

Multiple Objects Auction with Non-Unitary Demand: an Experimental Analysis

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1 Introduction

Examples in the use of auction mechanisms are the planned biddings for highway concessions and, in the New Power Sector Model scenario, power auctions from new generation developments.

Within the scope of the Ministry of Transportation, the highway concession program determines that the highway concession must be carried out through bidding¹.

Eight highway segments shall be auctioned for a twenty-year period. The winner is chosen in two phases. In the first phase, qualified bidders make bids for the Basic Toll Tariff. In this one, the bidder submitting the lowest value is ranked as the best. In case there are bidders whose Basic Tariff proposals were sufficiently close to the best ranked bidder's (in this case, higher by up to 10%), they shall be entitled to take part in the second phase as long as they accept the tariff proposed by the best ranked bidder.

In the second phase, qualified bidders would compete for the concession through payment value proposals to obtain the concession granting. The winner would be the one who, upon accepting the lowest tariff proposed in the first phase, makes a higher bid in this phase. The Resolution that determines the bidding mode states that in the second phase the granting proposals should be presented in successive bids.

A key point in establishing the bidding environment is that in some cases the segments to be auctioned are adjacent or contiguous; i.e. connected segments shall be assigned. Furthermore, some of these concessions refer to segments connected to others operated by utilities whose shareholders are companies that may take part in current biddings. This means that such environment is made up by both synergies among the objects to bid or between these and others whose operation has already been assigned in the past. In this case, there is asymmetry among bidders.

The concession bidding of highway segments is an example of multiple heterogeneous objects auctions. A key element to increase the complexity in this environment is the fact that more than one object can be assigned to each bidder (in this case, the concession agreement). Furthermore, the environment is asymmetric since there are players operating licenses connected to others that shall be bid. In the presence of synergies, the winner of a component making up a certain basket of goods obtains advantages vis-à-vis the others. Such asymmetry may lead to revenue loss for the auc-

¹In compliance with Resolution No. 006/CND, of 06/07/2005, published on 06/08/2005. For references access the site of the Ministry of Transportation (www.transportes.gov.br).

tioneer if objects are sequentially auctioned. In this case, there is an income transfer from society, in general terms, to a private player thus impairing the reputation of the policy maker².

In the existence of complementarities among objects, i.e. if the items are not independent, the task of bids submission becomes difficult as a result of concession assignment by means of a sequence of auctions. Therefore, it is worth investigating auction formats that allow players to submit bids that not only embody such synergies, but also attract bidders.

It is a fact known in the auction literature³ that the auction format affects the result both in terms of efficiency (capacity of assigning the object to the player who best values it) and revenue collection or cost reduction (in case of descending auctions). Therefore, the environment configuration allows recommendations regarding auction formats that best fit a specific context. However, such recommendations might have a greater implementation probability if they could be illustrated through situations able to reproduce the incentives existing in the environment under analysis.

This study reports the experimental analysis of alternative auction mechanisms, which means the electronic implementation of alternative formats of multiple objects auctions. The group of implemented auctions is the result of theoretical studies as well as the assessment of auction experience.

2 Experiment

The experiment carries out the comparative investigation of two mechanisms subject to use in the allocation of multiple heterogeneous objects: a hybrid auction of a single object used sequentially implemented and a simultaneous auction of multiple rounds. The performance of the said mechanisms is described below.

In each session, a group of I players, $i = 1, \dots, I$, competes for a group of K heterogeneous goods. The objects are auctioned either sequentially or simultaneously. For the sake of comparison, the number of objects to be auctioned equals eight. For each license taken individually, each bidder attributes a private value independently distributed as follows:

²Paul Klemplerer reports examples of flaws in auction design harming the reputation of economic policy makers. For further references, see Paul Klemplerer, "Auction: Theory and Practice", Princeton University Press, 2003.

