Optimal Road Pricing and Endogenous User Behavior

Gerd Meinhold and
Michael Pickhardt

Optimal Road Pricing ... Overview

- Motivation
- Braess‘s Paradox
- Road Pricing Model
- Policy Implications
- Concluding Remarks

Motivation

- In road pricing schemes, road user behavior is usually not taken into account
- Particularly true for the impact which different tolls on alternative routes may have on the route choice behavior of road users
- Hence, road tolls may turn out to be sub-optimal

Motivation

- The toll for heavy trucks charged on German motorways, but not on German highways, may serve as an example
  - In some areas, it has been observed that truck drivers choose highways rather then motorways
- Our model explains why rational, payoff maximizing road users may show such behavior patterns

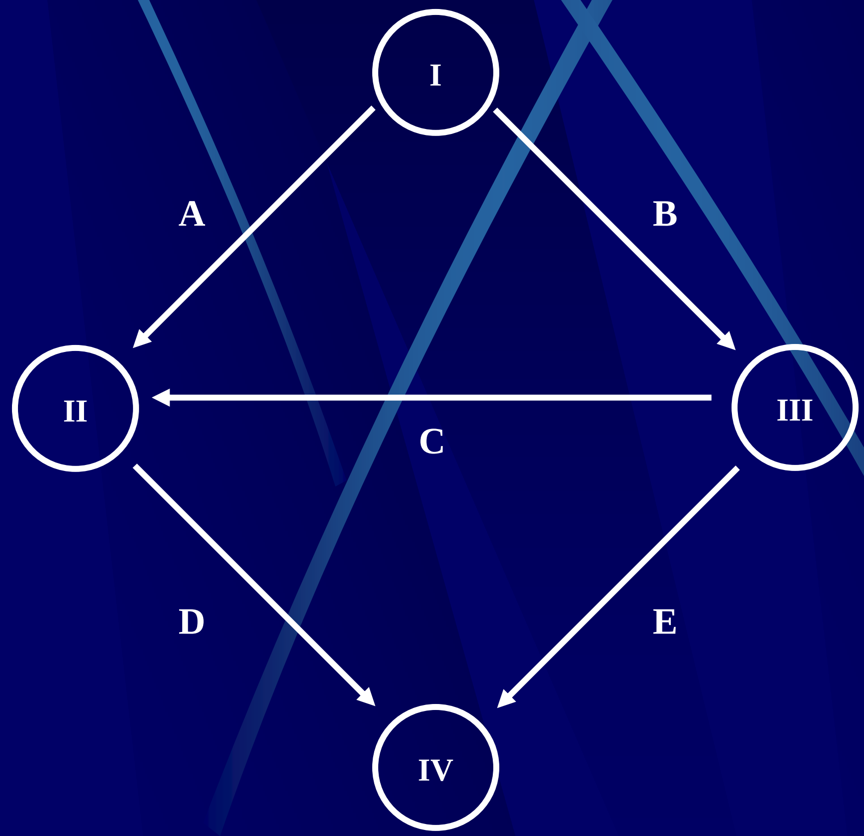
Motivation

- More precisely,we show that:
  - in a two-tier road network tolls on just one tier may cause Braess’s Paradox
  - a revenue maximizing toll exists that does not cause Braess’s Paradox

