A Modified Yardstick Competition Mechanism

by
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This paper analyzes a modified yardstick mechanism (MYC), whereby the yardsticks employed consists of a tariff basket and total costs. This mechanism has a significant information advantage: the regulator "only" needs to observe total costs and output of all firms within the competition mechanism. The modified yardstick competition mechanism can assure a socially optimal outcome allowing for spatial and second degree price discrimination, without increasing the informational requirements. We also introduce regulatory lags in the model. A systematic comparison between the results of traditional yardstick regulation, and modified yardstick regulation, is carried out. Finally, the applicability of the mechanism is discussed. (JEL: L51, L11, D40).

1 Introduction

The seminal paper by Shleifer [16] "A Theory of Yardstick Competition" has been followed by a large number of theoretical developments (and criticisms), but a relatively modest number of practical applications (in an even more modest number of countries, mainly the UK). The essence of yardstick competition is to deduce from all firms under the regulatory mechanism the regime for a particular firm "the price the regulated firm receives depends on the costs of identical firms" (Shleifer, [16], p. 319). The concept has been embraced by the theory of regulation under information asymmetry (for a survey see Chong [4]); but it has also been criticized for discouraging socially optimal investment (e.g. Dalen [6] and Sobel [17]).
In practice, yardstick competition has not become a success story among recent regulatory approaches. England and Wales' water and sewerage sectors are the most often cited example of real-world applications. But many other studies and sectors have stayed at timid attempts to embrace yardstick competition, only in order to continue without it (examples of electricity and public transport sectors). Assuming that information requirements may be one obstacle to the implementation

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of yardstick competition, we propose a modification of the yardstick competition mechanism introduced by Shleifer [16]. Yardstick competition as introduced by Shleifer can be interpreted as differentiated cost based price caps. Firms are allowed to charge a two-part tariff where the unit price is constrained from above by average (marginal) costs and the fixed (access) charge[1] is bounded from above by fixed costs. Of course, the main clue which makes this kind of cost related regulation viable is the introduction of a "shadow firm" representing the average all other local monopolists. Costs of this virtual firm are calculated by averaging over all variable and fixed costs except those of that firm the price caps of which are determined. This kind of regulation which introduces a virtual competition between otherwise unrelated local monopolists leads to a socially optimal allocation both with regard to output and to firm's internal cost structure. Even if firms are heterogenous the mechanism can be properly adapted to allow for local idiosyncrasies. Nevertheless, there remain some problems if the mechanism is put into practice. First, the mechanism can be strategically exploited. This is very well known in the literature and proposals have been made to overcome this kind of vulnerability. Tangeras [19] has introduced a collusion-proof mechanism; and Potters et. al[12] have analyzed the effectiveness of various kinds of yardstick competition systems within an experimental study where participants could collude within a repeated game setting. However, in the following we do not deal with these problems. Instead, we concentrate on some other issues associated with the basic yardstick competition model.

The mechanism requires that the regulatory authority can observe variable costs and fixed costs separately for each firm. This requires precise accounting rules which are able to exclude the manipulation of accounting. This kind of rules along with the proper monitoring system might be difficult to put into practice, in particular if monopolists are multiproduct/multiservice firms. Our mechanism proposed can do without this informational requirement as we will see later. In addition, yardstick competition does not allow for optimal price discrimination. The mechanism supports only an optimal uniform tariff. If we allow for consumer heterogeneity we know from the literature that an optimal non-linear tariff system can be found that sorts consumers optimally. For instance, Goldberg, Leland and Sibley [10] have shown that this tariff system implies an optimal deviation from marginal costs for all customer groups except that one with the highest demand[2]. Yardstick competition excludes optimal deviations from marginal costs by construction.

Our modifications proposed utilize regulation mechanisms introduced in the literature and combine them with the "shadow firm" approach of yardstick competition. Instead of regulating each element of the two part tariff separately, we impose a kind of tariff basket regulation and link it with yardstick competition. Armstrong and Sappington [1] analyze basket tariff regulation. A reference unit price is set by the regulatory authority and firms are allowed to charge a two part tariff. The regulation consists in that total revenue must not exceed revenue collected under the reference unit price. The main draw back of this regulation device is that the first best allocation cannot be reached. Nevertheless, this kind of regulation works

[1] Instead of a fixed charge Shleifer considers a transfer from a regulatory authority. Mathematically, this is equivalent.

[2] This property of self-selecting tariffs are very well understood and can be find in many advanced microeconomic textbooks. See e.g. Gravelle and Rees [11] or Wolfstetter [22].

better than average revenue regulation (see e.g. Armstrong, Cowan and Vickers [2]) within a static model framework. However, both regulation schemes require the authority to have a notion of how to set the reference unit price, i.e. the demand function must be known by the regulatory authority.

Our modified yardstick competition scheme takes the tariff basket regulation and inserts for the quantity weights average output of all monopolists, except that under regulation, and replaces revenue under the reference price by average total costs of all monopolists except that under consideration. As a result we arrive at a socially optimal Nash-Cournot equilibrium. In addition to the desirable allocative properties this modification requires no knowledge about demand or cost functions. The observability of output, prices and total costs of each local monopolist are sufficient to implement the scheme. Regulatory mechanism which do not rely on prior beliefs of the regulator are called non-bayesian (Vogelsang [20]). They are verifiable in the sense that they only utilize observables like costs, prices and ouput. Hence, yardstick competition is a member of this class.