³For further references, see Paul Milgrom, "Putting Auction Theory to Work", Cambridge University Press, 2003 and Flavio M. Menezes and Paulo K. Monteiro, "An Introduction to Auction Theory", Oxford University Press, 2004.

$$v_i(l_k) \sim U[\underline{v}_k, \bar{v}_k].$$

This piece of information is common knowledge at the beginning of the experimental session.

The design envisages the existence of scope economies or synergies between subsets of objects. The existence of positive synergies for a given subset of commodities means that to some pre-defined baskets the value of the joint possession of objects is higher than the sum of values separately assigned to the objects. Let $S \subseteq K$ be a subset of licenses presenting positive synergies. Without loss of generality, assume that for the entire subset $s \in S$,

$$v_i\left(\left(\bigcup_{k \in s} l_k\right)\right) = \alpha_s \left(\sum_{k \in s} v_i(l_k)\right) \geq \sum_{j=1}^k v_i(l_j) \quad \forall l_k \in s \quad (1)$$

where $v_i(l_1, \dots, l_j)$ is bidder i 's value for the basket (l_1, \dots, l_j) and $\alpha_s > 1$.

2.1 Treatments

The study tests two alternative mechanisms: a sequential auction and a simultaneous ascending auction.

2.1.1 Sequential Auction (Sequence of hybrid auctions for a single object)

In this treatment, the licenses are sequentially auctioned through an auction format similar to the one used in the Brazilian privatization process⁴. This is a sequential auction where one object is assigned in each auction. One can think of the object either as a license for the concession of a highway segment or as the right to explore a power generation development.

All players (potential buyers) submit sealed bids for a given object. After the opening of bids, if there are players whose bids are sufficiently close to the highest bid, the ones qualified to take part in the second phase will be the bidder with the highest bid and those bidders whose bids are higher than or equal to τ times the value of the highest bid. In the experiment $\tau = 0,9$ ⁵.

⁴For further references, see Dutra, J. and F.M. Menezes, "Hybrid Auctions", Economics Letters 77, 301-307, 2002.

⁵This value corresponds to the Ministry of Transportation proposal for highway concession biddings.

Thus, if there are bids that differ from the winning bid by less than 10%, the qualified bidders compete for the object in an ascending auction. In this auction, the reserve price, or the lowest admissible bid, is the highest bid submitted in the first phase. In order to take part in the second phase, qualified bidders should be willing to honor the payment represented by the highest bid of the first phase.

The second phase consists of an ascending auction⁶. In this auction, at each period the current price is equal to the current price of the preceding period plus a minimum increment denoted by δ^{\min} . Bidders may accept or reject the current price; if they reject it, they will be out of the bidding. The default situation is to accept the bid. The auction ends with one remaining player who is then declared the winner. The price to be paid is the price to which the next-to-last player leaves the auction.

2.1.2 Simultaneous Ascending Auction

Besides the sequential auction, in a second treatment the K licenses are allocated through a simultaneous ascending auction.

Simultaneous auctions have been championed and extensively used to assign commodities in cases where the values assigned by bidders are not independent. Although they might be susceptible to exposure problems (example), the relative simplicity of the bids formulation process makes this auction format proper for scenarios where it is desirable that players have an opportunity to reach a desired object aggregation.

In the auction⁷, the bids by players at each period must meet the budget constraint, i.e. the sum of their bids for the set of commodities they are competing for, $\sum_{k=1}^K b_{i(l_k)}^t$, cannot exceed the initially assigned income. Denote by \bar{w}_i^0 the initial endowment of the i -th player. Its budget constraint is given by:

$$\sum_{k=1}^K b_{i(l_k)}^t \leq \bar{w}_i^0, \quad i = 1, \dots, I; \quad \forall t \quad (2)$$

In order to avoid demand reduction (*insert footnote*), it is common practice to adopt an activity rule. In this case, the rule is a monotonicity

⁶For references on auctions, see F.M. Menezes and P.K. Monteiro “An Introduction to Auction Theory”, Oxford University Press, 2004.