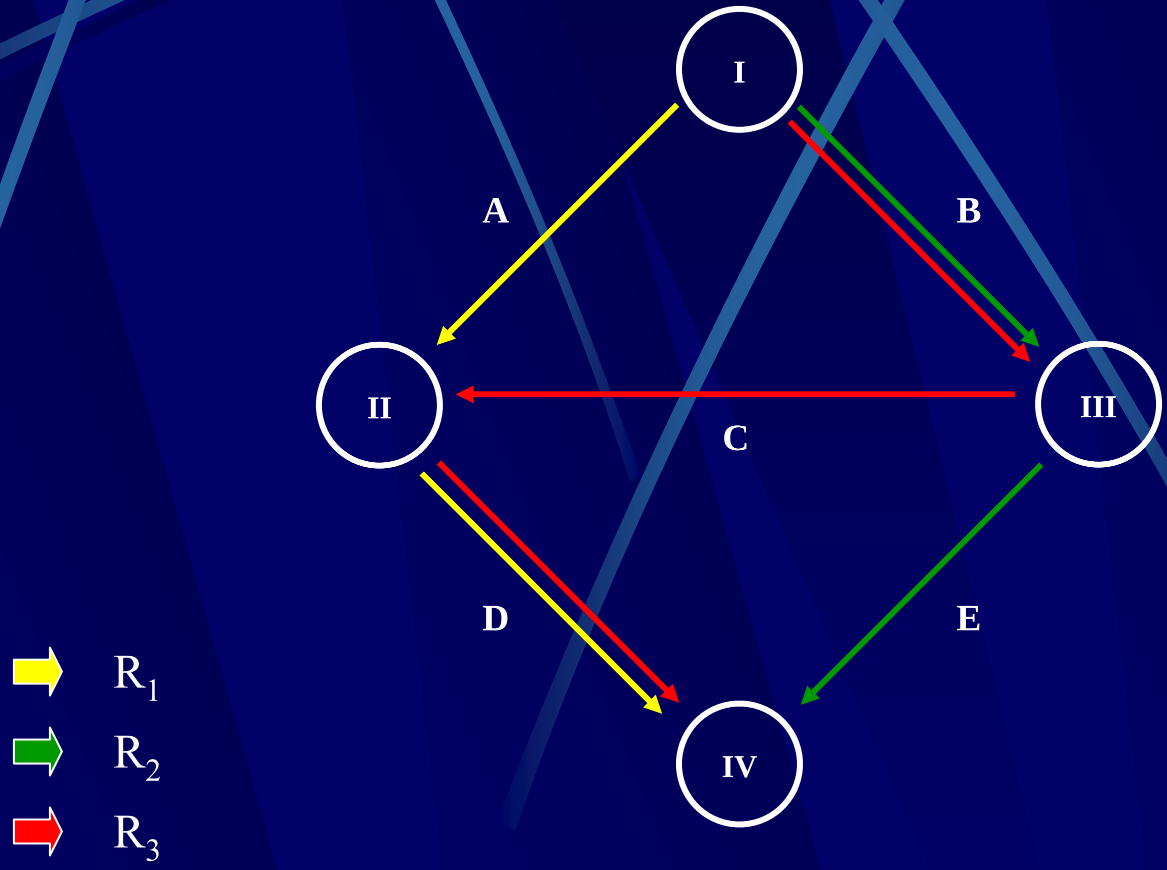
Braess's Paradox

- **Dietrich Braess** (1968). *Über ein Paradoxon aus der Verkehrsplanung*, Unternehmensforschung, x, pp. 258-268.
- **J.N.Hagstrom and R.A. Abrams** (2001). Characterizing Braess's Paradox for Traffic Networks, IEEE, Proceedings.
- **Pickhardt** (2006). Infraestructura de transportes y tarificación viaria en la Unión Europea, forthcoming in: *Información Comercial Española* (ICE), 831, pp.

