ESTIMATION OF THE MARGINAL COST OF HIGHWAY MAINTENANCE UNDER STOCHASTIC PAVEMENT DETERIORATION

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ABSTRACT

From an economic theory point of view, it is desirable that each vehicle pay its full (social) highway pavement costs, including not only its own operating costs, but also the costs that it causes to the highway agency and to the other vehicles. That can be achieved using highway pavement pricing strategies, such as marginal cost pricing. In order for marginal cost pricing to achieve its objectives, i.e. equity and economic efficiency, the estimates of marginal costs need to be accurate. This paper discusses the perpetual overlay indirect approach used for the estimation of marginal cost in highway pavement deterioration, and it questions three common assumptions. One of these assumptions is relaxed in this paper, namely the assumption that pavement deterioration is deterministic.

Keywords: marginal cost; pavement maintenance; stochastic pavement deterioration.

JEL Classification: D84, H41, H54, L92, R40, R48.
INTRODUCTION

A discrepancy exists in many countries between the user charges paid by different vehicles and the full highway pavement costs that each causes. This causes economic inefficiency, since vehicle users are not taking into account the costs that they cause but are paid for by others. From an economic theory point of view, it is desired that each vehicle pay its full (social) highway pavement costs, including not only its own operating costs, but also the costs that it causes to the highway agency and to the other vehicles. That can be achieved using highway pavement pricing. Three main steps are common to most pricing applications. First, the costs caused by each vehicle (or class of vehicles) should be estimated accurately. Second, a pricing strategy should be formulated such that the full cost that each vehicle imposes on the highway system (as estimated in the first step) is as close as possible to the user charges incurred. Third, this pricing strategy must be made feasible for real world implementation, in terms of public acceptability and technological capability. This paper focuses on the first step and attempts to improve the marginal cost estimates of highway pavement maintenance, rehabilitation and reconstruction (MR&R). The marginal cost of a good/service is defined as the increase in total cost that takes place when the quantity of this good/service that is produced increases by one unit, while the levels of production of all other goods/services are held constant.

Different highway agencies use different pricing strategies. In order to maximize social welfare and achieve economic efficiency, a highway agency should use marginal cost pricing, which is a pricing strategy that sets price equal to marginal cost. A marginal cost pricing strategy for pavement deterioration is both efficient and fair. Small, Winston and Evans show that when marginal cost pricing is used for pavement deterioration, and at the same time, pavements are constructed in an optimal way (i.e. in a way that minimizes the present value of all capital and maintenance costs), the welfare for each of the interest groups (truckers, shippers, railroads and highway contractors) increases. This increase in welfare takes place as the owners of heavy vehicles shift to less-damaging vehicle types. Furthermore, the pavement deficit (highway revenues minus the sum of capital and maintenance highway costs) is reduced significantly (Small et al. 1989).
The success of marginal cost pricing, as well as other pricing strategies, such as Boiteux-Ramsey pricing, in achieving their objectives, relies on having accurate estimates of the marginal costs. It is, therefore, not surprising that engineers and economists have devised many approaches for estimating marginal costs associated with highway pavements. Bruzelius has surveyed these approaches (Bruzelius 2004).

Traffic loading is the number of deterioration equivalence factors. Deterioration equivalence factors (DEF) express how damaging traffic is. A DEF increases pavement deterioration level by the same amount, regardless of which vehicle it comes from. For example, a vehicle with a traffic loading of \( n \) DEFs causes \( n/m \) times the amount of deterioration caused by another vehicle with \( m \) DEFs. The combined traffic loading from both of these vehicles is \( (n+m) \) DEFs. The expression used for the DEF depends on the type of deterioration being modeled.

In evaluating pavement deterioration, total vehicle weight does not sufficiently describe the pavement deterioration caused by a vehicle. The weights on the individual axles (known as axle loads) are better indicators of deterioration. The number of DEFs resulting from an axle is typically expressed as the axle load, divided by a standard load and then raised to a power \( p \), which depends on the type of deterioration being considered. The number of DEFs resulting from a vehicle, for a given type of deterioration, is the summation of the numbers of DEFs resulting from each of its axles.

One commonly used DEF is the equivalent single axle load (ESAL), which assumes that \( p \) equals 4. The number of ESALs resulting from a single axle equals the fourth power of the ratio of the axle load to the standard single axle load of 18 kips (80 kN).

