(p_I, p_E) - F_I - F_E \right] \end{aligned}$$

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[1] See Crew and Kleindorfer (forthcoming 2008) for a related discussion of such a weighted objective function in the context of price-cap regulation.

where $\lambda \geq 0$ is the Lagrange multiplier associated with the breakeven constraint, which measures the sensitivity of the optimal solution of the objective function to changes in required minimum profit level of the incumbent.

Note that $L\left(p_I, k, \lambda\right)$ is strictly concave in k for any fixed prices, and strictly concave in p_I for any fixed k (see Proof 1 and 2 in the Technical Appendix). Nevertheless, L is not jointly strictly concave i.e. the Hessian matrix (H) is not negative definite in the domain of L (see Proof 3 in the Technical Appendix).

However, given that $L\left(p_I, k, \lambda\right)$ is strictly concave in p_I for any fixed $k \geq 0$, it is possible to derive the optimal p_I from the necessary and sufficient first-order conditions. Assuming an interior solution for price, the optimal solution is characterized by $\frac{\partial L}{\partial p_I} = 0$, which yields:

$$p_I(k, p_E) = \frac{\left(\alpha_1 + \alpha_2 + \lambda\right)\left(a_I - ea_E\right) + \left(1 + \lambda\right)ep_E + \left(\alpha_2 + \alpha_3 + \lambda\right)c_I(k) - \alpha_3 ec_E}{\left(1 + \alpha_1 + \alpha_2 + 2\lambda\right)} \tag{4}$$

Putting the reaction function of the incumbent and of the entrant together (equations (3) and (4)) gives:

$$p_I^*(k) = \frac{\left(1 - e^2\right)}{\xi}\left[2\left(\alpha_1 + \alpha_2 + \lambda\right)\left(a_I - ea_E\right) + 2\left(\alpha_2 + \alpha_3 + \lambda\right)c_I(k) - 2\alpha_3 ec_E + \left(1 + \lambda\right)e\left(a_E - ea_I + c_E\right)\right] \tag{5}$$

$$p_E^*(k) = \frac{\left(1 - e^2\right)}{\xi}\left[e\left(\alpha_1 + \alpha_2 + \lambda\right)\left(a_I - ea_E\right) + e\left(\alpha_2 + \alpha_3 + \lambda\right)c_I(k) - \alpha_3 e^2 c_E + \left(1 + \alpha_1 + \alpha_2 + 2\lambda\right)\left(a_E - ea_I + c_E\right)\right] \tag{6}$$

where $\xi = \left(1 - e^2\right)\left[2\left(1 + \alpha_1 + \alpha_2 + 2\lambda\right) - \left(1 + \lambda\right)e^2\right] > 0$.

Since $p_I(k)$ is unique and feasible for every $k \geq 0$, the problem of $\max\left\{L\left(p_I, k, \lambda\right) \middle| p_I \geq 0, k \geq 0, \lambda \geq 0\right\}$ can be restated as $\max\left\{L\left(p_I^*(k), k, \lambda\right) \middle| k \geq 0, \lambda \geq 0\right\}$. The solution is recovered as $\left(p_I^*(k^*), k^*, \lambda^*\right)$ where k^* solves $\max\left\{L\left(p_I^*(k), k, \lambda\right) \middle| k \geq 0, \lambda \geq 0\right\}$.

The first order conditions for $\max\left\{L\left(p_I^*(k),k,\lambda\right)\middle| k\geq 0,\lambda\geq 0\right\}$ are:

$$\frac{\partial L\left(p_I^*(k),k,\lambda\right)}{\partial k}\leq 0;\quad \frac{\partial L\left(p_I^*(k),k,\lambda\right)}{\partial \lambda}\geq 0;\quad k\frac{\partial L\left(p_I^*(k),k,\lambda\right)}{\partial k}=0 \tag{8}$$

An interior solution $k>0$ obtains in (8) only if $\partial L\left(p_I^*(k),k,\lambda\right)/\partial k=0$, which implies (see Development 1 in the Technical Appendix):

$$c_I(k)=\frac{h_6\pm\sqrt{h_6^2-4\left(\alpha_2+\alpha_3+\lambda\right)h_7}}{2h_7} \tag{11}$$

where

$$h_6=\frac{\gamma}{b\left(1-e^2\right)}\left[\left(h_2-2h_1h_3\right)\left(a_I-ea_E\right)+e\alpha_3h_3c_E+\left(h_2-2h_1h_3-\alpha_3h_3\right)eh_5\right] \tag{10}$$

$$h_7=\frac{2\gamma}{b\left(1-e^2\right)}h_3\left(h_2-2h_1h_3\right)\left(2-e^2\right)-\frac{\alpha_3\gamma}{b(1-e^2)^2}h_3^2\left[e^2+\left(2-e^2\right)^2-2e^2\left(2-e^2\right)\right]$$

$$h_1=\alpha_1+\alpha_2+\lambda$$

$$h_2=\alpha_2+\alpha_3+\lambda$$

$$h_3=\frac{\alpha_2+\alpha_3+\lambda}{2\left(1+\alpha_1+\alpha_2+2\lambda\right)-\left(1+\lambda\right)e^2}$$

$$h_4=\frac{\left(1-e^2\right)}{\xi}\left[2\left(\alpha_1+\alpha_2+\lambda\right)\left(a_I-ea_E\right)-2\alpha_3ec_E+\left(1+\lambda\right)e\left(a_E-ea_I+c_E\right)\right]$$

$$h_5=\frac{\left(1-e^2\right)}{\xi}\left[e\left(\alpha_1+\alpha_2+\lambda\right)\left(a_I-ea_E\right)-\alpha_3e^2c_E+\left(1+\alpha_1+\alpha_2+2\lambda\right)\left(a_E-ea_I+c_E\right)\right]$$

The sign of h_6 and h_7 are examined here since they are determinant for the reasoning and computations that follow. It can be proved that h_6 is always positive. The first two terms of equation (10) are always positive, i.e. $\left(h_2-2h_1h_3\right)\left(a_I-ea_E\right)+e\alpha_3h_3c_E>0$. $\left(h_2-2h_1h_3-\alpha_3h_3\right)$

and h_5 are always positive as well. Therefore, h_6 is always positive. It is also easy to prove that h_7 is always positive.

The existence and uniqueness of a solution can be proved (see Proposition 1 and its Proof in the Technical Appendix). The solution to the maximisation problem, for duopoly, is given by $c_I(k) = \dfrac{h_6 - \sqrt{h_6^2 - 4(\alpha_2 + \alpha_3 + \lambda)h_7}}{2h_7}$.