⁷This restriction applies to the sequential auction as well.

requirement in the number of objects for which the bidder competes. Let N_i^t be the number of elements in the set of objects for which the i -th player submitted a bid in period t . Then the monotonicity rule requires that

$$N_i^t \leq N_i^{t-1} \quad \forall t \quad (3)$$

an i player who is said to be active for a set of licenses cannot increase the number of licenses to which the bid is submitted. Furthermore, at each new period only the bids meeting the following condition shall be accepted

$$b_{i,k}^t \geq \max_i \left\{ b_{i,k}^{t-1} \right\} + \delta_k^{\min}, \quad k = 1, \dots, K; \quad i = 1, \dots, I \quad (4)$$

i.e. price bids (in this case, granting value) must be non-decreasing. However, the bidder submitting the highest bid in a t round does not have to propose a higher bid in the subsequent round. In this case, at the beginning of the $(t + 1)$ -th round this one should be considered as active.

Conditions (2) – (4) must be met at each round so that player i may continue taking part in the auction.

The end of the auction is simultaneous; i.e. the auction continues as long as bids are still being submitted by at least one player for at least one object. That means the auction goes on while there is a price change for at least one product. For each object, the winner would be the bidder who would have submitted the highest bid at the moment the auction is over; prices to be paid will be the ones in force.

2.2 Auction Rules

Before the beginning of the auction, each bidder is informed of:

- the values assigned to each commodity;
- the values associated with pre-established combinations for which there are positive synergies.

2.2.1 Private Values

The first set of auctions is characterized by private values: for each object bidder i 's value is a number extracted with equal probability of a distribution that is of common knowledge.

In the case of the experimental design, it was established the existence of synergies for licenses 3, 4 and 5 as well as for the 7 and 8 combination;

the following valuations for commodities were experimentally implemented

^{8,9},

$$\begin{aligned}
 v_i \left(\bigcup_{k=3}^5 l_k \right) &= 1.44 \cdot \sum_{k=3}^5 v_i(l_k) \\
 v_i \left(\bigcup_{k=7}^8 l_k \right) &= 1.2 \cdot \sum_{k=7}^8 v_i(l_k)
 \end{aligned} \tag{5}$$

In 11 auctions the values assigned to the objects by bidders were private ones. In this case, all values were extracted with equal probability from the [30, 80] interval and such fact was common knowledge.

2.2.2 Almost-Common Values

In this second set of experimental sessions, a different form of assigning value to commodities was implemented. This structure describes the case where the value of the object has a common component for all bidders and this value is not known at the moment bids are submitted. The auctions motivating the current analysis are part of this group; i.e. there is a common component associated with the value of the object that is being sold.

For sessions numbered 12 to 17, players' values for the objects by were made up of two parts. In each auction, the value that participant i assigns to the k -th commodity, $V_{i,k}$, is equal to the sum of a common component (c_k), which is equal to all bidders, and of a private value ($x_{i,k}$), as follows:

$$V_{i,k} = x_{i,k} + c_k$$

The value of the common component of each commodity, c_k , $k = 1, \dots, 8$, is a number extracted with equal probability from the interval $[\underline{c}_k, \bar{c}_k]$; however, such value is not known until the end of the auction. The bidder observes only a signal, s_k , which can be understood as an estimate of the value of this common component; for each commodity, this signal $s_{i,k}$ is uniformly distributed in the interval $[c_k - z, c_k + z]$ and this is common

⁸The implemented values are based on a SEAE/MF Technical Note that reports estimates of synergies.

⁹An extension of the study leads to the analysis of the case of a group of players who hold a D license previously assigned to a \hat{i} participant in the auction so that $v_{\hat{i}}(C + D) > v_{\hat{i}}(C) + v_{\hat{i}}(D)$. This example characterizes the presence of asymmetries among players.

knowledge The private signal $x_{i,k}$ is a number uniformly distributed in the $[\underline{x}_k, \bar{x}_k]$ interval. Note that positive synergies for referred sets are still valid.