As FIGURE 1 shows, traffic loading affects MR&R cost indirectly. Among the approaches used in the literature to estimate MR&R marginal costs, the perpetual overlay indirect approach is the only approach that models all three links \( a, b \) and \( c \) in the figure, and that is why this approach has been chosen for this paper. Bruzelius just refers to it as the “indirect approach” (Bruzelius 2004). This approach assumes that pavement overlay (resurfacing) costs dominate MR&R costs and ignores all other MR&R costs. It uses an infinite analysis horizon and assumes that a pavement gets overlaid as soon it deteriorates to a predetermined trigger level. It first relates changes in traffic loading (additional ESAL) to changes in overlay frequency (an additional ESAL brings forward the future
overlays), and possibly changes in the overlay intensity (thicker overlays in anticipation of higher traffic loadings in the future). Then, it relates these changes in overlay frequency (and intensity) to MR&R marginal cost.

FIGURE 1 Indirect effect of traffic loading on MR&R cost.

The reviewed studies that follow this approach use ESAL as the DEF, but they define an additional ESAL in two different ways. The first way is to define the additional ESAL as being a one-time event, and then the MR&R marginal cost is defined as the change in the present value of all future overlay costs that results from this one-time increase in traffic loading by 1 ESAL (Newbery 1988; Transportation Research Board 1996). In other words, the loading increases this year, but the future annual loadings remain the same. This can be referred to as the one-time additional DEF. The second way is to define the additional ESAL as being an event that recurs annually, and then the MR&R marginal cost is defined as the change in the annualized cost of all future overlays, as a result of increasing the traffic loading by 1 ESAL this year and every year in the future (Lindberg 2002; Small et al. 1989; Vitaliano and Held 1990). This will be referred to as the recurring additional DEF. When the additional ESAL is defined this second way, it makes sense for the highway agency to strengthen the pavement structure in anticipation of higher traffic loadings in the future.

This approach makes three questionable assumptions. The first assumption is that pavement deterioration caused by an axle is proportional to the fourth power of the axle load. Using the ESAL as the DEF is the same as assuming that pavement deterioration caused by an axle is proportional to the axle load raised to the power four. This fourth power rule came from the AASHO Road Test, where deterioration was defined as loss of pavement serviceability, and the number of DEFs was found to be proportional to the fourth power of the axle loadings. The estimation procedures that came up with the fourth power rule were questionable (Bruzelius 2004, p. 45). Prozzi and Madanat obtained
a power of 4.15, with the same data set but using random effects estimation (Prozzi and Madanat 2004).

Different highway agencies use different definitions of deterioration. For example, deterioration might be defined as the increase in roughness, cracking, or rutting. Depending on the way deterioration is defined, the fourth power may not be suitable for expressing the DEF, in which case the traffic loading should not be expressed as the number of ESALs. Prozzi and Madanat obtain a power of 3.85 when deterioration is defined as roughness (Prozzi and Madanat 2004). Archilla and Madanat specify and estimate a model that predicts the increase in rut depth. Although the power for tandem axle load is close to 4 (3.89), the power for the single axle load is closer to 3 (2.98). The estimation of the latter numbers is possible because the model does not use predefined DEFs (e.g. number of ESALs) or structural numbers, both of which were originally estimated for serviceability models. Instead, the data completely determines the values of the model parameters (Archilla and Madanat 2000). Therefore, the use of ESAL as a deterioration equivalence factor is appropriate only when deterioration is defined as the loss in serviceability, and even in that case, it is still questionable.

The second assumption is that the only MR&R activity used by a highway agency is an overlay of constant intensity. Some studies assume that highway agencies overlay a given pavement using the same intensity every time it deteriorates to a constant, predetermined trigger level of pavement performance (Newbery 1990; Small et al. 1989). Recent research has shown that such a policy is optimal for both the finite horizon problem (Ouyang and Madanat 2006) and the infinite horizon problem (Li and Madanat 2002). However, in reality, a highway agency uses different types of MR&R activities, such as pothole repairs, crack seals, thin overlays, regular overlays and reconstruction; and it uses different triggers for different activities. Since each highway agency has its own MR&R policy, it is important to take into account this policy when determining MR&R marginal cost.

The third assumption is that pavement deterioration is deterministic, and as a result, the exact times of all future MR&R activities can be (exactly) predicted. This assumption is also questionable. Pavement deterioration is stochastic in nature, so the time interval between two consecutive MR&R activities (inter-activity time) is a random variable. The MR&R marginal cost function is not a linear
function of the inter-activity times, so the expected value of the MR&R marginal cost function is generally different from the value of the same function evaluated at the expected inter-activity times.

This paper relaxes the third assumption by accounting for the stochastic nature of pavement deterioration. It estimates the expected costs taken over the distribution of performance.