Adapting the previous expressions to the monopoly setup and following exactly the same reasoning, it is easy to prove the existence and uniqueness of equilibrium under monopoly. The optimal price and k under monopoly are given, respectively, by:

$$p_I^*(k) = \frac{(\alpha_1 + \alpha_2 + \lambda)a_I + (\alpha_2 + \alpha_3 + \lambda)c_I^*(k)}{(1 + \alpha_1 + \alpha_2 + 2\lambda)} \tag{13}$$

$$c_I^*(k) = \frac{h_8 - \sqrt{h_8^2 - 4(\alpha_2 + \alpha_3 + \lambda)h_9}}{2h_9} \tag{14}$$

where

$$h_8 = \frac{\gamma(\alpha_2 + \alpha_3 + \lambda)(1 + \lambda)}{b(1 + \alpha_1 + \alpha_2 + 2\lambda)}a_I \quad \text{and} \quad h_9 = \frac{\gamma(\alpha_2 + \alpha_3 + \lambda)^2(2 + 2\lambda - \alpha_3)}{b(1 + \alpha_1 + \alpha_2 + 2\lambda)^2}$$

Note that the optimal k corresponds to $k^* = \ln\left(c_{I0} / c_I^*(k)\right) / \gamma$. The optimal prices, under duopoly, can be obtained by substituting $c_I^*(k) = \left(h_6 - \sqrt{h_6^2 - 4(\alpha_2 + \alpha_3 + \lambda)h_7}\right) / 2h_7$ into equations (5) and (6). Under monopoly, the optimal price is obtained by replacing (14) into (13).

The analysis of the results arising from the model is performed numerically due to the complexity of the model.

In the next section the model is calibrated with data from the postal sector and in the section thereafter some computational results are presented.

3. Model calibration – Postal sector

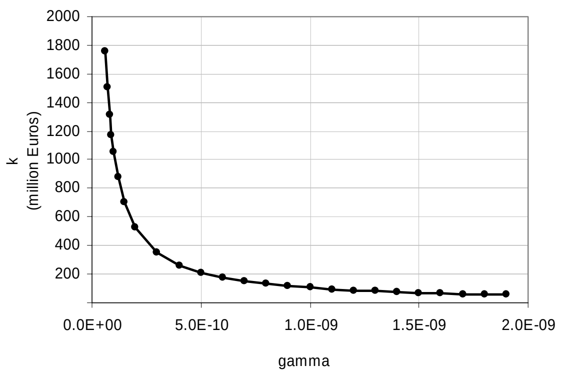
In this section, the model presented before is calibrated with data from the postal sector from France and United Kingdom.

Since there is no data available in order to estimate the parameter γ , ten different scenarios are studied. In all the scenarios the estimate of γ is based on the cost of achieving a reduction of the marginal cost of 10%. The resulting k for the values of γ considered are the followings (Table 1 and Figure 1):

Table 1 Scenarios for parameter γ

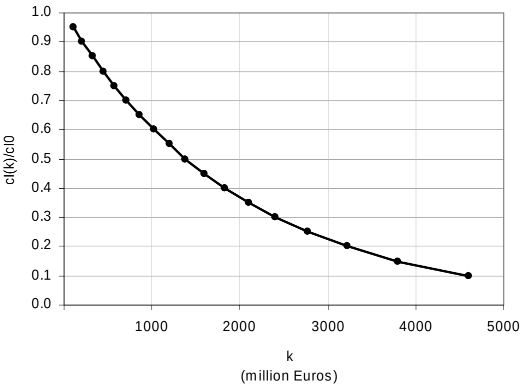
Scenario	$c_I(k) / c_{I0}$	gamma	k (million Euros)
1	0.9	3.0E-10	351
2	0.9	4.0E-10	263
3	0.9	5.0E-10	211
4	0.9	6.0E-10	176
5	0.9	7.0E-10	151
6	0.9	8.0E-10	132
7	0.9	9.0E-10	117
8	0.9	1.0E-9	105
9	0.9	1.1E-9	96
10	0.9	1.2E-9	88

Figure 1 Relationship between γ and k for a reduction of 10% in the marginal cost



The figure below depicts the relationship between the investment in new processes and technologies and the cost reduction obtained for $\gamma = 5E-10$, which is the value of γ chosen to calibrate the model.

Figure 2 Relationship between the investment in new processes and technologies and the cost reduction obtained (for $\gamma = 5E-10$)



In order to estimate the demand functions, parameters a and b are calibrated with data from the French and British postal market from 1997, when there was no competition yet. To calibrate b it is necessary to know the quantity, the average price and the price elasticity of demand. Once the value of b is known it is possible to compute a_I. Then, using equations (1) and (2), knowing a_I, e, the average price and the market share of the incumbent when the entrant charges the same price as the incumbent ($ms = q_I / (q_I + q_E)$) it is possible to find a_E, through the following equation (Dietl et al., 2005):

$$a_E = \frac{1}{1-e+\frac{e}{ms}}\left(a_I\left(e-1+\frac{e}{ms}\right)+p(1-e)\left(2-\frac{1}{ms}\right)\right)$$

For calibration purposes ms is assumed to be 70%.

The total volume of mail, including non-addressed mail, of the French (British) postal operator in 1997 was 25'770 (17'300) million objects. The average price in France (United Kingdom) in 1997 was approximately 0.55 euros (0.5 euros) (CTcon, 1998, Deutsche Post, 2007). Price elasticity of demand is assumed to be -0.6. The rationale for this assumption is as follows.

There is considerable divergence in the literature concerning price elasticity of demand in the postal sector. See Robinson (2007) for a review for price elasticity models for postal products. According to Robinson's extensive review of the literature, the price elasticity measures for postal products in various studies and countries has been found to be between -0.2 and -0.8. The value -0.6 was chosen, which is slightly above the middle point of the interval in order to account for the fact that the innovations considered here are likely to be focused on business products that have a higher elasticity (in absolute value).

Regarding the parameter e, since the services provided by the incumbent and the entrant are similar but not perfect substitutes it is assumed that $e = 0.7$. A sensitivity analysis for this parameter will be performed afterwards.

Regarding the supply side, the total operational costs of the French group were 9'848 million euros. Based on German data from 1998, the costs associated with the letter segment are assumed to be 75% of the total costs of the group. The operational costs linked to the letter segment of the British postal operator in 1997 were 6'603 million euros.

It is assumed that approximately 40% of the operational costs of the incumbent are fixed[2]. Therefore, the initial marginal cost of the French (British) incumbent is 0.17 euros (0.23 euros). The structure of competitors is more flexible than the structure of the historical operator. Hence, the percentage of fixed costs of the entrant is smaller than that of the incumbent. Competitors

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[2] This is a typical figure (in the medium run applicable to pricing decisions) for postal incumbents. See the discussion in Cohen et al. (2006) and d'Alcantara and Amerlynck (2006).

have a smaller infrastructure than the incumbent because the former has almost no private customers, has few postal outlets, sorting centres and delivery offices. Table 2 summarises the assumed cost structure of the incumbent and of the entrant.

Table 2 Cost structure of the incumbent and entrant

		Collection	Processing	Delivery	Overhead	Total
Percentage of total costs		10	30	55	5	100%
Fraction of variable costs	I	50	80	50	10	57%
	E	75	85	60	50	69%

Source: Adapted from Dietl et al. (2005)

The major differences in terms of costs between the entrant and the incumbent are that the entrant: 1) has a smaller infrastructure, which allows smaller overhead costs; 2) focuses on business customers, which allows the extensive use of computerized sorting in the printing stage; 3) pays lower wages than the incumbent and these represent the major share of the total costs (80%); and 4) is likely to have more recent technology. The wage premium is estimated to be approximately 15%.