The repeated application of regulatory schemes may enhance welfare considerably. Moreover, many dynamic regulatory schemes get by with fewer informational requirements than their twins in a static context. Many of these a non-baysian and rely exclusively on observable data. On the other hand, some additional strategic problems can emerge. Currier [5] has shown that strategic behaviour of firms under average-revenue-lagged regulation can lead to the unregulated global profit maximum. Also more sophisticated mechanisms like the famous VF-mechanism developed by Vogelsang and Finsinger [21] may be susceptible to strategic behavior of non-myopic firm (see Sappington [13]). This extends also to the very well known incremental surplus subsidy (ISS) mechanism developed by Sappington and Sibley [14]. This mechanism is not proof against abuse, i.e. the managerial exploitation for fringe benfits.

The dynamic version of our modified yardstick competition is a hybrid of Shleifer's mechanism and the dynamic subsidy scheme developed by Finsinger and Vogelsang [7]. We introduce a one-period lag in the regulatory constraint with regard to the quantity weights of the revenue basket and total costs. We can show that the dynamic version of the modified mechanism approaches the optimal steady state more slowly than the lagged Shleifer yardstick competition. Hence, welfare losses can be identified that must be set off against welfare increases due to optimal price discrimination. In contrast to other mechanism in the literature the dynamic version of our modified yardstick competition does work less optimal than that in the static version. The question remains why we have introduced a dynamic lagged version. The very reason for this exercise is, that we belief that to gather all information and to calculate the relevant constraints for all local monopolists requires time. Specifically, this applies for the case of heterogenous firms were the basis model has to be supplemented by an econometric analysis to allow for idiosyncrasies.

The remainder of this paper is structured in the following way: the next section develops the basic static model, and compares the information requirements of marginal-cost regulation with those of the one proposed in this paper. We find that the modified yardstick competition (MYC) leads to a socially optimal allocation, a Nash-equilibrium which is symmetric and unique. We are also able to show that the modified yardstick competition mechanism will lead to socially optimal allocation in the case of spatial price discrimination. Section 3 then develops the dynamic version of the model: we introduce regulatory lag, whereby the (regulated) price p_i depends on the observed cost c_i in the previous period. We distinguish two types of behavior: i) myopic behavior by the regulated companies, and ii) firms with a long-run planning horizon maximizing their present value ("strategic behavior"). Section 4 investigates the welfare properties of both regulation regimes during the transition period, whereas we confine ourselves to a linear model structure. Section 5 summarizes and provides an outlook on the potential use of the modified yardstick mechanism.

2 The basic static model

The following mathematical structure is based on the one-period model of Shleifer[16]. There are n identical local monopolies supplying a local market with goods, e.g. water services. The demand functions of the local markets are given by

$$x(p_i), \quad x'(p_i) < 0, x''(p_i) \geq 0; \quad i \in \{1, 2, \cdots, n\} \tag{1}$$

where p_i is the local price of firm i. Notice that we have assume identical demand functions. In the following we utilize the inverse demand function

$$p(x_i), \quad p'(x_i) < 0, p''(x_i) \geq 0, \quad i \in \{1, 2, \cdots, n\} \tag{2}$$

The cost function of firm i is

$$K(x_i, c_i) = c_i x_i + R(c_i), \quad i \in \{1, 2, \cdots, n\} \tag{3}$$

where c_i are marginal costs that are endogenous, i.e. firms can choose it's level. Reducing c_i lead to costs according to the function

$$R(c_i), R_c(c_i) < 0, \quad R_{cc}(c_i) > 0, \quad i \in \{1, 2, \cdots, n\} \tag{4}$$

These expenses are fixed costs. Without loss of generality, we also assume, that $R(c^{max}) = 0$, where c^{max} solves $R_c(c) \leq 0$. If firms minimize fix costs, they choose a high value of c, such that $R(c) = 0$.
All firms are identical with regard to technology and local production conditions.
The optimal allocation and the optimal tariff can be derived from the usual welfare maximization approach:

$$\max_{x_i, c_i} [\int_0^{x_i} p(v_i) dv_i - c_i x_i - R(c_i)] \quad \forall i \in \{1, 2, \cdots, n\} \tag{5}$$

The first order conditions that determine the optimal values $\{x^*, c^*\}$ are:

$$p(x_i) = c_i, \quad -x_i - R_c(c_i) = 0 \tag{6}$$

A global interior maximum requires that the welfare function is concave, i.e.

$$p'(x_i) < 0, \quad R_{cc}(c_i) > 0, \quad -p'(x_i) > 1/R_{cc}(c_i), \forall x_i, c_i \tag{7}$$

and

$$R_c(0) > x^{max}, \text{ where } p(x^{max}) = 0; \tag{8}$$

$$p(0) > \hat{c}(0) \tag{9}$$

where $\hat{c}(x_i)$ solves the first order condition $-x_i - R_c(c) = 0$. The following figure collects all these assumptions[3] and indicates the optimal allocation $\{x_i^*, c_i^*\}$ as well