**METHODOLOGY**

Consider a simple maintenance, rehabilitation and reconstruction (MR&R) policy, where a highway agency resurfaces (overlays) a pavement section each time it fails. Assume that the highway agency defines pavement failure as unacceptable serviceability.

Let random variable $\rho$ be the number of load applications to failure for an overlay cycle. Let $D$ be the structural number. Let $L_1$ be the nominal load in kips on the axle group. For example, $L_1$ equals 18 for an 18-kip single axle load, and it equals 32 for a 32-kip tandem axle load (Highway Research Board 1962). Let $L_2$ be a dummy variable indicating the configuration of the axle group: 1 for a single axle group, and 2 for a tandem axle group (Highway Research Board 1962). The AASHO Road Test, Report 5 (Highway Research Board 1962, equations 7 and 18) specifies and estimates the following model:

$\rho = 10^{5.93}(D + 1)^{9.36}L_2^{4.33} \div (L_1 + L_2)^{4.79}$

Let random variable $X$ be the number of ESALs to failure for an overlay cycle. By setting $L_1 = 18$ and $L_2 = 1$, each load application becomes an ESAL, and $\rho$ becomes $X$.

$X = \frac{10^{5.93}(D + 1)^{9.36}}{19^{4.79}}$  (1)

The AASHO model is misspecified; $L_1$ and $L_2$, which have different units, should not be summed. Furthermore, the methods used to estimate the model parameters are vague (Prozzi and Madanat
Prozzi and Madanat use a stochastic duration model, where the hazard rate follows a Weibull distribution. They re-estimate the AASHO model as follows (Prozzi and Madanat 2000):

\[
E[\rho] = \frac{10^{5.28} (D + 1)^{6.68} L_2^{2.62}}{(L_1 + L_2)^{0.03}}
\]

By setting \( L_1 = 18 \) and \( L_2 = 1 \), each load application becomes an ESAL, and \( \rho \) becomes \( X \):

\[
E[X] = \frac{10^{5.28} (D + 1)^{6.68}}{19^{0.03}}
\]

The (accelerated) Weibull model has the following probability density function (Liu 2003):

\[
f(x|\xi) = \lambda \theta (\lambda x)^{\theta - 1} \exp\left(\frac{\lambda x}{\beta}\right) \exp\left[-\left(\frac{\lambda x}{\beta}\right)^{\theta}\right], \text{ for } x \geq 0, \text{ and } 0 \text{ otherwise}
\]

Where, \( \lambda = 5.288 \times 10^{-6} \), \( \theta = 1.333 \), and

\[
z^\beta \lambda = -8.907 \ln(D + 1) - 3.493 \ln(L_2) + 4.040 \ln(L_1 + L_2) = -8.907 \ln(D + 1) + 4.040 \ln(19)
\]

A previous study (Haraldsson 2007, Essay I, pp. 3-25) took into account the stochastic nature of pavement deterioration; however, it only did so for the first overlay cycle. It treated the subsequent cycles as deterministic. Another previous study (Liu 2003) took into account the stochastic nature of pavement deterioration by treating \( X \) (for all overlay cycles) as a random variable. However, it made the unrealistic assumption that each future overlay cycle had the same number of ESALs to failure. This case study will relax this assumption by allowing each overlay cycle to have its own number of ESALs to failure.
Let random variable $X_i$, such that $X_i \geq 0$, denote the number of ESALs to failure for the $i$th overlay cycle. Let constant $L$, such that $L > 0$, be the annual traffic loading (ESAL/year). Then the duration of the $i$th overlay cycle is defined as random variable $T_i$ (year):

$$T_i = \frac{X_i}{L}$$  \hspace{1cm} (4)$$

Let $r$, such that $r > 0$, be the discount rate per annum, let $u$ be the overlay cost ($/mile$), and let $V$ be the present value of all future overlays. Assume for simplicity that there is only one highway lane. FIGURE 2 shows the cash flow diagram for all future overlays.

$$V = \sum_{n=1}^{\infty} u \exp\left( -r \sum_{i=1}^{n} T_i \right) = u \sum_{n=1}^{\infty} \prod_{i=1}^{n} \exp(-rT_i)$$ \hspace{1cm} (5)$$

Then,

$$E(V|\mathcal{Z}) = E\left[ u \sum_{n=1}^{\infty} \prod_{i=1}^{n} \exp(-rT_i) \mid \mathcal{Z} \right] = u \sum_{n=1}^{\infty} E\left[ \prod_{i=1}^{n} \exp(-rT_i) \mid \mathcal{Z} \right]$$ \hspace{1cm} (6)$$