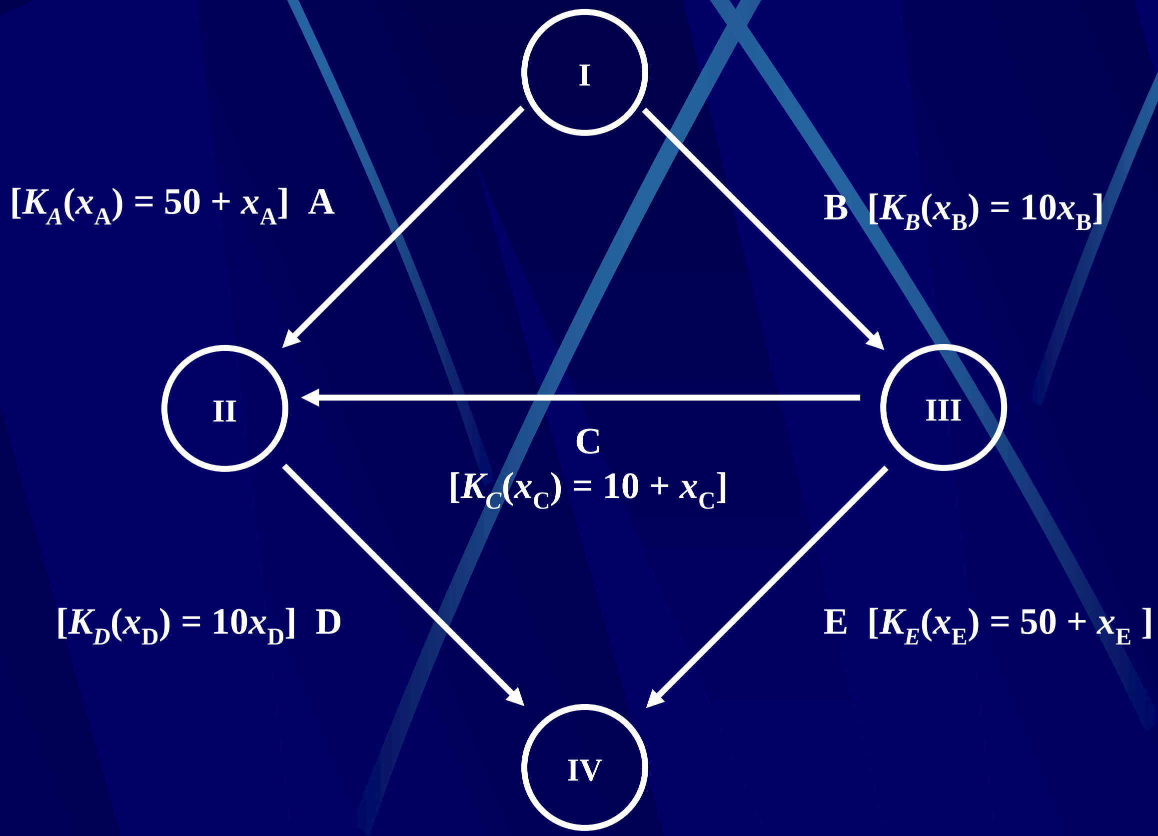
Braess's Paradox



Braess's Paradox



Braess's Paradox



Braess's Paradox

- Suppose that total flow X from supply node I to demand node IV is given, with $X = 6$.
- In this case, the equilibrium flow distribution over the three possible routes, $R_{1\text{-}3}$, is: $R_1 = R_2 = R_3 = 2$.
- Then, because of $x_A = x_C = x_E = 2$, and $x_B = x_D = 4$, each unit of flow has travel costs of 92 units of time and total travel costs amount to $(6 \cdot 92 =)$ 552 units of time.

Braess's Paradox

- Now assume that link *C* is blocked by an appropriate road toll or some other ruling, so that route 3 cannot be used anymore.
- The equilibrium flow distribution over the two remaining routes routes, $R_{1\text{-}2}$, is now: $R_1 = R_2 = 3$, which leads to $x_A = x_B = x_D = x_E = 3$,
- and yields travel costs of just 83 units of time for each unit of flow and total travel costs of $(6 \cdot 83 =)$ 498 units of time

Braess's Paradox

- Hence, introducing a congestion fee on certain parts of a transportation network (link *C* in Figure 1) may improve both individual and overall welfare, or in other words, may represent a Pareto-improvement.

- In modern transport economics the Braess Paradox is sometimes used to illustrate this point (e.g. see Hagstrom and Abrams 2001; Johansson and Mattsson 1995, p. 24-25).

Braess's Paradox

- Now look at the Braess Paradox the other way round, that is, by assuming that link *C* does not exist and that the equilibrium flow distribution $R_1 = R_2 = 3$ prevails.
- Adding new infrastructure to the existing road network, that is, link *C*,
- now leads to the seemingly paradoxical situation that rational, payoff maximizing road users will adjust their route choice in a way that leads to the new equilibrium flow distribution $R_1 = R_2 = R_3 = 2$,

Braess's Paradox

- which is characterized by higher individual and overall travel costs in terms of time to get from node I to node IV.
- This is why Braess called it a paradox, but effectively it is simply a situation in which the resulting Nash User Equilibrium is not a Pareto-optimum.