In order to account for these differences in terms of costs the entrant is assumed to have a cost saving of 30% in collection and processing. The cost saving in delivery is smaller, approximately 5%, because most business mailings are business-to-customer originating a huge number of delivery points. Also, innovation to reduce delivery costs is very limited since delivery depends basically on manpower (Dietl et al., 2005). Finally, it is assumed that the entrant has a cost saving of 33% in overhead costs (Dietl and Waller, 2002).

The results of the model, using the calibration just described, are presented and discussed in the next section.

4. Results

This section presents the results of a number of computational experiments for the model. Although the model was calibrated with data from the postal sector the results can, in general, be extrapolated to other network industries. Firstly, the influence of competition on the incentives to innovate is analysed. Then, the role of the different objectives of the incumbent on the incentives to innovate is investigated. Finally, the robustness of the results is tested for changes in the degree of service differentiation (e), the elasticity of innovation cost (γ), and the price elasticity of demand.

4.1 The effect of competition on innovation

Table 3 shows the main results of the model for the calibration values mentioned in the previous section, namely: incumbent's market share equal to 70%, degree of substitution (e) equal to 0.7, elasticity of innovation cost (γ) equal to 5E-10, and price elasticity of demand equal to -0.6.

The first column of Table 3 ($\alpha_1 = 1, \alpha_2 = 0, \alpha_3 = 0, \lambda \geq 0$) corresponds to maximizing revenue subject to the breakeven constraint. The second column ($\alpha_1 = 0, \alpha_2 = 1, \alpha_3 = 0, \lambda = 0$) refers to the profit maximization case. The third column ($\alpha_1 = 0, \alpha_2 = 0, \alpha_3 = 1, \lambda \geq 0$) regards the profit-constrained welfare maximizing case or Ramsey case. Finally, the fourth column ($\alpha_1 = 0, \alpha_2 = 0, \alpha_3 = 1, \lambda = 0$) corresponds to the unconstrained welfare maximizing case.

Table 3 Results for France (left) and UK (right) with e=0.7, incumbent's market share=70%, γ *=5E-10 , price elasticity of demand=-0.6*

		FR				UK			
Alpha1		1	0	0	0	1	0	0	0
Alpha2		0	1	0	0	0	1	0	0
alpha3		0	0	1	1	0	0	1	1
lambda		≥ 0	$= 0$	≥ 0	$= 0$	≥ 0	$= 0$	≥ 0	$= 0$
Monopoly									
average price		0.73	0.79	0.2	0.05	0.67	0.75	0.30	0.08
quantity		20'617	19'148	35'547	39'820	13'841	12'128	21'449	26'088
welfare		16'579	15'611	22'473	22'786	8'278	7'330	11'079	11'563
consumer surplus		7'560	6'521	22'473	28'202	4'614	3'543	11'079	16'393
profit		9'019	9'090	0	-5'416	3'665	3'788	0	-4'830
k		1'145	997	2'234	2'462	921	657	1'797	2'189
Duopoly									
average price	I	0.41	0.48	0.23	0.11	0.39	0.50	0.34	0.09
	E	0.42	0.45	0.36	0.32	0.44	0.48	0.43	0.34
quantity	I	22'633	19'857	30'280	35'160	16'020	12'852	17'524	25'358
	E	10'081	11'368	6'536	4'274	4'986	6'454	4'289	657
welfare		20'687	19'967	22'032	22'331	9'850	9'087	10'165	10'911
consumer surplus		16'600	14'932	21'995	26'053	9'473	7'779	10'374	16'060
profit	I	2'983	3'429	0	-3'315	426	945	0	-4'499
	E	1'105	1'606	36	-407	-49	363	-208	-650
k		1'030	769	1'895	2'269	912	472	1'258	2'144
k duopoly - k monopoly		-114	-228	-339	-193	-8	-184	-539	-45

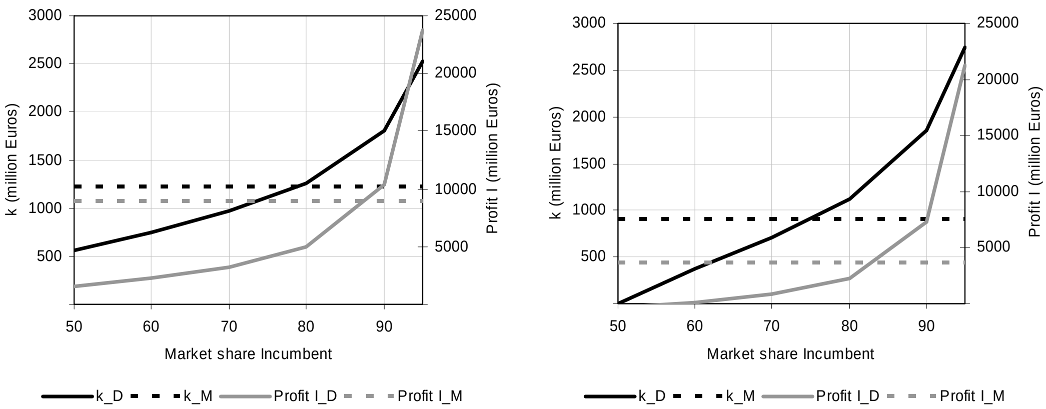
Units: prices are in euros, quantities are in millions of items and the remaining variables are in millions of euros.

Firstly, it is interesting to note that the investment in process innovations is always larger under monopoly than under duopoly, independent of the weight on welfare. However, when the objective is revenue or profit maximisation, prices are higher under monopoly than under duopoly while the quantity supplied and the social welfare are smaller under monopoly than under duopoly. The consumer surplus more than doubles under duopoly.

The market share of the incumbent has positive influences on innovation (and of course its profit), under duopoly (Figure 3), proving that the incentives to innovate strongly depend on the quantity supplied by the firm. With a larger mail volume, the gains from reducing the unit delivery cost by a given amount are larger, has found before by Gautier and Bloch (forthcoming 2008).

Figure 3 also reveals one of the most interesting conclusions of the model. In fact, the investment in innovation is larger under monopoly than under duopoly only if the market share of the incumbent does not exceed a certain threshold. In the French case this threshold corresponds to eighty percent and in the British case it corresponds to seventy five percent. If the incumbent preserves a certain market share, which value depends on the particular characteristics of the market, duopoly gives rise to more investment in innovation than monopoly. For concluding which market structure creates more incentives to innovate, the market share of the incumbent is determinant.

Figure 3 Effect of the incumbent's market share on k (α_1 *=0,* α_2 *=0.8,* α_3 *=0.2, e=0.7,* γ *=5E-10, elasticity=-0.6) – France (left) and UK (right)*



Competition may force firms to minimize cost for a given output but each firm produces less output, fails to achieve minimum efficient scale, and suffers excess unit cost (Kwoka, 2006). When the market share of the incumbent is sufficiently reduced, the net effect of competition on cost efficiency and, therefore on the incentives to process innovate, is small. This is because competition does not reduce operating costs sufficiently to offset the higher fixed costs of operation. When the incumbent preserves a certain market share, competition has a stronger positive effect on cost efficiency.