Assuming $\{T_i\}_{i \geq 1}$ are independent, or equivalently, $\{X_i\}_{i \geq 1}$ are independent:
Assume that \( \{T_i\}_{i \geq 1} \) are identically distributed, or equivalently, \( \{X_i\}_{i \geq 1} \) are identically distributed (they all have the same distributed as X). Then:

\[
E(V|z) = u \sum_{n=1}^{\infty} \prod_{i=1}^{n} E[\exp(-r.T_i)|z] \tag{7}
\]

Since \( 0 < (r.T_1) < \infty \), \( 0 < E[\exp(-r.T_1)|z] < 1 \). Therefore, the geometric series converges:

\[
E(V|z) = u \sum_{n=1}^{\infty} \left[ E[\exp(-r.T_i)|z] \right]^n \tag{8}
\]

As aforementioned, a previous study (Liu 2003) made the unrealistic assumption that \( T_i = T_k \) for all \( i, k \geq 1 \). That assumption leads to the following incorrect result:

\[
E(V|z) = u \frac{E[\exp(-r.T_i)|z]}{1 - E[\exp(-r.T_i)|z]} \tag{9}
\]

This incorrect result overestimates \( E(V|z) \), regardless of the distribution of the positive random variable \( T_1 \) (Anani 2007).

Dividing both the numerator and denominator of Equation (9) by \( E[\exp(-r.T_i)|z] \), and using Equation (4):
\[ E(V|z) = u \cdot \frac{1}{\left( E\left( \exp\left( \frac{-rX_1}{L} \right) |z \right) \right)^{-1}} = u \cdot \frac{1}{\left( g(L) \right)^{-1} - 1} \]  \tag{10}

Where,

\[ g(L) := E\left( \exp\left( \frac{-rX_1}{L} \right) |z \right) = \int_0^\infty \exp\left( \frac{-rX}{L} \right) f(x|z) dx \]  \tag{11}

The integral in Equation (11) converges (Anani 2007).

The expected annualized cost = \( E(r.V|z) = r \cdot E(V|z) \). Suppose that the additional ESAL is defined as a recurring additional DEF (Lindberg 2002; Small et al. 1989; Vitaliano and Held 1990).

Then, the MR&R marginal cost ($/ESAL/mile) is defined as follows:

\[ C' = \frac{\partial \left( r \cdot E(V|z) \right)}{\partial L} \]

Using this definition and Equation (10):

\[ C' = r \cdot u \cdot \frac{d}{dL} \left( \frac{1}{\left( g(L) \right)^{-1} - 1} \right) = r \cdot u \cdot \frac{d}{dL} \left( \frac{1}{1 - g(L)} \right)^2 \cdot \frac{d}{dL} g(L) \]  \tag{12}

Where, \( g(L) \) is given by Equation (11), and \( dg(L)/dL \) is given by Equation (14) below.

Define \( b := \exp(z'\beta) \). For typical values of \( D, z'\beta \) is finite. Therefore, \( b \) is strictly positive and finite. Using this definition, Equation (3) and Equation (11):

\[ \frac{d}{dL} g(L) = \lambda^\theta \cdot b \cdot \frac{d}{dL} \int_0^\infty x^{\theta-1} \cdot \exp\left( -\frac{rX}{L} - \lambda^\theta \cdot b \cdot x^\theta \right) dx \]  \tag{13}
The order of differentiation and integration in Equation (13) can be interchanged (Anani 2007). Then:

$$\frac{d}{dL} g(L) = \lambda^\theta \cdot \frac{\partial}{\partial L} \int_0^{\infty} x^{\theta-1} \exp \left( -\frac{r \cdot x}{L} - \lambda^\theta \cdot b \cdot x^\theta \right) dx = \frac{\lambda^\theta \cdot \partial \cdot r \cdot b \cdot x^\theta}{L^2} \int_0^{\infty} x^{\theta} \cdot \exp \left( -\frac{r \cdot x}{L} - \lambda^\theta \cdot b \cdot x^\theta \right) dx \quad (14)$$

Using Equations (3) and (14):

$$\frac{d}{dL} g(L) = \frac{r}{L^2} \int_0^{\infty} x \cdot \exp \left( -\frac{r \cdot x}{L} \right) f(x|x_0) dx = \frac{r}{L^2} \cdot E \left[ X_1 \cdot \exp \left( -\frac{r \cdot X_1}{L} \right) \right] \quad (15)$$

Numerical integration can be used for evaluating Equations (11) and (14). Then, these two values can be plugged into Equation (12) in order to obtain the MR&R marginal cost. This MR&R marginal cost, which takes into account the stochastic nature of pavement deterioration, will then be compared with the estimate of MR&R marginal cost that assumes that pavement deterioration is deterministic.