Road Pricing Model

- We continue to assume that travel costs are additive and linear, but we now use a generalized cost function $K_i(\cdot)$ for links *A* to *E*:

- $$K_i(x_i) = \alpha_i + \beta_i x_i \qquad (6)$$

- with: $i = A, \ldots, E$; $x_i, \alpha_i \geq 0$; $\beta_i > 0$; and $x_i, \alpha_i, \beta_i \in IR$.

- Again, x_i denotes the units of flow on links *A* to *E* and it is assumed that units of flow are homogenous in all relevant aspects

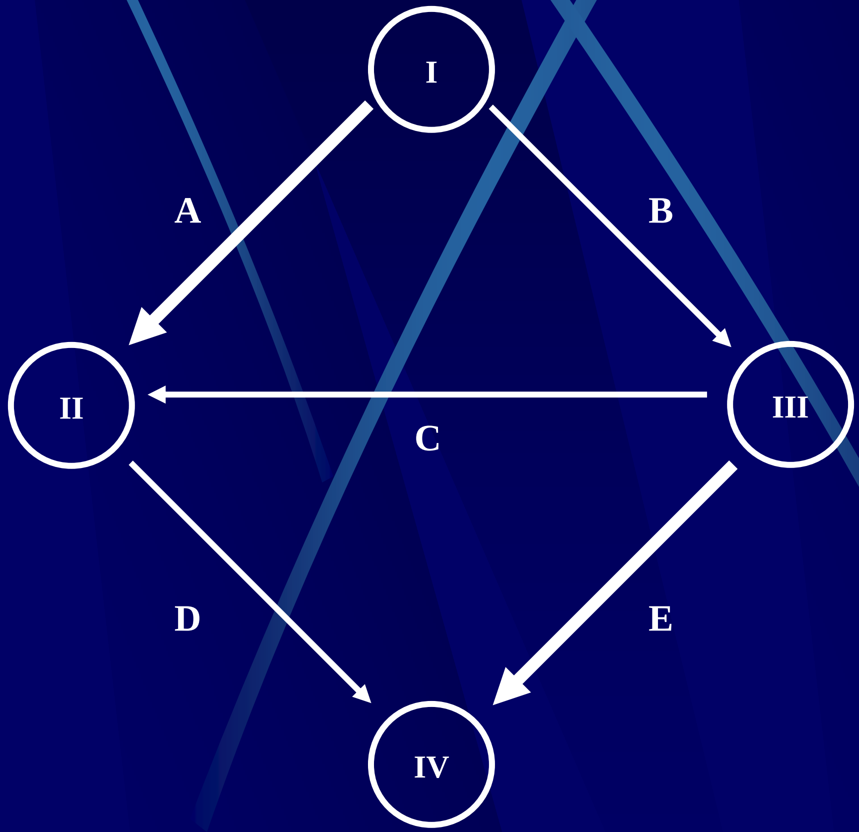
Road Pricing Model

- α_i represents a toll measured in monetary units which is due for using the *i*-th link,
- β_i represents a parameter that captures the impact of traffic flow intensity on travel costs with respect to the *i*-th link.
- The parameter β_i may in turn depend on a vector of parameters associated with the *i*-th link. Typically β_i would be measured in units of time, but for simplicity we assume that β_i is expressed in monetary units.

Road Pricing Model

- We now specify the transportation network shown in Figure 1 as a two-tier road transportation network,
- where links *A* and *E* represent motorways *M* and links *B*, *C* and *D* represent highways *H*.
- Next we assume that parameters α and β are identical on motorways and take the value, $\alpha_A = \alpha_E = \alpha_M$, and $\beta_A = \beta_E = \beta_M$.