TESTABLE HYPOTHESIS: In the presence of positive synergies, the simultaneous auction allows bidders with a higher probability to benefit from such synergies, thus assuring higher efficiency at the auction and higher revenue for the auctioneer.

2.2.3 Payoffs

Besides observing the information concerning values for the goods, at the beginning of the auction each bidder observes the initial prices of each object. Upon such information, the auction gets started.

In each experimental session, the players' earnings are made up of a participation fee, in the form of a fixed rate, plus their decision gains throughout the session. For each acquired commodity, the winner's gain is given by:

$$v_i(l_k) - p_k = v_i(l_k) - \max_{i \in I} \{b_{i,k}^t\}$$

where $b_{i,k}^t = b_i^t(l_k)$. The non-winning bidders earn nothing. Total gains in the session are equal to gains for the sum of obtained licenses, net of paid prices.

If the bidder managed to add the corresponding licenses to pre-established synergies, his gain for said combination is equal to the value for the combination, which is given by (1), net of the price paid for the combination, which is equal to the sum of paid prices. By way of illustration, if participant 1 won licenses 3, 4 and 5 his gain, π_1 , would be equal to:

$$\begin{aligned} \pi_1(l_3, l_4, l_5) &= v_1(l_3, l_4, l_5) - (p_3 + p_4 + p_5) \\ &= 1.44(v_{3,1} + v_{4,1} + v_{5,1}) - (p_3 + p_4 + p_5) \end{aligned}$$

At the end of the experimental session, the payment is made in cash. two different ways.

3 Results

Aggregated Results

Seventeen experimental sessions have been carried out. Out of these, 8 sessions consist of sequential auctions while the remaining ones are simultaneous auctions. In each auction, a group of bidders ranging from six to

eight competes for the ownership of eight commodities. As a whole, 136 commodities were auctioned, which stood for licenses.

The values assigned by bidders to the objects were private ones in 88 auctions. In this case, all values were extracted with equal probability from the $[30, 80]$ interval and such fact was of common knowledge.

For auctions numbered 89 to 136, in turn, the values assigned by players to the objects were made up of two parts (almost-common values); in each auction, the value that bidder i assigns to the k -th commodity, $V_{i,k}$, is equal to the sum of a common value component (c_k) and a private value ($x_{i,k}$) as follows:

$$V_{i,k} = x_{i,k} + c_k$$

The common value component of each commodity, c_k , $i = 1, \dots, 8$, is a number between 30 and 80, drawn with equal probability; however, this value is not observed by i . The bidder knows only a signal, $s_{i,k}$, which can be understood as a value estimate of this common component. For each commodity, this signal, $s_{i,k}$, is uniformly distributed in the $[c_k - 10, c_k + 10]$ interval. On the other hand, the private signal $x_{i,k}$ is a uniformly distributed number in the $[0, 30]$ interval.

Table 1 summarizes the results of the experimental design. The following conclusions can be drawn from said data:

- The mean efficiency of the simultaneous auction is relatively higher than that of the sequential auction;
- The revenue obtained with the implementation of the simultaneous auction is higher;
- The number of inefficient assignments (number of times in which the winner of the auction was a player who would not present the highest value for the commodity) is lower in the simultaneous auction;
- Mean gains of bidders in the simultaneous auction are comparatively higher than those in the sequential auction.

Table 1: Experimental Results (Mean per Treatment)

| Mechanism | Values | Profit/Potential Value ¹ | Value Appropriation ² |
|--------------|---------------|-------------------------------------|----------------------------------|
| Sequential | Private | - 4% | 92% |
| Sequential | Almost Common | 8% | 97% |
| Simultaneous | Private | 3% | 97% |
| Simultaneous | Almost Common | 2% | 99% |

Notes: (1) Bidders' mean profit as proportion of the maximum possible value;
(2) Value realized as proportion of the maximum possible value.