The expression for MR&R marginal cost with deterministic deterioration is a special case of Equation (12), where $X_i=E(X)$ for all $i \geq 1$, and the time interval between any two consecutive overlays, $T$, is:

$$T = \frac{E(X)}{L} \quad (16)$$

Where, $E(X)$ is given by Equation (2). Then, Equation (11) becomes:

$$g(L) = E \left( \exp \left( -\frac{r \cdot E(X)}{L} \right) \right) = \exp \left( -\frac{r \cdot E(X)}{L} \right) = \exp \left( -r \cdot T \right) \quad (17)$$

Then,
\[
\frac{d}{dL} g(L) = \frac{r.E(X)}{L^2} \cdot \exp\left(\frac{-r.E(X)}{L}\right) = \frac{r.T}{L} \cdot \exp\left(-r.T\right)
\] 

(18)

Then, Equation (12) becomes as follows (for deterministic deterioration):

\[
C' = \frac{r^2 \cdot u.T \cdot \exp(-r.T)}{L(1 - \exp(-r.T))^2} = \frac{r^2 \cdot u.T \cdot \exp(r.T)}{L(\exp(r.T) - 1)^2}
\] 

(19)

Where, \(T\) is given by Equation (16).

**COMPUTATIONS**

This section computes and compares the estimates of MR&R marginal cost obtained using the stochastic and deterministic assumptions. Equation (12) takes into account the stochastic nature of pavement deterioration. On the other hand, Equation (19) assumes that deterioration is deterministic.

The values of \(D\), \(r\) and \(L\) are varied. The value of \(u\) is not varied, since changing \(u\) by a (multiplicative) factor \(k\) simply changes both marginal cost estimates by the same factor \(k\).

MATLAB code is written to compute the marginal cost estimates and generate plots. TABLE 1 shows the values used for the computations and the corresponding results. As TABLE 1 shows, taking into account the stochastic nature of pavement deterioration increases the estimate of marginal cost estimate, but not the way the estimate is affected by the values of \(D\), \(r\) or \(L\). The marginal cost (stochastic or deterministic) moves in the opposite direction to \(D\) and \(r\), and in the same direction as \(L\) (et ceteris paribus). It makes intuitive sense for \(C'\) to move in the opposite direction to \(D\); a stronger pavement requires less frequent overlays. It also makes sense for \(C'\) to move in the opposite direction to \(r\), since a higher \(r\) reduces the present value of (the difference in) future overlay costs. Finally, \(C'\) is expected to move in the same direction as \(L\), for a higher \(L\) makes the overlay cycles shorter, thus increasing the present value of (the difference in) future overlay costs.
The last column of TABLE 1 shows the absolute percent difference between marginal cost estimates, which is defined as follows:

\[
\text{absolute percent difference} = \left| \frac{C'_\text{deterministic} - C'_\text{stochastic}}{C'_\text{stochastic}} \right| \times 100\% = \left(1 - \frac{C'_\text{deterministic}}{C'_\text{stochastic}}\right) \times 100\%
\]  

The second inequality is true because taking into account the stochastic nature of pavement deterioration increases the estimates of marginal costs using this model.