Hence, it is interesting to examine the results of the model for the different objectives, namely revenue maximisation, profit maximisation, welfare constrained maximisation, when the market share of the incumbent is, for example, 80% (Table 4). Welfare unconstrained maximisation is neglected here since it is not an interesting case.

Table 4 Results for France (left) and UK (right) with e=0.7, incumbent's market share=80%, γ =5E-10 , price elasticity of demand=-0.6

		FR			UK		
alpha1		1	0	0	1	0	0
alpha2		0	1	0	0	1	0
alpha3		0	0	1	0	0	1
lambda		≥ 0	= 0	≥ 0	≥ 0	= 0	≥ 0
Monopoly							
average price		0.73	0.79	0.20	0.67	0.75	0.30
quantity		20'61	19'148	35'547	13'841	12'128	21'449
welfare		15'57	15'611	22'473	8'278	7'330	11'080
consumer surplus		7'560	6'521	22'473	4'614	3'543	11'080
profit		9'019	9'090	0	3'665	3'788	0
k		1'145	997'	2'234	921	657'	1'797
Duopoly							
average price	I	0.46	0.51	0.20	0.45	0.53	0.28
	E	0.50	0.52	0.41	0.55	0.58	0.49
quantity	I	25'12	22'695	35'697	18'283	15'687	23'384
	E	4'707	5'833	0	106	1'309	0
welfare		19'16	18'294	21'754	9'524	8'576	10'977
consumer surplus		14'56	13'062	22'493	8'116	6'661	11'637
profit	I	4'932	5'353	0	2'068	2'534	0
	E	-337	-121	-739	-660	-618	-660
k		1'240	1'036	2'126	1'177	871	1'780
k duopoly - k monopoly		95	39	-108	256	214	-17

Units: prices are in euros, quantities are in millions of items and the remaining variables are in millions of euros.

Now, the difference between the investment in process innovation under monopoly and under duopoly depends on the weight of welfare. When the objective of the firm is revenue or profit maximisation, investment in innovation under duopoly is larger than under monopoly.

In short, the market share of the incumbent determines whether competition has a positive effect or not on innovation. When the incumbent preserves a large market share, duopoly is revealed to be more favourable to innovation than monopoly. On the contrary, if the market share of the incumbent is sufficiently reduced, then the incumbent invests less in innovation under duopoly than under competition. The threshold value that determines if innovation under monopoly is larger or not than under duopoly differs according to the sector and market characteristics. The role of revenue, profit, and welfare maximisation is analysed in more detail below.

4.2 The role of revenue, profit and welfare maximisation

The largest value of investment in innovation occurs when the objective is exclusively to maximize welfare (Table 3 and Table 4). The lowest level of investment is registered when the incumbent is exclusively concerned with profit maximization. Moreover, the incentive to innovate when the single objective is social welfare maximization disregarding the profit of the incumbent is larger than the incentive to innovate when incumbent's breakeven constraint is considered. For UK the investment in innovation almost triplicates from profit maximization to the profit-constrained welfare maximizing case.

Contrarily to what could be expected, a larger level of investment is registered under the revenue maximization case than under the profit maximization case (Table 3 and Table 4). This happens because when maximising revenue the incumbent does not take into account the cost of innovation, which is the fact that prevents the incumbent from investing more when the objective is profit maximisation. Also, more innovation results in smaller marginal costs, therefore the incumbent decreases the price in order to increase the quantity supplied. The price decreases up to the point where the product of the quantity supplied and the price is maximized.

As expected prices and profits are much lower under the welfare maximizing cases than under revenue or profit maximization, whereas quantities and welfare are larger. Welfare maximisation is the case that provides larger incentives to innovate associated with larger welfare, namely consumer surplus.

Figure 4 depicts the evolution of investment in innovation according to changes in the alphas (holding other parameters constants) when the incumbent has seventy percent of market share. α_1 is fixed along each line in the graph but it changes across lines. α_3 is equal to α_1 and α_2 subtracted from one ($\alpha_3 = 1 - \alpha_1 - \alpha_2$). The figure shows that as α_2 increases, holding α_1 constant, the incentives to innovate decrease. Also, an increase in α_1, for α_2 constant, originates a decrease in k. Moreover, the smaller the sum of α_1 and α_2 relatively to α_3, the smaller investment in innovation under both monopoly and duopoly. In other words, an increase in α_3, holding α_1 constant, stimulates innovation until a certain point. This means that regulation that has the effect of motivating the incumbent to place greater weight on welfare maximisation favours innovation. As noted before, when the market share of the incumbent is seventy percent, there is more stimulus to innovate under monopoly than under duopoly.

Figure 4 Effect of α_1, α_2 and α_3 on k (e=0.7, market share I=70%, γ =5E-10, elasticity=-0.6) – France (left) and UK (right)

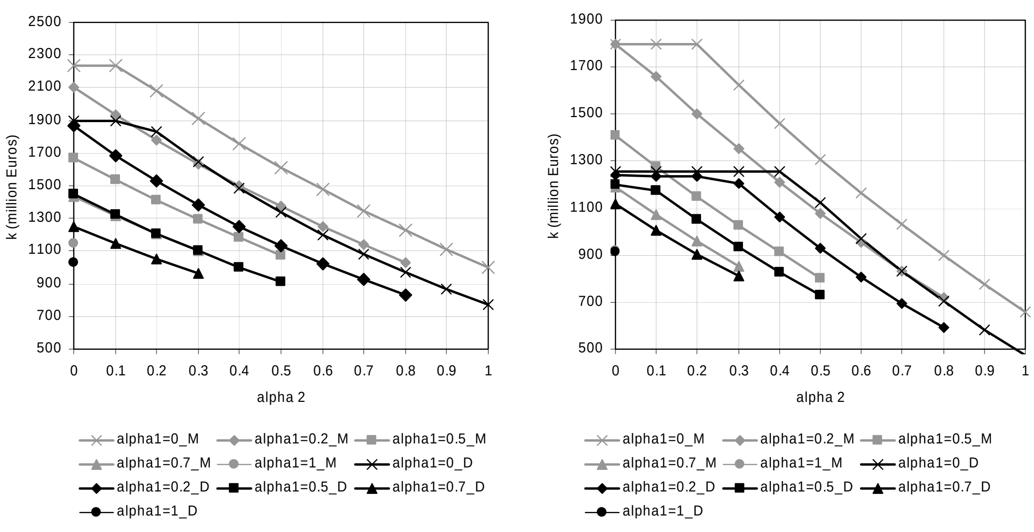
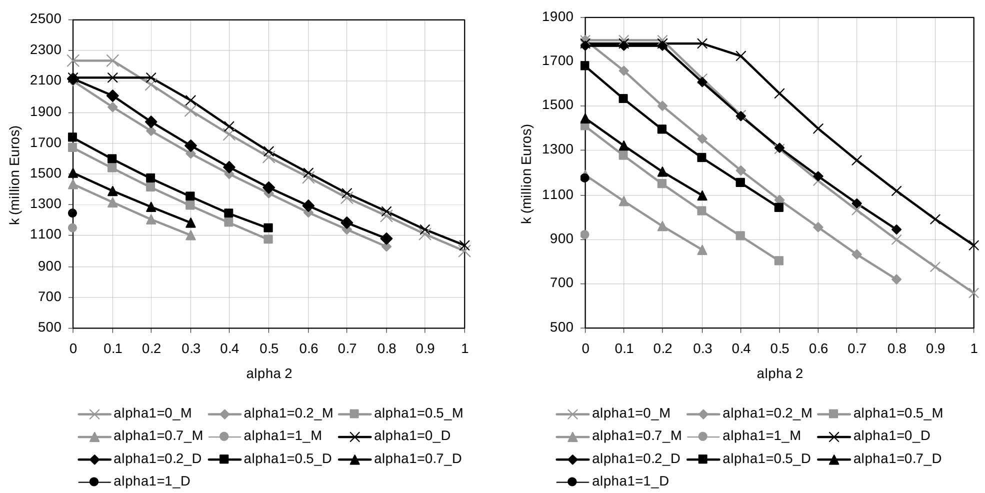