According to results, the simultaneous auction is higher both in terms of efficiency and revenue guarantee (without necessarily resulting in losses to bidders).

This is the conclusion of the experimental study carried out in an environment meant to reproduce a group of characteristics found in the scenario of highway concessions auctions forecast for the second semester of 2005.

3.1 Individual Behavior

Analysis results of bidders' individual behavior are presented below. Table 2 shows data on the bids as proportion of players' values in the auction. It is inferred that players are more aggressive in the case of simultaneous auctions and such behavior is more intense the higher the synergy degree among goods. As described in the experimental design, the absence of synergy refers to goods 1, 2 and 6; while the weak and strong synergies refer to baskets (l_7, l_8) and (l_3, l_4, l_5) respectively.

Table 2: Bid Behavior (Bid as value proportion).

| Mechanism | Private Values | | Almost-Common Values | | General Mean | |
|-----------|----------------|--------------|----------------------|--------------|--------------|--------------|
| | Sequential | Simultaneous | Sequential | Simultaneous | Sequential | Simultaneous |
| Absent | 0.93 | 0.84 | 0.91 | 0.74 | 0.93 | 0.81 |
| Weak | 0.88 | 0.97 | 0.95 | 1.03 | 0.94 | 0.99 |
| High | 1.02 | 1.21 | 1.00 | 1.23 | 1.03 | 1.22 |

A more aggressive bidding behavior is favored by the auctioneer's once it allows a higher appropriation of the value bidders assign to the objects. In the case of highway concessions this means a higher rent extraction in the form of an increase in the collected revenue (as granting). Table 3 shows

the percentage variation of bids as value proportion relatively to the case in which goods do not present synergy. Data denote an advantage of the simultaneous auction over the sequential one; an advantage that is even more stressed in the case of almost-common values.

Table 3: Bid as value proportion (% Variation).

| Mechanism | Private Values | | Almost-Common Values | | General Mean | |
|-----------|----------------|--------------|----------------------|--------------|--------------|--------------|
| | Sequential | Simultaneous | Sequential | Simultaneous | Sequential | Simultaneous |
| Weak | -5.00 | 13.00 | 4.00 | 29.00 | 1.00 | 18.00 |
| High | 9.68 | 44.05 | 9.89 | 66.22 | 10.75 | 50.62 |

Base: Absence of synergy

Mean equality tests for the case of private values are shown in Table 4. At a 5% level, means are statistically different for goods 1 and 2 (absence of synergy) and for good 4.

Table 4: Means Difference Test – Private Values

| Good | Mean per Mechanism | | |
|--------|--------------------|--------------|----------|
| | Sequential | Simultaneous | p- value |
| Good 1 | 0.978 | 0.836 | 0.03 |
| Good 2 | 0.980 | 0.841 | 0.00 |
| Good 3 | 1.194 | 1.251 | 0.61 |
| Good 4 | 1.009 | 1.279 | 0.02 |
| Good 5 | 0.954 | 1.113 | 0.16 |
| Good 6 | 0.871 | 0.833 | 0.65 |
| Good 7 | 0.990 | 0.992 | 0.98 |
| Good 8 | 0.875 | 0.955 | 0.34 |

In the case of almost-common values, data reported in Table 5 show an advantage of the sequential auction in the case of no synergy (Goods 1 and 6). On its turn, at the 5% significance level, the simultaneous auction reveals a more aggressive bidding behavior under strong synergy. Note that in the case of Good 3, said means do not differ under the statistical viewpoint. Such behavior is expected considering that in the sequential auction players show more aggressiveness in the auction of the first object of the basket. This strategy ends up by reducing the participation of other players in subsequent auctions (of basket objects) since only the winner of commodity 3 may then carry out the involved synergies¹⁰. Figure 2 shows these arguments..

¹⁰For further references, see (theoretical reasoning).

