Endogenous risk in public-private partnerships

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Abstract

This study characterises the optimal public-private partnership for investment and operation of an essential utility, when market risk and synergies between building and operating activities coexist. Relying on real options techniques, we show that the firm does not properly internalise the synergies between activities when it expects to renegotiate whenever the market falls and continuing the operation becomes unprofitable. We find that the effort decision depends on the relative importance of the positive externality with respect to both expectation and volatility of the market. Furthermore, in contrast with Jensen and Meckling (1976), our results reveal that, as the renegotiation decision is endogenous to the initial effort decision, equity financing may be preferable to debt financing.

Keywords: Stochastic demand; public-private partnership; renegotiation

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1 Introduction

Essential facilities, such as infrastructures for long-distance transportation services, involve significant investment costs. Under some circumstances, public transfers, which are ultimately financed by taxpayers, can be avoided if the government awards exclusive franchising contracts to private consortia that are willing to invest. One may think about concession contracts for highways, or Eurotunnel project and other recent private financing in railways.

The literature about public-private partnerships (hereafter, PPPs) has examined the opportunity of private involvement in essential facility projects. However, nowadays, the optimal allocation of activities between public and private sector is largely debated. In practice, European governments involve private firms in infrastructure activities without a clear understanding of the associated long-term implications. The key feature of the debate is that building and operating essential facilities involve increasing returns technology. Competition on the market is unfeasible and market operation needs be "controlled" by public authorities.

There are two related directions in the literature about financing essential facilities, with a serious gap in between. The first one focuses on the market side. Some studies discuss how the downstream market should be regulated for the firm to make optimal investment decisions (see Caillaud and Tirole, 2004). Others (the PPP literature) focus on the optimal contracting in the presence of market risk, which induces renegotiation. Relevant examples are given by concession contracts in highway infrastructures (see Engel, Fischer and Galetovic, 1997, 2001, 2003; Guasch et Al, 2003). It is noteworthy that, in all such studies, the risk of investment does not depend on firm's actions.

The second domain includes works about the optimal PPP organization, in presence of synergies between different stages of an essential facility project. Suggestive studies are Hart (2002) and Bennet and Iossa (2002). Optimality depends on the importance of the synergies between building and operating activities in the presence of non-contractable effort at the infrastructure building stage.

Observe that, in practice, the governments involve private firms in infrastructure activities for financial considerations. Private consortia make investments by using private funds and subsequently recover the initial cost from market returns. This may make private consortia performing such activities socially desirable, even in the absence of synergies between building and operating. However, in case the market falls significantly in the future, the firm can credibly threat to renounce to the essential activity in which case renegotiation and public transfers may materialize. Therefore, the advantage of private involvement in such projects is in that public transfers can be *delayed*. On the other hand, if synergies between activities exist, the firm's effort may reduce the risk of public subsidies since the operating flows from the project are expected to be higher.

The arguments above suggest that the optimal organization of a PPP is exposed to market risks. Henceforth, a clear link between market and organizational perspective exists, which the literature, so far, has not precisely identified. The aim of the present study is to combine the two orientations we have been illustrating. More precisely, we examine the optimal financing of PPP projects, taking into account that both the positive synergies between building and operating activities (the organizational perspective) and the exogenous risk (the market perspective) coexist. The latter is common knowledge, whereas the former depends on the firm's effort.

As for the methodology, we rely upon the real option techniques. This is appropriate because the firm has the option to reduce or not the volatility of the financial risk while exerting unobservable effort according to the contract it receives. Moreover, the firm has the option to renegotiate whenever the market falls and continuing the activity becomes unprofitable. The government has an option to delay the provision of public funds by involving a private firm in contracting. When a sufficiently low market demand is realized and the firm credibly threats to renounce to activity, the government decides either to accept a contract renegotiation or to expropriate the firm's investment. The social consequence is the same in any of the two cases: public transfers take place in the industry *ex post*. The optimal decision at the evaluation stage of the project is to offer the firm a rent that reduces the possibility of such subsidies. Thus, the option value of the government trades-off the *ex-ante* rent against the risk of providing higher subsidies *ex post*. The amount of the rent that is given up to the firm crucially depends on the relative importance of the positive externality between the building and operating activities as compared to the (exogenous) market risk of the project.

In the context described above, we examine the optimal capital structure of the firm that finances the project. Jensen and Meckling (1976) and a few subsequent studies argue that debt financing is always preferable to equity financing at inducing the firm to exert optimal effort at the building stage. Our results contrast with their findings because we add the renegotiation issue to the moral hazard problem. In our framework, the firm has an incentive to *postpone* renegotiation whenever the risk is shared with external equity holders. Consequently, the firm better internalizes the positive externality in exerting effort when the project is equity financed, rather than debt financed. In this perspective, an interesting connection can be identified between PPP studies and financial studies *à la* Leland (1994) about the incentives of the firm to undertake more or less risky projects, according to the way the risk is shared between equity holders and debtors.