Figure 5 also analyses how changes in the alphas impact innovation, but for an incumbent's market share equal to eighty percent. The figure shows that if the weight given to revenue (α_1) is zero and the weight given to profit (α_2) does not exceed the two percent then competition does not favour innovation. In all the other cases (which are the great majority), competition has a positive effect on innovation.

Figure 5 Effect of α_1, α_2 and α_3 on k (e=0.7, market share I=80%, γ =5E-10, elasticity=-0.6) – France (left) and UK (right)

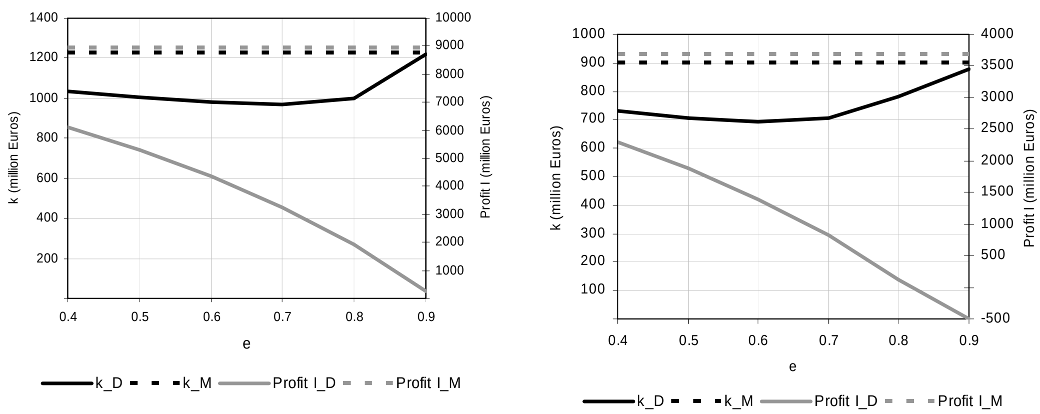


As before, the incentives to innovate are still decreasing as α_2 (α_1) increases, holding α_1 (α_2) constant. This confirms that an increase in the weight associated with welfare (α_3) stimulates the investment in innovation. This conclusion is independent of the market share of the incumbent.

4.3 Sensitivity analysis for differentiability (e), elasticity of innovation cost (γ) and demand elasticity

The impact of the degree of differentiability of the services offered by the incumbent and by the entrant, i.e. parameter e, is analysed here (Figure 6). It is important to recall that the degree of differentiation decreases as e increases.

Figure 6 Effect of parameter e on k (α_1 =0, α_2 =0.8, α_3 =0.2, market share I=70%, γ =5E-10, elasticity=-0.6) – France (left) and UK (right)

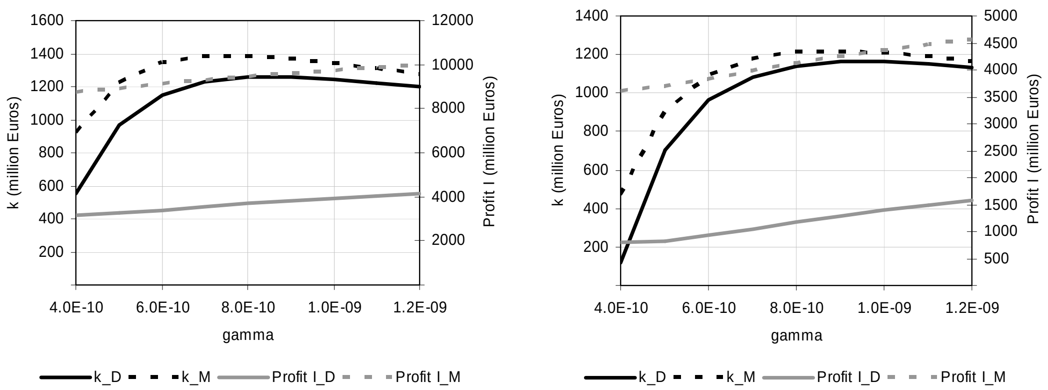


As expected, the incumbent's profit increases as the degree of differentiation increases. Interestingly, k first decreases in e and after a point it increases in e. This change is registered when e equals 0.7 (approximately) in the French case, and when e equals 0.6 (approximately) in the British case. These points correspond to the points where the quantity supplied by the entrant attains its maximum (the quantity supplied by the entrant has the shape of a parabola with the concavity turned down). Investment in innovation under duopoly gets very close to that under monopoly when the service of the two operators is very homogeneous. In fact, the quantity supplied by the incumbent is always increasing in e. A larger market provides the conditions for the incumbent to invest more in innovation.

It can be concluded that the results of the model are not altered by changes in the degree of differentiability of the services.

It is also essential to analyse the robustness of the main results to changes in the elasticity of innovation cost (γ). Clearly, profits increase has γ increases (Figure 7). A smaller γ implies that for the same investment, a larger cost reduction is attained.

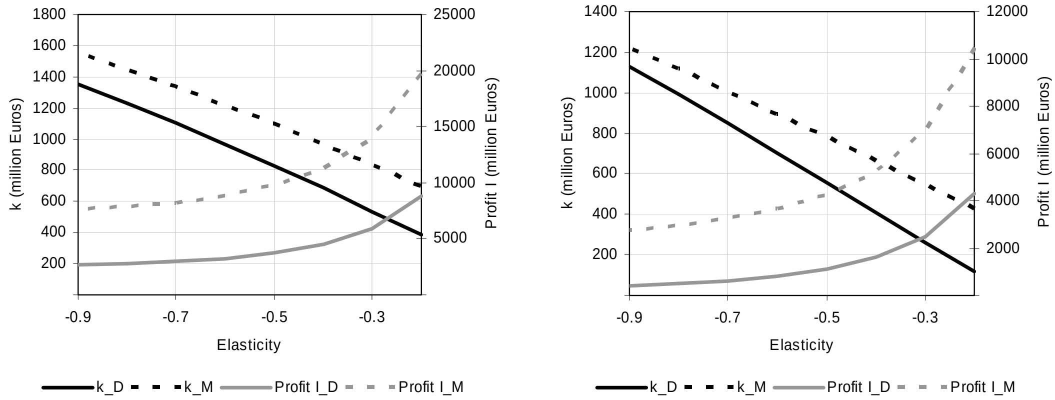
Figure 7 Sensitivity analysis γ (α_1=0, α_2=0.8, α_3=0.2, e=0.7, market share I=70%, elasticity=-0.6) – France (left) and UK (right)



The investment in innovation is not monotonic in the elasticity of innovation cost. In the French case, k is increasing in γ until $\gamma = 8$E-10 whereas in the British case this threshold corresponds to $\gamma = 9$E-10. After those points k decreases as γ increases. Nevertheless, innovation under monopoly is always larger than under duopoly.