Table 5: Means Difference Test – Almost-Common Values

| Mean per Mechanism | | | |
|--------------------|------------|--------------|----------|
| Good | Sequential | Simultaneous | p- value |
| Good 1 | 0.894 | 0.658 | 0.00 |
| Good 2 | 0.878 | 0.799 | 0.33 |
| Good 3 | 1.109 | 1.202 | 0.56 |
| Good 4 | 0.949 | 1.232 | 0.02 |
| Good 5 | 0.950 | 1.262 | 0.01 |
| Good 6 | 0.935 | 0.734 | 0.00 |
| Good 7 | 0.996 | 1.018 | 0.77 |
| Good 8 | 0.947 | 1.087 | 0.19 |

The analysis of individual data reveals a comparative edge of the simultaneous auction format vis-à-vis the sequential one. In the presence of positive synergies, bids by players in the simultaneous auction reflect the increase of the value associated with the ownership of pre-established combinations of commodities. In the sequential auction, in turn, the uncertainty as to the capacity of reaching the desired aggregation of commodities compel players to adopt a more conservative behavior in which joint ownership is not priced. Figures 1 and 2 show these observations.

Finally, a regression analysis of bids is shown in Tables A1, A.2 and A.3. The inclusion of several interaction dummies between the simultaneous mechanism and goods confirm the presented arguments especially in the case of almost-common values (Table A.2 attached). Such evidence takes on greater importance taking into account that this value structure is closer to reality.

4 Conclusions

The National Privatization Council determined that the concession for a group of eight highway segments should be carried out through bidding. Resolution No. 006/05 establishes that the auction should take place in two phases. In the first one, a competition occurs by means of the Basic Toll Tariff. In case of bidders with proposals sufficiently close to the lowest proposed tariff, the competition goes on to a second phase. In this phase, players submit a granting value proposal. The winner of the auction is the player who, upon acceptance of the lowest Basic Tariff, offers a payment of a higher granting value.

The mechanism initially proposed by the Ministry of Transportation for the bidding is a sequential auction: the concessions would be allocated in a pre-determined order. However, two elements provide uniqueness to the concession environment. Firstly, the bidding involves some segments that are connected or close to others that have already been bid. Estimates by the Ministry of Finance report the existence of considerable scale economies in the joint operation of connected segments. Furthermore, some involved segments are close to others that were the object of concessions in the past. In this case, there is asymmetry among bidders as such companies already holding concession agreements can operate the segments under more favorable conditions (at lower cost).

Both elements presented herein justify the comparison of the auction format proposed by the Ministry of Transportation with a simultaneous auction. This comparison was carried out in an experimental environment developed to reproduce conditions similar to the ones presented. The results confirm a superiority of the simultaneous auction whenever there are positive synergies: it was possible to simplify the bid submission task, thus obtaining higher revenue (in the granting phase) and higher efficiency (choosing the bidder who assigns a higher value to the object).

A Appendix

Outcomes from estimated regressions aiming to assess the determinants of players' bids in the auction are presented below.

Table A.1 - Bid Determinants – Private Values

| Variables | Coefficient | p- value |
|------------------------|-------------|----------|
| Private Value | 0.96 | 0.00 |
| Mechanism | -5.72 | 0.03 |
| Good 2 | -0.10 | 0.93 |
| Good 3 | 12.73* | 0.00 |
| Good 4 | 3.07 | 0.39 |
| Good 5 | -0.87 | 0.87 |
| Good 6 | -4.64* | 0.03 |
| Good 7 | 1.83 | 0.64 |
| Good 8 | -4.05 | 0.13 |
| Mechanism *Good 2 | -2.17 | 0.49 |
| Mechanism *Good 3 | 5.32 | 0.35 |
| Mechanism *Good 4 | 14.70* | 0.01 |
| Mechanism *Good 5 | 13.84* | 0.04 |
| Mechanism *Good 6 | 1.51 | 0.73 |
| Mechanism *Good 7 | 3.96 | 0.47 |
| Mechanism *Good 8 | 9.03* | 0.04 |
| Constant | 1.10 | 0.73 |
| Number of Observations | 624 | |
| F(16, 607) | 32.34 | |
| Prob > F | 0.00 | |
| R ² | 0.41 | |