Road Pricing Model



Road Pricing Model

- Likewise, we assume that parameter β is identical on highways and takes the value, $\beta_B = \beta_C = \beta_D = \beta_H$.
- The parameter α is set equal to zero on links *B* and *D*, $\alpha_B = \alpha_D = 0$, but may take positive values on the traverse link *C*, with $\alpha_C \geq 0$.
- Moreover, we define the units of flow or number of vehicles traveling on route R_j as x_j, with $j = 1, 2, 3$, and the total number of units of flow is *X*, with $X \geq x_i$ and $X = x_1+x_2+x_3$.

Road Pricing Model

- Based on equation (6) the travel costs associated with the three conceivable routes $R_{1\text{-}3}$ can then be expressed as a function of the links which each route involves:
-
- $K_{R1}(x_A, x_D) = \alpha_A + \alpha_D + \beta_A x_A + \beta_D x_D$ (7)
- $K_{R2}(x_B, x_E) = \alpha_B + \alpha_E + \beta_B x_B + \beta_E x_E$ (8)
- $K_{R3}(x_B, x_C, x_D) = \alpha_B + \alpha_C + \alpha_D + \beta_B x_B$
- $+ \beta_C x_C + \beta_D x_D$ (9)

Road Pricing Model

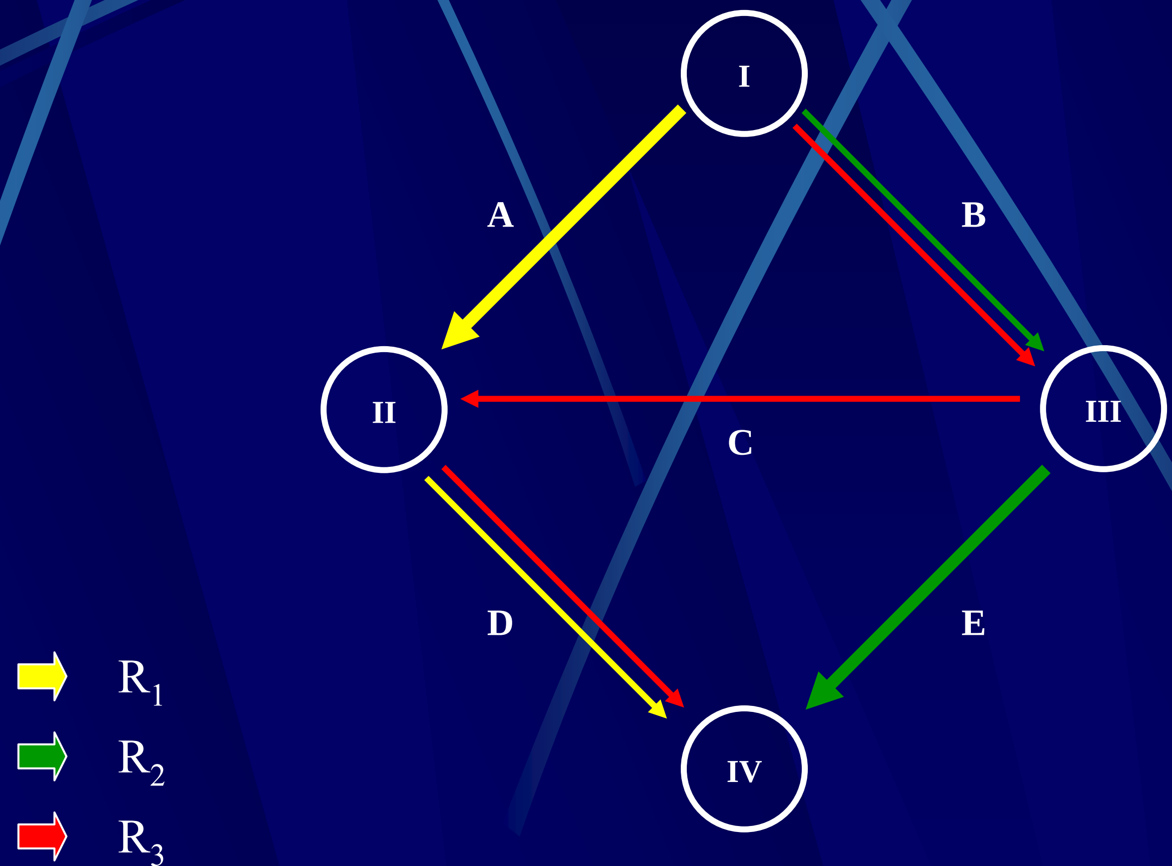
- Further, the assumptions made so far and the route definitions allow us to rewrite equations (7) to (9) in the following way:

- $$K_1(x_1, x_3) = \alpha_M + (\beta_M + \beta_H)x_1 + \beta_H x_3 \qquad (11)$$

$$K_2(x_2, x_3) = \alpha_M + (\beta_M + \beta_H)x_2 + \beta_H x_3 \qquad (12)$$

$$K_3(x_3) = \alpha_C + X\beta_H + 2\beta_H x_3 \qquad (13)$$

Road Pricing Model



Road Pricing Model

- Travel costs associated with using the *j*-th route now depend on the units of flow on a certain route
- thus, the specifications shown in (11) to (13) allow for analyzing endogenous road user behavior
- Finally, following Braess we assume that demand for transport services is given, or in other words, that *X* units of flow need to get from supply node I to demand node IV irrespectively of the values α and β may have.

Road Pricing Model

- Hence, in our setting this assumption implies that demand for transport services is perfectly price elastic.
- We also refrain from assuming a budget constraint for each unit of flow.
- These assumptions allow us to focus exclusively on route choice behavior.

Road Pricing Model

- Given the set of equations (11) to (13) and the assumptions made so far, four questions are of interest.

- What are the conditions for:
  - (i) a Nash User Equilibrium,
  - (ii) total cost minimum,
  - (iii) a revenue maximum, and
  - (iv) when do the former three conditions coincide?

Numerical Examples

Parameter values: X =10; β_H = 3; β_M = 1

- **Case 1**: No tolls, that is, $\alpha_C = \alpha_M = 0$

Allocation $x_1/x_2/x_3$	Values
5/5/0	Nash
5/5/0	200 (T-cost min.)

Numerical Examples

Parameter values: X =10; β_H = 3; β_M = 1

- **Case 2**: Toll on motorway, α_C=0f, α_M= 40

Allocation	Values
2/2/6	Nash
4/4/2	580 (T-cost min.)
(2/2/6)	660 (T-cost)
(2/2/6)	160 (revenue)

Numerical Examples

Parameter values: X =10; β_H = 3; β_M = 1

- **Case 3**: Toll* on motorway, α_C=0f, α_M= 30

Allocation	Values
3/3/4	Nash
4.5/4.5/1	495 (T-cost min.)
(3/3/4)	540 (T-cost)
(3/3/4)	180 (revenue)

Numerical Examples

- Case 2:

  excess burden = 660-200-160=300

- Case 3:

  excess burden = 540-200-180=160

Numerical Examples

Parameter values: X =10; β_H = 3; β_M = 1

- **Case 4**: Toll* highway C , $\alpha_C \geq$ 20, α_M=30f

Allocation	Values
5/5/0	Nash
5/5/0	500 (T-cost min.)
(5/5/0)	300 (revenue)

Numerical Examples

- Case 2:
  excess burden = 660-200-160=300

- Case 3:
  excess burden = 540-200-180=160

- Case 4:
  excess burden = 500-200-300=0

Numerical Examples

Parameter values: X =10; β_H = 3; β_M = 1

- **Case 5**: , α_C=0, α_M=10

Allocation	Values
5/5/0	Nash
5/5/0	300 (cost min.)
(5/5/0)	100 (revenue)

Numerical Examples

- Case 5:

  excess burden = 300-200-100=0

- Case 5 shows the maximal toll on motorways that does not cause an excess burden (Braess Paradox), if α_C is fixed and equal to zero.

Nash User Equilibrium

$$x_1 = \frac{\alpha_C - \alpha_M + 2X\beta_H}{(\beta_M + 3\beta_H)} \qquad (14)$$

- Values for x_2 and x_3 follow from any value calculated for x_1, according to $x_1 = x_2$ and $x_3 = X - 2x_1$.

Nash User Equilibrium

- For any set of given values of the parameters α, β, and X, a Nash User Equilibrium flow distribution exists and can be calculated from (14).