The major contribution of the present work resides in that it provides important insights about the reasons why many governments rely upon PPPs and, at the same time, it sheds light on the drawbacks which show up in terms of implementation.

The paper is organized as follows. Section 2 presents the market, the technological context of the essential facility project and the payoff functions. In Section 3, the no renegotiation benchmark is presented, as similar to the previous literature of PPP about the optimal organization. In Section 4, we examine the problem of the firm that exerts all the activities in the essential facility project. The issue of optimal leverage of the firm that is socially optimal is explored in Section 5. The last Section summarizes the results and concludes.

2 The market demand, the firm and the payoffs

The government offers to a firm a monopoly franchising contract for building and operating the transportation service. We are interested in understanding the optimal public-private partnership in an uncertain market. We assume without loss of generality that the contract should last over the entire life of the asset. However, the contract ends within a finite period whenever the firm decides to renounce to the activity. By contract end we mean either renegotiation or expropriation of the firm. What matters is that starting from this period the government subsidizes the subsequent losses in operating the infrastructure.

2.1 The market demand

We assume that the tastes of consumers are represented by a stochastic variable y_t on time interval $t \in [0, \infty)$. Uncertainty is represented by a complete probability space $(\Omega, \mathcal{F}, P)$. The demand y_t follows a geometric Brownian motion with drift $\alpha > 0$ and volatility $\sigma \geq 0$, so that

$$dy_t = \alpha y_t dt + \sigma y_t dz_t. \tag{1}$$

y_0 is the value it takes at the current date $t_0 = 0$, while z_t is a simple Brownian motion defined on $(\Omega, \mathcal{F}, P)$. The flow of information on which decisions are based is expressed by the filtration $(\mathcal{F}_t)_{t \geq 0} = (\sigma \{y_t | v \leq t\})_{t \geq 0}$ generated by $(y_t)_{t \geq 0}$.

We note that the demand evolution defined in (1) is exogenous to the industry which means that it cannot be controlled through price regulation[1].

2.2 The firm

Building the infrastructure requires for an initial cost I at $t = 0$. If the firm does not allocate own resources in its activity so that it is entirely debt financed, the cost

[1] This assumption can be relaxed to the situation in which exogenous and endogenous demand coexist, but this generalisation is beyond the scope of our analysis.

of operation is

$$C(e) = rI + C - \delta e \tag{2}$$

rI is the rate of debt to be paid to the creditor during each period of operation, provided that r, such that $0 \leq r \leq 1$ is the risk free interest rate in the economy. $c - \delta e$ is the maintenance cost of infrastructure. $c > 0$ is the fixed part of the maintenance cost (independent of firm's action), while $\delta > 0$ is the slope of cost reduction in the initial effort for the quality of infrastructure. This is the positive externality between building and operation activity from which bundling the two phases of the project in one consortium is preferable to separating them. For simplicity, we denote in what follows by C^d the expression $C^d = rI + C$ so that in case the firm is debt financed,

$$C(e) = C^d - \delta e$$

In case the firm is equity financed, so that the firm keeps a share ν from its activity, the operational cost reduces to

$$C^{\nu}(e) = C - \delta e.$$

If not otherwise stated, we assume in what follows that the activity is debt financed.

The effort e costs the firm $\psi(e)$. We assume for simplicity that the effort can take two possible states, which are $e = 0$ and $e = 1$, so that $\psi(0) = 0$ and $\psi(1) = \psi$, where $\psi > 0$.

2.3 The payoffs

The payoff of the firm, as derived in Appendix A.1 is

$$V(y/e) = \frac{y}{r-\alpha} - \frac{C(e)}{r} - \psi(e) - \left(\frac{y}{y_b}\right)^{\beta_2} \left(\frac{y_b}{r-\alpha} - \frac{C(e)}{r}\right), \tag{3}$$

under the condition that the initial demand is high enough, $y > y_b$, so that the project is undertaken.

β_2 in (3) is the negative root of the quadratic equation $\beta(\beta-1)\frac{\sigma^2}{2} + \alpha\beta = r$ (our notation follows the one of Dixit and Pyndick, 1994). It is essential for our subsequent analysis to remark that β_2 is very low when the volatility σ of the demand is low while it tends to zero when σ becomes very large.

Let us explain now the various terms of (3). $y/(r-\alpha) - C(e)/r$ is the discounted flow of expected net market revenues, if the firm does not renounce to the activity before the end of the contract duration. $y/(r-\alpha)$ are expected market revenues (that are dynamic) as evaluated at current demand on the market $y_0 = y$. $C(e)/r$ are the expected flows of (constant) operational costs, discounted with the risk-free rate of return r. By exerting effort, the firm bears the disutility $\psi(e)$.