Notes: (1) Mechanism is a dummy with value 1 if the auction is simultaneous;
(2) ben_i is a dummy variable with value 1 for the i -th good, $i = 2, \dots, 8$;
(3) $Mechanism * good_i$ is the interaction of the i -th good with the mechanism.

Table A.2 - Bid Determinants – Almost-Common Values

| Variables | Coefficient | p- value |
|------------------------|-------------|----------|
| Private Value | 1.30* | 0.00 |
| Mechanism | -24.62* | 0.00 |
| Good 2 | 0.37 | 0.92 |
| Good 3 | 18.92* | 0.00 |
| Good 4 | 7.04 | 0.14 |
| Good 5 | 7.92 | 0.06 |
| Good 6 | 8.87* | 0.03 |
| Good 7 | 6.05 | 0.31 |
| Good 8 | 3.66 | 0.50 |
| Mechanism *Good 2 | 6.19 | 0.40 |
| Mechanism *Good 3 | 18.51* | 0.05 |
| Mechanism *Good 4 | 30.76* | 0.00 |
| Mechanism *Good 5 | 30.32* | 0.00 |
| Mechanism *Good 6 | -2.74 | 0.65 |
| Mechanism *Good 7 | 14.98 | 0.06 |
| Mechanism *Good 8 | 18.11* | 0.02 |
| Constant | 49.96* | 0.00 |
| Number of Observations | 288 | |
| F(16, 271) | 17 | |
| Prob > F | 0.00 | |
| R ² | 0.44 | |

Notes: (1) Mechanism is a dummy with value 1 if the auction is simultaneous;
(2) bem_i is a dummy variable with value 1 for the i -th good, $i = 2, \dots, 8$;
(3) $Mechanism * good_i$ is the interaction of the i -th good with the mechanism.

Tabela A.3. Bidding Determinants

| Variables | Coefficient | p- value |
|------------------------|-------------|----------|
| Private Value | 0.05 | 0.10 |
| Mechanism | -15.74* | 0.00 |
| Good 2 | -1.77 | 0.58 |
| Good 3 | 12.72* | 0.01 |
| Good 4 | 5.90 | 0.15 |
| Good 5 | -1.86 | 0.70 |
| Good 6 | -2.21 | 0.55 |
| Good 7 | 0.50 | 0.91 |
| Good 8 | -5.83 | 0.13 |
| Mechanism *Good 2 | 5.03 | 0.31 |
| Mechanism *Good 3 | 12.89* | 0.04 |
| Mechanism *Good 4 | 18.43* | 0.00 |
| Mechanism *Good 5 | 26.87* | 0.00 |
| Mechanism *Good 6 | 3.42 | 0.53 |
| Mechanism *Good 7 | 12.16* | 0.04 |
| Mechanism *Good 8 | 18.18* | 0.00 |
| Constant | 58.60* | 0.00 |
| Number of Observations | 912 | |
| F(16, 895) | 7.97 | |
| Prob > F | 0.00 | |
| R ² | 0.12 | |

Notes: (1) Mechanism is a dummy with value 1 if the auction is simultaneous;
(2) bem_i is a dummy variable with value 1 for the i -th good, $i = 2, \dots, 8$;
(3) $Mechanism * good_i$ is the interaction of the i -th good with the mechanism.

The results of Table A.3 show that, in the presence of synergies the mechanism-good interaction presents a significant positive effect from a statistical viewpoint. Asterisks indicate statistically significant parameters at 5% level.

Figures 1 and 2 present bid data as a proportion of the value the bidders assign to the objects in cases of private and almost-common values respectively.