**TABLE 1 Computations**

| Comp. No. | D | r | L | u | Approx. E[T] | C' stochastic | C' deterministic | %|Difference| |
|-----------|---|---|---|---|-------------|----------------|------------------|-----------------|
| 1         | 2 | 0.02 | 10,000 | 100,000 | 4           | 2.8008         | 2.5541           | 9%              |
| 2         | 2 | 0.02 | 100,000 | 100,000 | <1          | 2.8017         | 2.5554           | 9%              |
| 3         | 2 | 0.02 | 1,000,000 | 100,000 | <1          | 2.8017         | 2.5554           | 9%              |
| 4         | 2 | 0.05 | 10,000 | 100,000 | 4           | 2.7961         | 2.5472           | 9%              |
| 5         | 2 | 0.05 | 100,000 | 100,000 | <1          | 2.8017         | 2.5553           | 9%              |
| 6         | 2 | 0.05 | 1,000,000 | 100,000 | <1          | 2.8017         | 2.5554           | 9%              |
| 7         | 2 | 0.08 | 10,000 | 100,000 | 4           | 2.7881         | 2.5346           | 9%              |
| 8         | 2 | 0.08 | 100,000 | 100,000 | <1          | 2.8016         | 2.5552           | 9%              |
| 9         | 2 | 0.08 | 1,000,000 | 100,000 | <1          | 2.8017         | 2.5554           | 9%              |
| 10        | 5 | 0.02 | 10,000 | 100,000 | 400         | 0.0189         | 0.0005           | 97%             |
| 11        | 5 | 0.02 | 100,000 | 100,000 | 40          | 0.0266         | 0.0236           | 11%             |
| 12        | 5 | 0.02 | 1,000,000 | 100,000 | 4           | 0.0273         | 0.0249           | 9%              |
| 13        | 5 | 0.05 | 10,000 | 100,000 | 400         | 0.0144         | 0.0000           | 100%            |
| 14        | 5 | 0.05 | 100,000 | 100,000 | 40          | 0.0248         | 0.0180           | 27%             |
| 15        | 5 | 0.05 | 1,000,000 | 100,000 | 4           | 0.0272         | 0.0248           | 9%              |
| 16        | 5 | 0.08 | 10,000 | 100,000 | 400         | 0.0124         | 0.0000           | 100%            |
| 17        | 5 | 0.08 | 100,000 | 100,000 | 40          | 0.0232         | 0.0113           | 51%             |
| 18        | 5 | 0.08 | 1,000,000 | 100,000 | 4           | 0.0271         | 0.0247           | 9%              |
| 19        | 8 | 0.02 | 10,000 | 100,000 | 6000        | 0.0005         | 0.0000           | 100%            |
| 20        | 8 | 0.02 | 100,000 | 100,000 | 600         | 0.0011         | 0.0000           | 100%            |
| 21        | 8 | 0.02 | 1,000,000 | 100,000 | 60          | 0.0017         | 0.0015           | 15%             |
| 22        | 8 | 0.05 | 10,000 | 100,000 | 6000        | 0.0004         | 0.0000           | 100%            |
| 23        | 8 | 0.05 | 100,000 | 100,000 | 600         | 0.0008         | 0.0000           | 100%            |
| 24        | 8 | 0.05 | 1,000,000 | 100,000 | 60          | 0.0016         | 0.0008           | 47%             |
| 25        | 8 | 0.08 | 10,000 | 100,000 | 6000        | 0.0003         | 0.0000           | 100%            |
| 26        | 8 | 0.08 | 100,000 | 100,000 | 600         | 0.0007         | 0.0000           | 100%            |
| 27        | 8 | 0.08 | 1,000,000 | 100,000 | 60          | 0.0014         | 0.0003           | 78%             |

\[\%|\text{Difference}| = -\left(\frac{C'_\text{deterministic} - C'_\text{stochastic}}{C'_\text{stochastic}}\right) \times 100\%\]
The results from TABLE 1 will first be confirmed analytically for the deterministic case, and then shown graphically, in more detail, for the stochastic case. It is easier to understand these patterns analytically for the deterministic case, i.e. Equation (19).

First, the effect of changes in D on the (deterministic) C’ will be analyzed. E[X] moves in the same direction as D et ceteris paribus (Equation (2)), and T moves in the same direction as E[X] (Equation (16)). Thus, T moves in the same direction as D. The relationship between T and C’ needs a little more work but can be obtained as follows from Equation (19):

\[
\frac{\partial C'}{\partial T} = \frac{r^2 u}{L} \left( \frac{\exp(rT)}{(\exp(rT) - 1)^2} + \frac{(rT)\exp(rT)}{(\exp(rT) - 1)^2} - \frac{2(rT)(\exp(rT))^2}{(\exp(rT) - 1)^3} \right)
\]

This can be expressed as the product of four terms:

\[
\frac{\partial C'}{\partial T} = \left( \frac{r^2 u}{L} \right) \left( \frac{1}{(\exp(rT) - 1)^3} \right) \{\exp(rT)\} \{(1 - rT)\exp(rT) - 1 - rT\}
\]

Again, r>0, u>0, L>0 and T>0. It is easy to see that the first three terms are positive. Using the Taylor expansion for \(\exp(rT)\), the fourth term equals the following:

\[
(1 - rT)\exp(rT) - 1 - rT = (1 - rT)\left( \sum_{i=0}^{\infty} \frac{(rT)^i}{i!} \right) - 1 - rT = \sum_{i=0}^{\infty} \frac{(rT)^i}{i!} - \sum_{i=0}^{\infty} \frac{(rT)^{i+1}}{i!} - 1 - rT
\]

\[
= \sum_{i=0}^{\infty} \frac{(rT)^i}{i!} - \sum_{i=1}^{\infty} \frac{(rT)^i}{(i-1)!} - 1 - rT = -rT - \sum_{i=1}^{\infty} \frac{1}{i!} \left( \frac{1}{i-1} \right) (rT)^i
\]