- That is, if $0 \leq x_1 \leq X/2$ holds, an interior solution results where in equilibrium all three routes are used.

Nash User Equilibrium

- Yet, if $x_1 > X/2$ holds, a corner solution results where in equilibrium only two routes are used, with $x_1 = x_2 = X/2$ and $x_3 = 0$.

$$X \leq \frac{2(\alpha_C - \alpha_M)}{(\beta_M - \beta_H)} \qquad (15)$$

Nash User Equilibrium

- Likewise, if $x_1 < 0$ holds, another corner solution results where in equilibrium only one route is used, with $x_1 = x_2 = 0$ and $x_3 = X$.

$$X \geq \frac{\alpha_C - \alpha_M}{-2\beta_H} \qquad (16)$$

Cost Minimum

- FOC for a minimum is:

$$\frac{\partial \hat{K}}{\partial x_1} = 4x_1(\beta_M + 3\beta_H) + 2(\alpha_M - \alpha_C - X4\beta_H) \quad (23)$$

- Setting (23) zero and rearranging yields:

$$x_1 = \frac{\alpha_C - \alpha_M + X4\beta_H}{2(\beta_M - 3\beta_H)} \quad (24)$$

Cost Minimum

- Equating (14) and (24) and rearranging yields:
- $$\alpha_M = \alpha_C \qquad (26)$$
- Hence, if (26) holds an interior solution emerges that represents both a Nash User Equilibrium and the minimum of total costs.
- Also, the two conceivable corner solutions will have similar properties.

Revenue Maximum

- Depending on the result for x_1 that is calculated from (14), three cases can be distinguished:

- *1.* $$\breve{x}_1 \le 0 \Rightarrow x_1 = 0 \Rightarrow G(\alpha_C) = \alpha_C X$$

- *2.* $$\breve{x}_1 \ge \frac{X}{2} \Rightarrow x_1 = \frac{X}{2} \Rightarrow G(\alpha_M) = \alpha_M X$$

Revenue Maximum

- *3.*

$$0 \le x_1 \le \frac{X}{2} \;\Rightarrow\; x_1 = \frac{\alpha_C - \alpha_M + 2\beta_H X}{(\beta_M + 3\beta_H)} \;\Rightarrow$$

$$G(\alpha_C, \alpha_M) = 2(\alpha_M - \alpha_C)\frac{\alpha_C - \alpha_M + 2\beta_H X}{(\beta_M + 3\beta_H)} + \alpha_C X$$

- *It follows from these three cases that in equilibrium G depends on just α_C and α_M.*

Revenue Maximum

- Then, if we continue to assume that demand for traffic services *X* is perfectly price elastic and assume that the network provider can set both α_C and α_M,

- the network provider simply has to set $\alpha_C = \alpha_M$ and all three cases coincide, with:

$$G(\alpha_M) = \alpha_M X$$

Revenue Maximum

- Hence, under these circumstances the network provider could set α_C and α_M infinitely high and the revenue maximum would approach infinity.

- Note that this case coincides with a vignette solution.

- Yet, this is an unrealistic setting. A more policy relevant setting emerges if the network provider can set either α_C or α_M, but not both!

Revenue Maximum

- Suppose that α_C, the toll on the traverse highway link is fixed, and that the network provider can freely set the toll on motorways, α_M.

- This case coincides with the 2005 introduction of a toll on German motorways, if $\alpha_C = 0$.

Revenue Maximum

- The condition for a revenue maximum now is:

$$\alpha_M = \beta_H X + \alpha_C \qquad (34)$$

- Substituting (34) into (14) and rearranging yields:

$$\breve{x}_1 = X \frac{\beta_H}{(\beta_M + 3\beta_H)} \qquad (35)$$

Revenue Maximum

- Because of β_H, β_M and $X > 0$, it follows that $x_1 > 0$ holds for any calculated from (35).
- Likewise, it can be shown that $x_1 \leq X/2$ holds for any calculated from (35), because rearranging yields $0 \leq \beta_H + \beta_M$, which is true as β_H, $\beta_M > 0$.
- Thus, if α_C is fixed, the revenue maximizing x_1 can be calculated from (34) and a Nash User Equilibrium emerges that represents an interior solution according to (14).