The remaining term in (3) is a "correction" term expressing how the financial value of the project changes when the demand falls at some $y_b < y$ and the activity becomes unattractive for being continued. Indeed, the firm renounces to the activity once y_b realizes and save expected losses of size $y_b/(r-\alpha) - C(e)/r < 0$. Since the firm can decide to renounce to activity in such conditions, this term increases the overall value of the project by eliminating downside flows that are below a certain level. The value of this term is weighted by the discount term $(y/y_b)^{\beta_2}$ in the space of realizations of the stochastic demand y_t, as based on the initial value $y_0 = y$. An additional remark is useful to be made here: $\psi(e)$ does not enter in this term because the disutility of effort is "sunk" at the moment when the firm decides to renounce to activity.

The social function, as derived in Appendix A.1 is written as

$$W(y/e) = S\frac{y}{r-\alpha} - \frac{C(e)}{r} - \psi(e) + \left(\frac{y}{y_b}\right)^{\beta_2}\left(\frac{y_b}{r-\alpha} - \frac{C(e)}{r}\right). \tag{4}$$

$Sy/(r-\alpha)$ is the expected social value that derives from the flow of market demand, such that $S > 1$. The costs incurred by the firm, $C(e)$ and $\psi(e)$ are part of the social function. Moreover, the social value of the project reduces in the expected losses $y_b/(r-\alpha) - C(e)/r < 0$. Its justification is that any transfer in the sector from public funds is costly.

3 The no-renegotiation benchmark

The social value of the project reduces in any transfer that is provided to the firm during the contract. Therefore, at social optimum,

$$W(y/e) = S\frac{y}{r-\alpha} - \frac{C(e)}{r} - \psi(e).$$

Consequently, the social optimal effort is

$$\frac{\delta}{r} \geq \psi. \tag{5}$$

This expression says that it is socially optimal if the firm internalizes the reduction in operational cost in its effort decision.

The public and private interests would coincide in our model if the market were no uncertain. Indeed, if $\sigma = 0$, then the demand never falls in the region $y_t \leq y_b$. The payoff of the firm reduces to

$$V(y/e) = \frac{y}{r-\alpha} - \frac{C(e)}{r} - \psi,$$

and the effort $e = 1$ is chosen whenever (5) is satisfied.

Some studies have already shown that in procurement contracts for essential facilities, the agent internalizes the positive externality between initial effort and its operation cost whenever the payment for construction is linked to the resulting cost of operation. The solution adopted was to decentralize both activities to one entity rather than separated agents (Hart, 2002; Benneth and Iossa, 2002).

On the other hand, the governments try to avoid public transfers so that the agent that builds and operates is financed from the financial market (debitors, equity holders), while it obtains its payoff directly from the (uncertain) market, those motion is defined by (1). With debt financing, the payoff of the firm is the closed form expression of $V(y)$ in (3). Therefore, the bundling of building and operating activities in one consortium is optimal also in the absence of technological synergies, in that public transfers are avoided. This is the second literature of public private partnerships, that of monopoly franchising contracts. The central point of such studies is that the contracts are renegotiated whenever the market is less optimistic than expected at contracting stage.

In our case, the consortium has incentives to renounce to activity whenever the demand falls in the region $y_t \leq y_b$, as expressed in (3), where y_b is to be determined in the problem of the firm. In order to allow for the continuation of the public activity, public transfers must take place in the project for demand values in this region. We call in what follows the value y_b that triggers public transfers as the renegotiation trigger.

We examine in what follows the issue of renegotiation in firm's decision about the unobservable effort e.

4 The problem of the firm

The problem of the firm is of backward induction. Based on the riskiness of the market, the firm determines first the probability of renegotiation. Subsequently, it decides the quality of the infrastructure to be built, provided that it can save on construction cost on its own benefit.

4.1 Second decision variable: the renegotiation trigger

Leland (1994) shows that a firm investing in a project declares bankruptcy when a certain value $y_b < y$ of the stochastic variable realizes. In our case, the firm can credibly threat to renounce to the activity once it is no longer expected to provide positive flows of revenues. Indeed, it can be easily shown that the payoff $V(y)$ decreases in y_b until a certain level while subsequently it increases. The optimal y_b

in (3), denoted $y_b(e)$ solves

$$y_b(e) = \frac{\beta_2}{\beta_2 - 1}\frac{C(e)}{r}(r - \alpha) \tag{6}$$

The interpretation of this solution is the following. The firm prefers to renounce to the activity once the expected market return is below the maintenance cost by the ratio $\frac{\beta_2}{\beta_2-1} < 1$. This decision is taken at the period τ when the market demand passes the value $y_b(e)$ for the first time. We define the stochastic time τ as $\tau \equiv \inf\{t \geq 0 \ s.t. \ y_t = y_b\}$. After τ, the firm saves instantaneous negative returns of expected size $y_t - C(e) < 0$ at any $t > \tau$ such that $y_t \leq y_b$.

The value y_b that solves (6), as in financial papers à la Leland (1994), will be called in what follows the renegotiation trigger. The particularity of our approach is that once y_b realizes, the project is not abandoned because it is socially valuable. The contract is renegotiated and the firm is allocated public funds. Also, the firm has an additional decision variable that adds to y_b, which is the choice of the initial effort. Indeed, we notice easily from (6) that $y_b(1) < y_b(0)$. The social interest is that the firm exerts the effort $e = 1$ so that renegotiation and consequently public subventions are avoided.