Finally, Figure 8 shows the sensitivity of the model's results to changes in price elasticity of demand.

Figure 8 Sensitivity analysis for price elasticity of demand (α_1 =0, α_2 =0.8, α_3 =0.2, e=0.7, market share I=70%, γ =5E-10) – France (left) and UK (right)



It is possible to observe that the larger the price elasticity of demand is, the smaller k is, under both monopoly and duopoly. For all the elasticity values analysed, the relationship between investment in innovation under monopoly and duopoly does not change. This confirms the robustness of the model's results to changes in the price elasticity of demand.

In conclusion, the main results of the model are robust to changes in the degree of service differentiation, gamma, and the price elasticity of demand.

5. Concluding remarks

The model presented in this paper investigates the incumbent's optimal investment in innovation under monopoly and duopoly.

The results show that the incumbent has greater incentives to innovate under monopoly than under duopoly, when its market share duopoly decreases significantly. The size of the decrease in market share required to trigger lower innovation in the face of competition depends in a complicated manner on all the model parameters. Of course, even when levels of investment in innovation are higher under monopoly, there is a price to pay in form of the usual deadweight losses of monopoly. How large these are will depend on the “commercial orientation”, i.e., objective function, of the incumbent. In particular, when all the weight is given either to revenue or profit maximisation, prices are higher under monopoly than under duopoly while the quantity supplied and social welfare are smaller.

If the market share of the incumbent is above a certain level, then it is duopoly that gives rise to more incentives to innovate (except when welfare maximisation is the sole objective or has a sufficiently high weight in the incumbent’s objective function).

It is also proven that the incentives to innovate decrease as the weight given to revenue and/or to profit increase. In other words, the more regulation can move the incumbent to act as a welfare maximiser, the larger the investment in innovation is. This conclusion (that regulatory intervention which motivates the incumbent to act like a welfare-maximising firm favours innovation) is independent of the market share of the incumbent.

Although the results were derived from the model calibration for a specific sector, the postal sector, the model development is general, and the conclusions above apply to other network industries. Nonetheless a number of future research topics are suggested by this. These have to do with both detailed empirical work, in light of these results, as well as extending the theory to encompass oligopolistic settings, a more general investment game by all competitors (not just the incumbent as analyzed here), more general cost functions, multi-product firms, dynamics, and a number of other topics that are only barely touched by this analysis.

Technical Appendix

Proof 1: $L\left(p_I, k, \lambda\right)$ is strictly concave in k for any fixed prices

$$\frac{\partial L}{\partial k} = \left(\alpha_2 + \alpha_3 + \lambda\right)\left[\gamma c_I(k) q_I(p_I, p_E) - 1\right]$$

$$\frac{\partial^2 L}{\partial k^2} = -\left(\alpha_2 + \alpha_3 + \lambda\right)\gamma^2 c_I(k) q_I(p_I, p_E) < 0$$

Proof 2: $L\left(p_I, k, \lambda\right)$ is strictly concave in p_I for any fixed k

$$\frac{\partial L}{\partial p_I} = \frac{1}{b(1-e^2)}\left[\left(\alpha_1 + \alpha_2 + \lambda\right)\left(a_I - e a_E - 2p_I + e p_E\right) + \left(\alpha_2 + \alpha_3 + \lambda\right) c_I(k) + \alpha_3\left(-p_I + \left(p_E - c_E\right)e\right)\right]$$

$$\frac{\partial^2 L}{\partial p_I^2} = \frac{-2\left(\alpha_1 + \alpha_2 + \lambda\right) - \alpha_3}{b(1-e^2)} < 0$$

Proof 3: L is not jointly strictly concave i.e. the Hessian matrix (H) is not negative definite in the domain of L.

For H to be negative definite, its first order leading principal minor has to be negative and the second order leading principal minor has to be positive. It is easy to verify that the first order leading principal minor is negative:

$$H = \begin{pmatrix} \dfrac{\partial^2 L}{\partial k^2} & \dfrac{\partial^2 L}{\partial k \partial p_I} \\ \dfrac{\partial^2 L}{\partial p_I \partial k} & \dfrac{\partial^2 L}{\partial p_I^2} \end{pmatrix}$$

$$= \begin{pmatrix} -\left(\alpha_2 + \alpha_3 + \lambda\right)\gamma^2 c_I(k) q_I(p_I, p_E) & -\dfrac{\left(\alpha_2 + \alpha_3 + \lambda\right)\gamma c_I(k)}{b(1-e^2)} \\ -\dfrac{\left(\alpha_2 + \alpha_3 + \lambda\right)\gamma c_I(k)}{b(1-e^2)} & -\dfrac{2\left(\alpha_1 + \alpha_2 + \lambda\right) + \alpha_3}{b(1-e^2)} \end{pmatrix}$$

However, the second order leading principal minor is not always positive:

$$\begin{vmatrix} -(\alpha_2+\alpha_3+\lambda)\gamma^2 c_I(k) q_I(p_I,p_E) & -\frac{(\alpha_2+\alpha_3+\lambda)\gamma c_I(k)}{b(1-e^2)} \\ -\frac{(\alpha_2+\alpha_3+\lambda)\gamma c_I(k)}{b(1-e^2)} & -\frac{2(\alpha_1+\alpha_2+\lambda)+\alpha_3}{b(1-e^2)} \end{vmatrix} =$$

$$= \frac{(\alpha_2+\alpha_3+\lambda)\gamma^2 c_I(k) q_I(p_I,p_E)\left[2(\alpha_1+\alpha_2+\lambda)+\alpha_3\right]}{b(1-e^2)} - \left[\frac{(\alpha_2+\alpha_3+\lambda)\gamma c_I(k)}{b(1-e^2)}\right]^2$$

<u>Development 1</u>:

From equation (5) it is possible to obtain:

$$\begin{aligned} L\left(p_I^*(k),k,\lambda\right) = & \frac{1}{b(1-e^2)}\left[(\alpha_1+\alpha_2+\lambda)p_I^*(k)-(\alpha_2+\alpha_3+\lambda)c_I(k)+\alpha_3 a_I\right]*\left(a_I-ea_E-p_I^*(k)+ep_E^*(k)\right)+ \\ & +\frac{\alpha_3}{b(1-e^2)}\left[a_E-c_E-\frac{e}{(1-e^2)}\left(a_I-ea_E-p_I^*(k)+ep_E^*(k)\right)\right]*\left(a_E-ea_I-p_E^*(k)+ep_I^*(k)\right)- \\ & -\frac{\alpha_3}{2b(1-e^2)^2}\left[\left(a_I-ea_E-p_I^*(k)+ep_E^*(k)\right)^2+\left(a_E-ea_I-p_E^*(k)+ep_I^*(k)\right)^2\right]- \\ & -(\alpha_2+\alpha_3+\lambda)k-(\alpha_2+\alpha_3+\lambda)F_I-\alpha_3 F_E \end{aligned} \quad (7)$$

The first order conditions for $\max\left\{L\left(p_I^*(k),k,\lambda\right)\middle| k\geq 0,\lambda\geq 0\right\}$ are:

$$\frac{\partial L\left(p_I^*(k),k,\lambda\right)}{\partial k}\leq 0;\quad \frac{\partial L\left(p_I^*(k),k,\lambda\right)}{\partial \lambda}\geq 0;\quad k\frac{\partial L\left(p_I^*(k),k,\lambda\right)}{\partial k}=0 \quad (8)$$

An interior solution $k>0$ obtains in (8) only if $\partial L\left(p_I^*(k),k,\lambda\right)/\partial k=0$. From (7) it is obtained:

$$\frac{\partial L\left(p_I^*(k),k,\lambda\right)}{\partial k}=$$

$$=\frac{1}{b(1-e^2)}\left\{\begin{array}{l}\left[\left(\alpha_1+\alpha_2+\lambda\right)\frac{\partial p_I^*(k)}{\partial k}+\gamma\left(\alpha_2+\alpha_3+\lambda\right)c_I(k)\right]\left(a_I-ea_E-p_I^*(k)+ep_E^*(k)\right)+\\+\left[\left(\alpha_1+\alpha_2+\lambda\right)p_I^*(k)-\left(\alpha_2+\alpha_3+\lambda\right)c_I(k)+\alpha_3 a_I\right]\left(-\frac{\partial p_I^*(k)}{\partial k}+e\frac{\partial p_E^*(k)}{\partial k}\right)\end{array}\right\}+$$

$$+\frac{\alpha_3}{b(1-e^2)}\left\{\begin{array}{l}-\frac{e}{(1-e^2)}\left(-\frac{\partial p_I^*(k)}{\partial k}+e\frac{\partial p_E^*(k)}{\partial k}\right)\left(a_E-ea_I-p_E^*(k)+ep_I^*(k)\right)+\\+\left[a_E-\frac{e}{(1-e^2)}\left(a_I-ea_E-p_I^*(k)+ep_E^*(k)\right)-c_E\right]\left(-\frac{\partial p_E^*(k)}{\partial k}+e\frac{\partial p_I^*(k)}{\partial k}\right)\end{array}\right\}-$$

$$-\frac{\alpha_3}{b(1-e^2)^2}\left\{\begin{array}{l}\left(a_I-ea_E-p_I^*(k)+ep_E^*(k)\right)\left(-\frac{\partial p_I^*(k)}{\partial k}+e\frac{\partial p_E^*(k)}{\partial k}\right)+\\+\left(a_E-ea_I-p_E^*(k)+ep_I^*(k)\right)\left(-\frac{\partial p_E^*(k)}{\partial k}+e\frac{\partial p_I^*(k)}{\partial k}\right)\end{array}\right\}-\left(\alpha_2+\alpha_3+\lambda\right)$$

The optimal prices can be re-written as follows (in order to make computations easier):

$$p_I^*(k)=h_4+2h_3c_I(k)$$

$$p_E^*(k)=h_5+eh_3c_I(k)$$

where

$$h_3=\frac{\alpha_2+\alpha_3+\lambda}{2\left(1+\alpha_1+\alpha_2+2\lambda\right)-\left(1+\lambda\right)e^2}$$

$$h_4=\frac{\left(1-e^2\right)}{\xi}\left[2\left(\alpha_1+\alpha_2+\lambda\right)\left(a_I-ea_E\right)-2\alpha_3 ec_E+\left(1+\lambda\right)e\left(a_E-ea_I+c_E\right)\right]$$

$$h_5=\frac{\left(1-e^2\right)}{\xi}\left[e\left(\alpha_1+\alpha_2+\lambda\right)\left(a_I-ea_E\right)-\alpha_3 e^2 c_E+\left(1+\alpha_1+\alpha_2+2\lambda\right)\left(a_E-ea_I+c_E\right)\right]$$

Defining $h_1=\alpha_1+\alpha_2+\lambda$ and $h_2=\alpha_2+\alpha_3+\lambda$, and knowing that $\partial p_I^*(k)/\partial k=-2\gamma h_3 c_I(k)$ and $\partial p_E^*(k)/\partial k=-e\gamma h_3 c_I(k)$ gives:

$$\frac{\partial L\left(p_I^*(k),k,\lambda\right)}{\partial k}=c_I(k)\left(h_6-h_7c_I(k)\right)-\left(\alpha_2+\alpha_3+\lambda\right) \tag{9}$$

where

$$h_6=\frac{\gamma}{b\left(1-e^2\right)}\left[\left(h_2-2h_1h_3\right)\left(a_I-ea_E\right)+e\alpha_3h_3c_E+\left(h_2-2h_1h_3-\alpha_3h_3\right)eh_5\right] \tag{10}$$

$$h_7=\frac{2\gamma}{b\left(1-e^2\right)}h_3\left(h_2-2h_1h_3\right)\left(2-e^2\right)-\frac{\alpha_3\gamma}{b(1-e^2)^2}h_3^2\left[e^2+\left(2-e^2\right)^2-2e^2\left(2-e^2\right)\right]$$

The sign of h_6 and h_7 are examined here since they are determinant for the reasoning and computations that follow. It can be proved that h_6 is always positive. The first two terms of equation (10) are always positive, i.e. $\left(h_2-2h_1h_3\right)\left(a_I-ea_E\right)+e\alpha_3h_3c_E>0$. $\left(h_2-2h_1h_3-\alpha_3h_3\right)$ and h_5 are always positive as well. Therefore, h_6 is always positive. It is also easy to prove that h_7 is always positive.

The zeros of the quadratic on the right hand side of equation (9) are:

$$c_I(k)=\frac{h_6\pm\sqrt{h_6^2-4\left(\alpha_2+\alpha_3+\lambda\right)h_7}}{2h_7} \tag{11}$$

<u>Proposition 1</u>: A solution exists to the problem $\max\left\{L\left(p_I^*(k),k,\lambda\right)\middle|k\geq 0,\lambda\geq 0\right\}$ (and therefore to the original problem). The optimal solution $k^*\geq 0$ is the following:

i) If $h_6^2\leq 4\left(\alpha_2+\alpha_3+\lambda\right)h_7$, then $k^*=0$

ii) If $h_6^2>4\left(\alpha_2+\alpha_3+\lambda\right)h_7$, define $\hat{k}$ and $\hat{\lambda}$ as the k and λ, respectively, corresponding to the negative root in (11), namely, corresponding to $c_I(\hat{k})=\frac{h_6-\sqrt{h_6^2-4\left(\alpha_2+\alpha_3+\lambda\right)h_7}}{2h_7}$; then k^* is the solution to:

$L\left(p_I^*(k^*),k^*,\lambda^*\right)=\max\left\{L\left(p_I^*(\hat{k}),\hat{k},\hat{\lambda}\right),L\left(p_I^*(0),0,\lambda^*\right)\right\}$. In particular, if an interior solution obtains, then $k^*=\hat{k}$.