Since the fourth term is the sum of negative terms, it is negative. As a result, \(\delta C'/\delta T\) is negative, and C’ moves in the opposite direction to T. Since T moves in the same direction as D, and C’ moves in the opposite direction to T, C’ moves in the opposite direction to D.
Next, the effect of changes in $L$ on the (deterministic) $C'$ will be analyzed. $E[X]$ is constant for fixed $D$ (Equation (2)), and for typical values of $D$, $E[X]>0$. Plugging Equation (16) into Equation (19):

$$C' = \frac{r^2.u.T.\exp(r.T)}{L(\exp(r.T) - 1)} = \frac{r^2.u.E[X]}{L^2} \left( \frac{\exp\left(\frac{r.E[X]}{L}\right)}{\exp\left(\frac{r.E[X]}{L}\right) - 1} \right)^2$$

$$\frac{\partial C'}{\partial L} = \left\{ \frac{r^2.u.E[X]}{L^2} \left( \frac{\exp\left(\frac{r.E[X]}{L}\right)}{\exp\left(\frac{r.E[X]}{L}\right) - 1} \right)^2 - 2. \frac{E[X]r^2.u.E[X]}{L^2} \left( \frac{\exp\left(\frac{r.E[X]}{L}\right)}{\exp\left(\frac{r.E[X]}{L}\right) - 1} \right)^2 + 2. \frac{E[X]r^2.u.E[X]}{L^2} \left( \frac{\exp\left(\frac{r.E[X]}{L}\right)}{\exp\left(\frac{r.E[X]}{L}\right) - 1} \right)^2 \right\}$$

Using Equation (16) to replace $E[X]$, and rearranging, this can be expressed as the product of two terms:

$$\frac{\partial C'}{\partial L} = \left\{ \frac{r^2.u.E[X]}{L^2} \left( \frac{\exp\left(\frac{r.E[X]}{L}\right)}{\exp\left(\frac{r.E[X]}{L}\right) - 1} \right)^2 \right\} \left( E[X]r^2.u.E[X] \right) - 2. \frac{E[X]r^2.u.E[X]}{L^2} \left( \frac{\exp\left(\frac{r.E[X]}{L}\right)}{\exp\left(\frac{r.E[X]}{L}\right) - 1} \right)^2 + 2. \frac{E[X]r^2.u.E[X]}{L^2} \left( \frac{\exp\left(\frac{r.E[X]}{L}\right)}{\exp\left(\frac{r.E[X]}{L}\right) - 1} \right)^2$$

The first term is positive (using $r>0$, $T>0$ and $u>0$). Using the Taylor expansion for $\exp(rT)$, the second term equals the following:
\[ r.T + 2 + (r.T - 2)\exp(r.T) = r.T + 2 + (r.T - 2)\sum_{i=0}^{\infty} \frac{(r.T)^i}{i!} \]

\[ = r.T + 2 + \sum_{i=0}^{\infty} \frac{(r.T)^i}{i!} + \sum_{i=0}^{\infty} -2\frac{(r.T)^i}{i!} = r.T + \sum_{i=1}^{\infty} \frac{(r.T)^i}{(i-1)!} + \sum_{i=1}^{\infty} -2\frac{(r.T)^i}{i!} \]

\[ = r.T + \sum_{i=1}^{\infty} \left( \frac{1}{(i-1)!} - \frac{2}{i!} \right) (r.T)^i = \sum_{i=3}^{\infty} \left( \frac{1}{(i-1)!} - \frac{2}{i!} \right) (r.T)^i \]

(20)

Since the second term is the sum of positive terms, it is positive. As a result, \( \frac{\delta C'}{\delta L} \) is positive, and \( C' \) moves in the same direction as \( L \).

Next, the effect of changes in \( r \) on the (deterministic) \( C' \) will be analyzed. Using Equation (19):

\[ \frac{\partial C'}{\partial r} = \frac{u.T}{L} \left( \frac{2}{(\exp(r.T) - 1)^2} + \frac{r^2.T.\exp(r.T)}{(\exp(r.T) - 1)^2} - 2 \frac{r^2.T.(\exp(r.T))^3}{(\exp(r.T) - 1)^3} \right) \]

This can be written as the product of two terms:

\[ \frac{\partial C'}{\partial r} = \left\{ \frac{u.r.T.\exp(r.T)}{L.(\exp(r.T) - 1)^3} \right\} \left\{ -2 - r.T + (2 - r.T)\exp(r.T) \right\} \]

The first term is positive (using \( u>0, r>0, T>0 \) and \( L>0 \)). The second term, which is the negative of the LHS of Equation (20), is negative. As a result, \( \frac{\delta C'}{\delta r} \) is negative, and \( C' \) moves in the opposite direction to \( r \).