4.2 First decision variable: the effort choice

Let us move now to the choice of quality of the infrastructure to be built. We denote by ΔV the difference between the payoffs of the firm when it exerts and when it does not exert effort. Thus, by evaluating (3) at $e = 1$ and $e = 0$, we can write

$$\Delta V = \frac{\delta}{r} - \psi - \frac{1}{\beta_2 - 1}\left(\frac{y}{y_b(0)}\right)^{\beta_2}\frac{C^d}{r}\left(\left(\frac{C^d}{C^d - \delta}\right)^{\beta_2 - 1} - 1\right). \tag{7}$$

The firm exerts effort $e = 1$ whenever $\Delta V \geq 0$, or

$$\frac{\delta}{r} \geq \psi + \frac{1}{\beta_2 - 1}\left(\frac{y}{y_b(0)}\right)^{\beta_2}\frac{C^d}{r}\left(\left(\frac{C^d}{C^d - \delta}\right)^{\beta_2 - 1} - 1\right). \tag{8}$$

This is the key inequality that drives to the understanding of firm's incentives to exert unobservable effort in our model.

We note first that if no uncertainty exists, then $\beta_2 \to -\infty$ and the condition (8) reduces to

$$\delta/r \geq \psi,$$

as in the benchmark case. Our result differs from previous studies that characterize public-private relationships as procurement contracts, in that it allows for private investment, followed by market risk and possible renegotiation. Provided that the firm decides freely to renounce to activity once the market is low enough and reaches the value $y_b < y$, the incentive power of the synergies between the activities is weakened with respect to the benchmark case. The firm benefits from maintenance cost reduction just until the renegotiation occurs. Afterwards, any operational cost is in charge of the public budget. The probability of renegotiation is therefore part of its effort decision. The additional term that is added to ψ on the right hand side above expresses this idea. We can thus establish the following Proposition.

Proposition 1 *In an uncertain market, a firm running a monopoly franchising contract expects to renegotiate it with the government whenever the market falls to a certain level. Therefore, the firm does not internalize optimally at the building stage of the project the synergies between building and operating activities, since it can limit its financial risk through ex post action.*

We explain in what follows the incentive of the inequality above with respect to the default parameters of the model.

4.3 The determinants of effort decision

We show in what follows the sensitivity of firm's effort choice with respect to the default parameters. The inequality (8) is easier satisfied when

a) the initial demand y is higher

b) the positive externality between activities is higher

c) the volatility of the market reduces.

Let us treat first the initial demand y. It can be easily shown that $V'(y/e=0) < V'(y/e=1)$. The intuition of this inequality is straightforward: in case the information $y_0 = y$ available at the evaluation stage of the project increases the payoff of the firm increases faster when it exerts effort with respect to the opposite case. Figure 1 shows the expected value function $V(y/e)$ at different values of the initial demand y that filtrates the information about the market cash flows, as evaluated at $e = 0$ and 1.

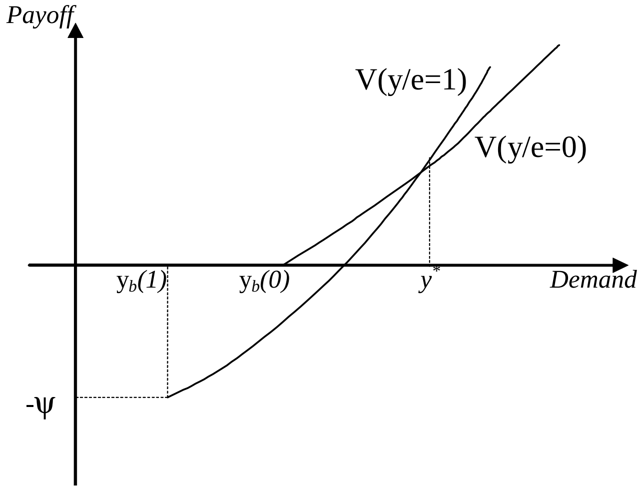

Figure 1: Payoff of the firm when either $e = 0$ or 1

As Figure 1 shows, the firm is not available to internalize the synergies between building and operating activities in its effort decision as long as the initial demand y is such that $y < y^*$. The key feature of this result is that the firm can transfer the costs from operation to the government whenever the market falls and renegotiation takes place. Therefore, as the disutility ψ is "sunk" at renegotiation period, high risk increases the probability of renegotiation and consequently it limits the incentives to make effort in maintenance cost reduction.

Let us look now at δ. It is very intuitive that the incentive of the firm to make effort increases as δ gets higher. Indeed, $V'(y/e=1)$ increases in δ, while $V'(y/e=0)$ remains unchanged, so that the threshold y^* reduces, as shown in Figure 2.