Revenue Maximum

- Now suppose that α_M, the toll on motorways is fixed, and that the network provider can freely set the toll on the traverse highway link, α_C.
- This case coincides with the current introduction of a toll on selected German highways, if $\alpha_M > 0$.

Revenue Maximum

- The condition for a revenue maximum now is:

$$\alpha_C = \alpha_M + \frac{X}{4}(\beta_M - \beta_H) \qquad (37)$$

- Substituting (37) into (14) and rearranging yields:

$$x_1 = \frac{X}{4}\frac{(\beta_M + 7\beta_H)}{(\beta_M + 3\beta_H)} \qquad (38)$$

Revenue Maximum

- Again, because of β_H, β_M and $X > 0$, it follows that $x_1 > 0$ holds for any calculated from (38).
- Likewise, if $\beta_H \leq \beta_M$, it can be shown that $x_1 \leq X/2$ holds for any x_1 calculated from (38), because rearranging yields $0 \leq \beta_M - \beta_H$, which is true as $\beta_H \leq \beta_M$ holds.
- Thus, if α_M is fixed and $\beta_H \leq \beta_M$ holds the revenue maximizing x_1 can be calculated from (37) and a Nash User Equilibrium emerges that represents an interior solution according to (14).

Revenue Maximum

- Yet, if $\beta_H \geq \beta_M$, it can be shown that $x_1 \geq X/2$ holds for any x_1 calculated from (38), because rearranging now yields $0 \leq \beta_H - \beta_M$, which is true as $\beta_H \geq \beta_M$ holds.
- Therefore, if α_M is fixed and $\beta_H \geq \beta_M$ holds, a Nash User Equilibrium would occur if (15) holds and for maximizing revenue it would be sufficient if x_1 satisfies (15), which is less strong then (37).

Revenue Maximum

- What is the condition for the maximum revenue that allows a Nash User Equilibrium to coincide with a total cost minimum?
- Again, policy relevant settings emerge if the network provider can set either α_C or α_M, but not both.

Revenue Maximum

- Now suppose that $\beta_H \geq \beta_M$ holds, that is, traveling on highways is more time consuming than on motorways:

- In this case, the minimum total cost Nash User Equilibrium is a corner solution, with $x_1 = x_2 = X/2$ and $x_3 = 0$,

- And the following conditions emerge:

Revenue Maximum

- $\alpha_M^* = \alpha_C + X/2\ (\beta_H - \beta_M)$ if α_C fixed (42)
- $\alpha_C^* = \alpha_M - X/2\ (\beta_H - \beta_M)$ if α_M fixed (43)
- Where α_M^* is the highest permissible α_M and α_C^* the lowest permissible α_C
- Alternatively, if $\beta_H < \beta_M$ holds, that is, traveling on motorways is strictly more time consuming than on highways, $\alpha_M = \alpha_C$, is always required

Policy Implications

- Road user behavior, in particular route choice behavior, should be incorporated in road pricing schemes

- Benevolent governments should be interested in avoiding a Braess Paradox and the associated excess burdens

- Relevant parameter values in (42) and (43) can be empirically determined

Policy Implications

- Essentially, for $\alpha_C = 0$, the maximum toll α_M^* is determined by the efficiency gap of the two road tiers (β_H - β_M)
- In other words, our version of the Braess Paradox shows that the road network provider should not charge more than the efficiency gap,
- which could be interpreted as an economic limit for road tolls in a multi tier road network

Policy Implications

- Also, our results show that a uniform toll on motorways is appropriate only if the values for the parameters *X*, β_H, and β_M are the same throughout the network.

- Otherwise, Braess Paradoxes may emerge locally.

Concluding Remarks

- Detecting a Braess Paradox a priori in large road networks is a rather difficult problem
- Yet, Hagstrom and Abrams (2001) have pioneered computational approaches for solving such problems
- A posteriori detection in a market fashion may serve as an alternative

Concluding Remarks

Thank You!