FIGURE 3 through FIGURE 7 show, in more detail than TABLE 1, how the marginal cost that is estimated using the stochastic assumption varies with \( D, r \) and \( L \). The marginal cost moves in the opposite direction to \( D \) et ceteris paribus (FIGURE 4 and FIGURE 6). The marginal cost moves in the opposite direction to \( r \) et ceteris paribus (FIGURE 3 and FIGURE 7). The marginal cost moves in the same direction as \( L \) et ceteris paribus (FIGURE 3 and FIGURE 5).
FIGURE 8 through FIGURE 10 show, in more detail, the absolute percent differences. Both the table and figures show that the absolute percent difference moves in the opposite direction to L, and in the same direction as r and D, et ceteris paribus.

![Stochastic marginal cost estimates (fixing D=5).](image)

FIGURE 3 Stochastic marginal cost estimates (fixing D=5).
FIGURE 4 Stochastic marginal cost estimates (fixing $r=0.05$).

FIGURE 5 Stochastic marginal cost estimates (fixing $r=0.05$, zoomed in).
FIGURE 6 Stochastic marginal cost estimates (fixing $L=100,000$).

FIGURE 7 Stochastic marginal cost estimates (fixing $L=100,000$, zoomed in).
FIGURE 8 Absolute percent difference between marginal cost estimates (fixing D=5).

FIGURE 9 Absolute percent difference between marginal cost estimates (fixing r=0.05).
FIGURE 10 Absolute percent difference between marginal cost estimates (fixing L=100,000).

ADDITIONAL COMPUTATIONS

Not all of the input values for the previous computations are reasonable, since they might correspond to (expected) overlay cycles, $E[T_i]$, that are either too short or too long. It should be noted that $E[T_i]$ is referred to as $T$ in the deterministic case. In order to know whether it is important to take into account the stochastic nature of pavement deterioration, reasonable pairs of $D$ and $L$ are used for the following computations. Given a value of $L$, a highway agency is assumed have chosen a value of $D$ that leads to reasonable $E[T_i]$ of 5 or 10 years. The product of $L$ and $E[T_i]$ gives $E[X]$, which can be used to find $D$ using Equation (2) (the values of $D$ are rounded to the nearest tenth). A reasonable value of $r$ is 0.05. The computations are summarized in TABLE 2. Therefore, an increase of about 10% can be expected when the stochastic nature of deterioration is taken into account.
TABLE 2 Additional Computations

| Comp. No. | E[Ti] (years) | D | r  | L       | U       | C’ stochastic | C’ deterministic | %|Difference|
|-----------|---------------|---|----|---------|---------|---------------|-----------------|------|
| 1         | 5             | 2.1| 0.05| 10,000  | 100000  | 2.2436        | 2.0426          | 9
| 2         | 5             | 3.4| 0.05| 100,000 | 100000  | 0.2161        | 0.1968          | 9
| 3         | 5             | 5.2| 0.05| 1,000,000| 100000  | 0.0218        | 0.0199          | 9
| 4         | 10            | 2.5| 0.05| 10,000   | 100000  | 0.9871        | 0.8900          | 10
| 5         | 10            | 3.9| 0.05| 100,000  | 100000  | 0.1043        | 0.0943          | 10
| 6         | 10            | 5.9| 0.05| 1,000,000| 100000  | 0.0106        | 0.0096          | 10

CONCLUSIONS

The perpetual overlay indirect approach seems to be the most promising approach for reasonably estimating pavement MR&R marginal cost. However, it makes the three aforementioned unrealistic assumptions, and therefore it has room for improvement. This paper has relaxed one of these assumptions, namely the assumption that pavement deterioration is deterministic. (The authors are currently working on other studies to relax the other two assumptions.)

Both stochastic and deterministic marginal costs move in the opposite direction to D and r, and in the same direction as L (et ceteris paribus). For a reasonable range of input data, the stochastic marginal cost is higher. The absolute percent difference (between marginal costs) moves in the opposite direction to L, and in the same direction as D and r, et ceteris paribus. For typical values of r and reasonable combinations of D and L, the stochastic marginal cost is nearly 10% higher than deterministic marginal cost. As a result, a deterministic marginal cost pricing strategy would make the price paid by each vehicle fall short of the additional MR&R cost that it causes. Furthermore, if demand is not very elastic, using deterministic marginal costs can reduce highway agency revenue. Therefore, it is important to take the stochastic nature of deterioration into account.

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