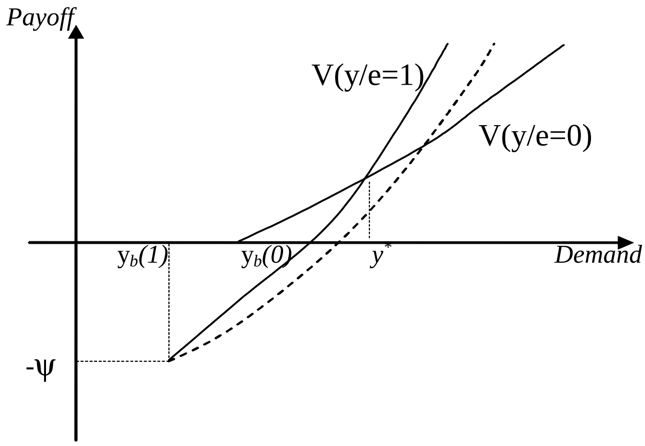

Figure 2: Payoff of the firm and changing value of δ

The implication of this result is the following: lower threshold y^* means higher market risk. When the firm benefits better from positive externality between the activities, this technological efficiency compensates for market risk so that the incentive for $e = 1$ is easier satisfied.

We[2] are left with the parameter σ^2, expressing the market riskiness. The right hand side of inequality (8) is increasing in market riskiness. Therefore, the firm's incentive to make effort reduces, as the probability of default increases. At higher riskiness σ^2, the firm exerts effort if δ is also higher. We can establish thus the following Proposition.

Proposition 2 *The firm's incentive to internalize the synergies between activities depends on the extent of the externality parameter with respect to the volatility of the market.*

We illustrate the link between the technological externality and market uncertainty in a concrete example.

4.4 Example: Positive externality and market volatility in the firm's decision

We assume that $r = 0.05$ and $\alpha = 0.02$. The initial demand is $y = 10$, while the operational cost is $C(e) = C^d - \delta e = 10 - \delta e$, where $e = 0$ or 1. δ can take any value between $r\psi$ and 10. The disutility of effort is $\psi = 50$. Figure 2 illustrates the payoffs $V(y/e = 0)$ and $V(y/e = 1)$ against the volatility parameter σ^2. $V(y/e = 1)$ differs according to the value of the externality parameter δ that is 1, 3, 5 or 9.

At $\delta = 1$, the condition $\delta/r \geq \psi$ that has been explain in the benchmarks case is not satisfied. Therefore, exerting effort is not optimal as the firm obtains a payoff $V(y/e = 1)$ that is below $V(y/e = 0)$ at any level of uncertainty, as clear in Figure 2.

At $\delta = 3$, the two payoffs cross at $\sigma^2 = 0$ because $\delta/r = \psi$. This level of externality is not enough for inducing $e = 1$ if the uncertainty exists.

$\delta = 5$ and $\delta = 9$ are two cases in which $e = 1$ is optimal for some degrees of uncertainty. By comparing the two cases, it is evident that higher value of externality parameter reflects a higher probability of effort $e = 1$. At $\delta = 9$ the function $V(y/e = 1)$ is almost flat in uncertainty parameter because the externality is very high and the probability of renegotiation goes close to zero.

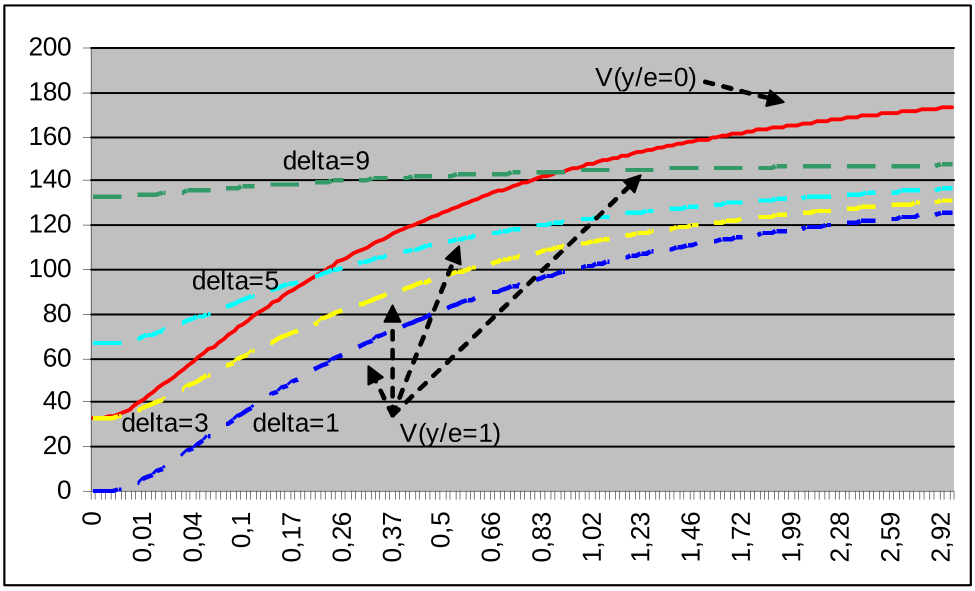

Figure 2

As the inequality (8) suggests, there exists an optimal threshold δ^* at any given level of the volatility σ^2 and at specific values of the other default parameters. In the example just announced, we find that δ^* is linearly increasing in σ^2 as illustrated in Figure 3 below.