Proof 3: Proof of proposition 1

When the profit constraint is imposed (e.g. as in the profit-constrained welfare maximising case or Ramsey case) the existence of a solution is proved using the Weierstrass Theorem and noting the continuity of $\prod_I\left(p_I^*(k),k\right)$ and the fact that from $\prod_I\left(p_I^*(k),k\right)$ attention can be restricted to the compact set $k\in\left[0,\overline{k}\right]$, where

$$\overline{k}=\frac{1}{b\left(1-e^2\right)\left[2\left(1+\alpha_1+\alpha_2\right)-e^2\right]^2}\left\{\begin{array}{l}\left[2\left(\alpha_1+\alpha_2\right)a_I+2\left(\alpha_2+\alpha_3\right)c_{I0}+e\left(a_E+c_E\right)\right]*\\ *\left[2\left(1+\alpha_1+\alpha_2\right)a_I+e^2\left(\alpha_1+\alpha_2\right)a_I+e^2\left(\alpha_2+\alpha_3\right)c_{I0}+e\left(1+\alpha_1+\alpha_2\right)\left(a_E+c_E\right)\right]\end{array}\right\}$$

There clearly is a k such that there is at least some price for which profits are greater than or equal to zero. One such k is $k=0$, since that definitely leads to $\prod_I\left(p_I^*(k),k\right)\geq 0$ and, therefore, to non-negative profits in the original objective function. If k is larger than $\overline{k}$, there is no feasible price that will allow $\prod_I\left(p_I^*(k),k\right)\geq 0$, and consequently, the incumbent cannot breakeven in the original maximisation problem.

When the profit constraint is not imposed, i.e., $\lambda=0$ (e.g. as in the welfare-maximising case) the existence of a solution is proved using, again, the Weierstrass Theorem and noting the continuity of the maximand corresponding to (7) and the fact that from that maximand attention can be restricted to the compact set $k\in\left[0,\overline{\overline{k}}\right]$, where

$$\bar{\bar{k}} = \frac{1}{b\left(1-e^2\right)\left(\alpha_2+\alpha_3\right)}\left\{\frac{\left(\alpha_1+\alpha_2\right)}{2\left(1+\alpha_1+\alpha_2\right)-e^2}\left[2\left(\alpha_1+\alpha_2\right)a_I+2\left(\alpha_2+\alpha_3\right)c_{I0}+e\left(a_E+c_E\right)\right]+\alpha_3 a_I\right\}*$$

$$*\left\{\frac{e}{2\left(1+\alpha_1+\alpha_2\right)-e^2}\left[e\left(\alpha_1+\alpha_2\right)a_I+e\left(\alpha_2+\alpha_3\right)c_{I0}+\left(1+\alpha_1+\alpha_2\right)\left(a_E+c_E\right)\right]+a_I\right\}+$$

$$+\frac{\alpha_3 a_E}{b\left(1-e^2\right)\left(\alpha_2+\alpha_3\right)}\left\{\frac{e}{2\left(1+\alpha_1+\alpha_2\right)-e^2}\left[2\left(\alpha_1+\alpha_2\right)a_I+2\left(\alpha_2+\alpha_3\right)c_{I0}+e\left(a_E+c_E\right)\right]+a_E\right\}$$

The feasible solution $k=0$ establishes a lower bound on the maximand corresponding to (7) since it leads to non-negative maximand, and therefore, the original objective function is also non-negative. For k larger than $\bar{\bar{k}}$, the solution for the maximand corresponding to (7) is lower than zero and, therefore, lower than the value of the maximand at $k=0$. Hence, the original objective function is also negative for k larger than $\bar{\bar{k}}$.

Given the existence of a solution and the differentiability of the objective function, if the hypothesis in i) holds, then the quadratic in equation (9) is negative (it certainly is negative for $c_I(k)=0$ and if it ever became positive, it would have to cross the horizontal axis, giving rise to at least one zero on the right hand side of equation (9)). Thus, under i), it is clear that the optimal solution must be $k^*=0$.

If there is an interior solution, i.e. $k^*>0$, then the right hand side of equation (9) must equal zero giving rise to the two roots in equation (11). It is easily verified that the second order condition $\partial^2 L\left(p_I^*(k),k,\lambda\right)/\partial k^2 \le 0$ can only be fulfilled at the negative root in (11). This can be showed by computing:

$$\frac{\partial^2 L\left(p_I^*(k),k,\lambda\right)}{\partial k^2} = -\gamma c_I(k)\left(h_6-h_7 c_I(k)\right)+\gamma h_7 c_I^2(k) = \gamma\left[h_7 c_I^2(k)-\left(\alpha_2+\alpha_3+\lambda\right)\right]$$

where it is used the fact that $c_I(k)\left(h_6-h_7 c_I(k)\right)=\left(\alpha_2+\alpha_3+\lambda\right)$ at an interior solution. Therefore, $\partial^2 L\left(p_I^*(k),k,\lambda\right)/\partial k^2 \le 0$ if and only if $h_7 c_I^2(k) \le \left(\alpha_2+\alpha_3+\lambda\right)$. Using again the fact that $c_I(k)\left(h_6 - h_7 c_I(k)\right) = \left(\alpha_2 + \alpha_3 + \lambda\right)$, i.e. $h_7 c_I^2(k) = c_I(k)h_6 - \left(\alpha_2 + \alpha_3 + \lambda\right)$, the second order condition holds if and only if $h_6 c_I(k) \le 2\left(\alpha_2 + \alpha_3 + \lambda\right)$. Computing further it obtains:

$$h_6 c_I(k) = h_6 \left(\frac{h_6 \pm \sqrt{h_6^2 - 4\left(\alpha_2 + \alpha_3 + \lambda\right)h_7}}{2h_7} \right) \le 2\left(\alpha_2 + \alpha_3 + \lambda\right)$$

if and only if

$$h_6^2 - 4\left(\alpha_2 + \alpha_3 + \lambda\right)h_7 \pm h_6\sqrt{h_6^2 - 4\left(\alpha_2 + \alpha_3 + \lambda\right)h_7} \le 0 \qquad (12)$$

Equation (12) can only hold for the negative root. Thus, if an interior solution obtains, it must be at the negative root of (11).

Finally, note that it is not possible in general to rule out the boundary solution, so that the optimal solution in case ii) occurs at the point k^* at which $L\left(p_I^*(k), k, \lambda\right)$ is maximized on the boundary (i.e. at $k = 0$) or in the interior (i.e. at $k = \hat{k}$). **QED**

Nevertheless, it seems reasonable to assume that the optimal solution is interior for this problem.

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