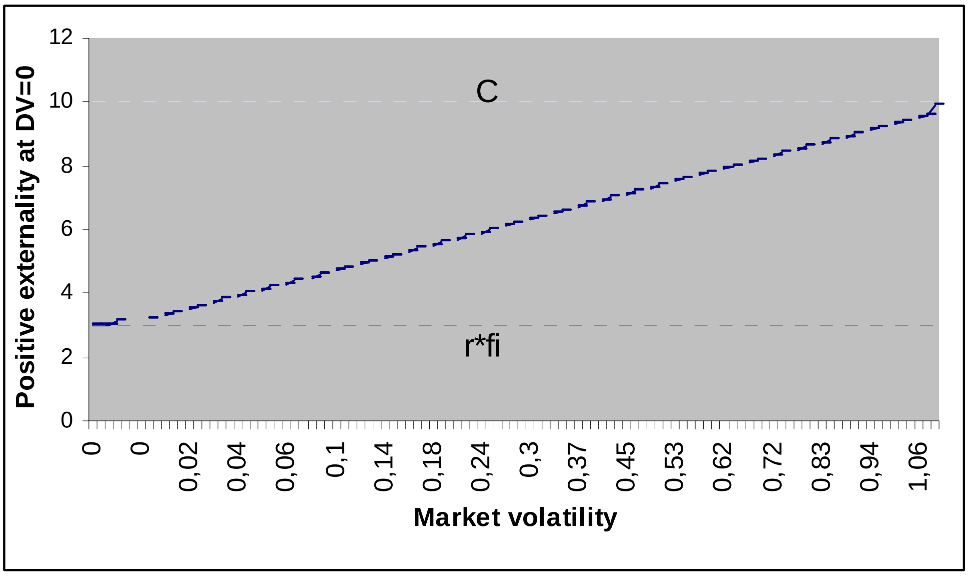

Figure 3

δ^* at which $\Delta V = 0$ increases in market volatility between the certainty threshold $r\psi$ at $\sigma^2 = 0$ and C^d , which is the fixed part of the operational cost. Therefore $\delta^* = C^d$ is the hypothetical case in which the positive externality is such that the firm bears no operational cost by exerting effort. Consequently, no renegotiation would take place.

5 Equity versus debt financing (still preliminary)

We have assumed all around the previous analysis that the firm is entirely debt financed. The literature has highlighted that the effort decision of the firm depends on the way the project is financed. Jensen and Meckling (1976) and other subsequent studies have demonstrated that in a two-period setting and without renegotiation, debt financing is socially desirable whenever synergies between activities exist. The renegotiation issue has not been examined in such studies. Intuitively, as the threat of renegotiation depends on the residual market flows once the investment is sunk, equity financing maybe a better option than debt financing since the expected losses from the market are to be shared by the firm with outside shareholders holders. We examine this issue in what follows.

We assume without loss of generality that no corporate tax is issued to the firm.

We denote by ν the share of the profits that are kept by the firm, in case the project is entirely equity financed. In this case, the value function denoted $V^{\nu}(y/e)$ is written

$$V^{\nu}(y/e) = \nu\left(\frac{y}{r-\alpha} - \frac{C^{\nu}(e)}{r}\right) - \psi(e) - \nu I - \nu\left(\frac{y}{y_b}\right)^{\beta_2}\left(\frac{y_b}{r-\alpha} - \frac{C^{\nu}(e)}{r}\right). \quad (9)$$

The interpretation of (9) is straightforward: the firm shares with outside shareholders the cash flows of the project and the investment cost I, while it bears alone the disutility $\psi(e)$, that are of size $y_t - C_t$ at any t, while it bears alone the unobservable effort $\psi(e)$. cost of operation is different than the one at previous analysis. The firm does not need to pay debt interest, so that

$$C^{\nu}(e) = C - \delta e.$$

We notice that the renegotiation trigger that is chosen by the firm has the same formula as at previous case

$$y_b^{\nu}(e) = \frac{\beta_2}{\beta_2 - 1}\frac{C^{\nu}(e)}{r}. \quad (10)$$

By comparing (10) with (6) we find that $y_b^{\nu}(e) < y_b(e)$. This is the key issue of this part of our analysis. Since the threat of renegotiation is based on the residual net returns that are expected from operating the infrastructure, the firm can credibly threat to renounce to activity *later* with respect to the case when it were debt financed. Under debt financing, the renegotiation takes place when the market flows do not cover both the maintenance cost plus the debt to be paid to the creditor. The firm does not share the profits but it must pay a fixed sum whenever it decides to continue the activity. The key issue of this difference is that with equity financing, the investment cost is "sunk" and it cannot be recovered by renouncing to operation activity when the market falls.

The analysis of effort decision is similar to the previous case. We calculate the difference between the payoff function of the firm when it exerts and when it does not exert effort. Therefore, the firm decides $e = 1$ whenever

$$\nu\frac{\delta}{r} \geq \psi + \frac{\nu}{\beta_2 - 1}\left(\frac{y}{y_b^{\nu}(0)}\right)^{\beta_2}\frac{C^{\nu}}{r}\left(\left(\frac{C}{C-\delta}\right)^{\beta_2-1} - 1\right) \tag{11}$$

A close attention needs to be provided to this inequality, as compared to (8). If no uncertainty exists, (11) reduces to

$$\nu\frac{\delta}{r} \geq \psi. \tag{12}$$

Indeed, without market uncertainty, it is less likely that the firm chooses $e = 1$ with equity rather than with debt financing. Jensen and Meckling (1976) have already examined this issue. Their argument has been repeated by Dewatripont and Legros (2005). As evident in (12) the reason why debt is socially preferable is that in case external shareholders finance (partially) the project, the firm must share with them the externality if its effort while it bears alone the disutility ψ.

However, under the market risk, a new decision variable determines the effort decision of the firm, which is the probability of renegotiation. We calculate the difference between the right hand sides of (11) and (8), which is

$$\begin{aligned}&-\frac{\delta}{r}+\\&+\nu\frac{\delta}{r}-\frac{1}{\beta_2-1}\left(\frac{y}{y_b(0)}\right)^{\beta_2}\frac{C^d}{r}\left(\left(\frac{C^d}{C^d-\delta}\right)^{\beta_2-1}-1\right)\left[\nu\frac{C}{C^d}\left(\frac{y_b(0)}{y_b^{\nu}(1)}\right)^{\beta_2}\frac{\left(\frac{C}{C-\delta}\right)^{\beta_2-1}-1}{\left(\frac{C^d}{C^d-\delta}\right)^{\beta_2-1}-1}-1\right]\end{aligned}$$

The first line of this expression is negative. The second line, is positive and increasing in ν. Therefore, according to the values of the default parameters, there exists an optimal threshold of the share ν at which the firm is indifferent between equity or debt financing in exerting effort. If the share ν is higher than this threshold, equity financing dominates debt financing in requiring $e = 1$. We can thus write the following proposition.

Proposition 3 *In case a firm engaged in a public private partnership does not* *bear the market risk and consequently no renegotiation is expected, debt financing is always preferable. By contrast, in the presence of market risk and renegotiation, equity financing may be preferable to debt financing as it induces the firm to make proper effort in building essential facilities.*

6 Conclusion

Our results can be summarized as follows. Involving private firms in public projects maybe socially desirable if the market alone can finance the project. However, the synergies between the activities are not well internalized in the programme of the private firm since the risk of the project can be transferred to the public budget through renegotiation. The firm has incentive to exert proper effort in case the externality in cost reduction by exerting unobservable effort is high enough with respect to the market risk. Otherwise, the public authority should give a rent to the firm and the optimal mechanism should be properly examined. This issue has been left for further research.

The capital structure that is optimal in the public-private partnership is crucial in this analysis. Consolidated consortium that are able to (partially) finance the project with their funds or attract outside equity financing may be socially desirable. They are weaker than smaller firms in threatening to quit the contract once their "funds" are sunk in the project. This result is in contradiction with the previous literature of public-private partnerships in that the probability of renegotiation in case of bad market conditions is part of the firm's programme.

We have assumed for simplicity that the demand evolves independently and cannot be "controlled" through regulation. Otherwise, the cost of public subsidies when the market falls should be traded-off against the cost of extracting more willingness to pay from consumers. We only focused on the case that extracting more willingness to pay through changing regulation is not a valuable instrument for solving the budget balance of an investor in essential facility.

7 Appendix

A1. Functional form of the payoffs

Le us take any payoff $F(y_t)$ based on the stochastic process (1); any such value satisfies the standard Ito's lemma

$$\alpha y_t F_{y_t} dt + \frac{1}{2}\sigma^2 y^2 F_{y_t y_t} - rF + A_0 = 0. \tag{13}$$

The general solution of (13) at $y_t = y$ is written

$$F(y) = A_0 + A_1 y^{\beta_1} + A_2 y^{\beta_2}. \tag{14}$$

$\beta_1 > 1$ and $\beta_2 < 0$ are known constants (see Dixit and Pyndick, 1994), while A_0, A_1, A_2 are determined by the boundary conditions of (13). The remaining terms show how the payoff function $F(y)$ changes when *ex post* actions are taken at some $y_t > y$ or $y_t < y$. In our case, *ex post* actions are relevant just when the demand is below the initial one, in the region $y_t < y$. Thus, if $y \to \infty$, $F(y) = A_0$. We must set in this case $A_0 = 0$, so that

$$F(y) = A_0 + A_2 y^{\beta_2} \tag{15}$$

We express in what follows the concrete functional form of $F(y)$ for both the firm's payoff and for the social one. We find for each case the specific value of the constants A_0 and A_2.

The payoff of the firm

By the properties of the geometric Brownian motion (1), it implies that the expected discounted flow of the demand from operating infinitely the infrastructure is $y/(r-\alpha)$. If the expected drift α of the demand is high and $r-\alpha$ reduces, higher market revenues are expected. The firm gets instantaneous subsidy s and bears instantaneous costs $C(e)$ during each period of operation. Hence, $s/r - C(e)/r$ is added to the payoff of the firm. Also, the firm bears an initial disutility $\psi(e)$ that remains "sunk" once the infrastructure is built. Thus, $A_0 = \frac{y}{r-\alpha} - \frac{C(e)}{r} - \psi(e) + \frac{s}{r}$. It follows that the payoff of the firm is written as

$$V(y) = \frac{y}{r-\alpha} - \frac{C(e)}{r} - \psi(e) + \frac{s}{r} + A_2 y^{\beta_2}.$$

A_2 is found from the remaining boundary condition of (14). Indeed, when the demand reaches a certain $y_b < y$, the firm renounces to finance the operation activity of infrastructure. We denoted by τ in the main text the expected period when $y_\tau = y_b$. We can thus write that at any $t > \tau$, the value of the project is V_b, such that $V_b = -\psi(e)$. Indeed, once the renegotiation takes place, the firm losses the sunk cost $\psi(e)$ but it does not bear losses from operation. The absolute and relative values of $V(y)$ between the regions $y < y_b$ and $y > y_b$ must be equal at $y = y_b$. Therefore,

$$\begin{aligned} \frac{y_b}{r-\alpha} - \frac{C(e)}{r} + \frac{s}{r} + A_2 y_b^{\beta_2} &= 0 \\ \frac{y_b}{r-\alpha} + \beta_2 A_2 y_b^{\beta_2} &= 0 \end{aligned}$$

From the first equation, we can write

$$A_2 = y_b^{-\beta_2}\left(\frac{C(e)}{r} - \frac{s}{r} - \frac{y_b}{r-\alpha}\right),$$

so that the closed form functional form of the payoff $V(y)$ defined in the region $y > y_b$ is

$$V(y) = \frac{y}{r-\alpha} - \frac{C(e)}{r} - \psi(e) + \frac{s}{r} - \left(\frac{y}{y_b}\right)^{\beta_2}\left(\frac{y_b}{r-\alpha} - \frac{C(e)}{r} + \frac{s}{r}\right),$$

as in the main text.

The social payoff

The functional form of the social function $W(y)$ derives analogously. The free constant in this case is $A_0 = S\frac{y}{r-\alpha} - C(e) - \psi(e) - \frac{s}{r}$, so that $W(y)$ is written as

$$W(y) = S\frac{y}{r-\alpha} - \frac{C(e)}{r} - \psi(e) - \frac{s}{r} + A_2 y^{\beta_2}.$$

$S\frac{y}{r-\alpha} - \frac{C(e)}{r} - \psi(e)$ is the net social benefit of the project if no renegotiation takes place and no transfers are realized between the government and the firm. s/r is the subsidy that solves the participation and eventually the incentive constraint of the firm. $A_2 y^{\beta_2}$ is the continuation function that shows how the social payoff changes when the renegotiation takes place.

A_2 is found from the boundary condition of (14). When the demand reaches a certain value $y_b < y$ and the firm renounces to finance the operation, the social value function becomes $W_b(y)$, such that

$$W_b(y) = S\frac{y}{r-\alpha} - \frac{C(e)}{r} - \psi(e) + \frac{y}{r-\alpha} - \frac{C(e)}{r}.$$

We notice how $W_b(y)$ differs from $W(y)$: the subsidy s/r for participation and eventual rent is replaced by $y_b/(r-\alpha) - C(e)/r < 0$, the direct financing of operation. The absolute and relative values of the two payoff expressions must be equal at y_b. Therefore,

$$\begin{aligned} -\frac{s}{r} + A_2 y_b^{\beta_2} &= +\frac{y_b}{r-\alpha} - \frac{C(e)}{r} \\ \beta_2 A_2 y_b^{\beta_2} &= \frac{y_b}{r-\alpha} \end{aligned}$$

From the first equation,

$$A_2 = y_b^{-\beta_2}\left(\frac{y_b}{r-\alpha} - \frac{C(e)}{r} + \frac{s}{r}\right),$$

so that the social payoff $W(y)$, at any $y > y_b$ is written as

$$W(y) = S\frac{y}{r-\alpha} - \frac{C(e)}{r} - \psi(e) - \frac{s}{r} + \left(\frac{y}{y_b}\right)\left(\frac{y_b}{r-\alpha} - \frac{C(e)}{r} + \frac{s}{r}\right),$$

which is the expression shown in the main text.

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