[3] All these assumptions also underly the model of Shleifer.

as the optimal tariff $\{p_i^*, \pi_i^*\}$.

picture 1 (The optimal solution)

To verify that the hatched area under $\hat{c}(x)$ is equal to π_i^* recall that the price p_i^* is equal to $\hat{c}$. Hence, fix costs must be covered by the access fee $\pi_i^* = R(c^*)$. From $-x - F_c(c) = 0$ we can calculate the inverse of $\hat{c}(x_i)$ which we refer to as $\hat{x}(c_i)$. Utilizing the first order condition with respect to c yields (subscribt i omitted):

$$\int_{c^*}^{c^{max}} x(v)dv = -\int_{c^*}^{c^{max}} F_c(u)dv = R(c^*) - R(c^{max}) = R(c^*) \tag{10}$$

In his seminal article Shleifer has shown that separate[4] price caps for the unit price p_i and the access fee π_i according to

$$p_i = \frac{1}{n-1}\sum_{j\neq i} c_j, \quad \text{and} \quad \pi_i = \frac{1}{n-1}\sum_{j\neq i} R(c_j) \quad \forall i \in \{1, 2, \cdots, n\} \tag{11}$$

lead to a first best outcome that is the result of maximizing the social welfare function in each location

Establishing this kind of yardstick competition requires that the regulation authority observes all prices and all costs separated according to unit costs and (common) fix costs, i.e. the information set I^{YC} necessary to enforce the mechanism is

$$I^{YC} = \{p_i, \pi_i, c_i, R(c_i), \forall i \in \{1, 2, \cdots, n\} \tag{12}$$

Notice, that the regulator needs only to know ex post values and no functional relationships like demand or cost functions. Regulation mechanisms of this kind are called non-Bayesian because no à priori knowledge on the part of the regulator is needed.[5]

However, the regulator is required to understand how variable and fixed costs are calculated and must secure that cost information is correct, i.e. separated according to these rules. This problem can be aggravated if firms serve different customers and are allowed to discriminate prices. Take as an example the water sector. Usually local drinking water supply and the sewerage system is based on a spatial distribution

[4] Shleifer derives second - best prices under a linear tariff system as well. In this case optimal unit prices equal total average costs. Yarstick competition supports this outcome.

[5] For a recent overview of different regulation mechanisms see Armstrong and Sappington [1] .

system. To guarantee optimal conveyance investments spatial price discrimination should be allowed.[6] If the local private drinking water provider or a privately run sanitation system discriminates user services in terms of distribution costs the regulatory authority needs to know the exact cost structure in terms of spatial location of each user group which in turn increases the information requirements considerably. Beyond the information problems of spatial price discrimination the problem of regulating non-linear tariffs remains within the regulation scheme (11). If the regulation authority would allow second degree price discrimination in the presence of heterogenous customers the yardstick mechanism (11) can not be implemented because marginal prices should no be restricted to marginal costs as it is well known from theory.[7]

In the following we extend the modified yardstick competition mechanism to spatial and second degree price discrimination without increasing the informational requirements on the part of the regulatory authority.

We start with the basic model introduced by Shleifer. Let us define average total costs

$$\bar{K}_{-i} = \frac{1}{n-1}\sum_{j\neq i} K(x_j, c_j) = \frac{1}{n-1}\sum_{j\neq i}[c_j x_j + R(c_j)], \quad \forall i \in \{1, 2, \cdots, n\} \tag{13}$$

and average output

$$\bar{x}_{-i} = \frac{1}{n-1}\sum_{j\neq i} x_j, \quad \forall i \in \{1, 2, \cdots, n\} \tag{14}$$

These averages do not contain the values of firm i.

To calculate these values and to impose the regulation mechanism the regulatory authority must be able to observe the ex post values of total costs, quantities of services sold and, of course, the tariff of each firm. The set of necessary information we call I^{MYC} and it contains

$$I^{MYC} = \{p_i, \pi_i, x_i, K_i, \forall i \in \{1, 2, \cdots, n\} \tag{15}$$

The regulation constraint that triggers the competition process is

$$p_i \bar{x}_{-i} + \pi_i \leq \bar{K}_{-i}, \quad \forall i \in \{1, 2, \cdots, n\} \tag{16}$$

The calculated revenue of firm i must not exceed it's calculated costs. Notice that firm i cannot manipulate these values. In contrast to the mechanism proposed by Shleifer firms under the MYC-mechanism have two degree of freedom when choosing p_i, π_i, c_i under the regulation constraint (16). The maximization program of firm i is

$$\max_{x_i, \pi_i, c_i} [p(x_i)x_i + \pi_i - c_i x_i - R(c_i)], \quad \text{s.t.} \quad (16) \tag{17}$$

[6] Chakravorty, Hochman and Zilberman [3] have shown that spatially uniform prices lead to suboptimal investments.

[7] Optimal second degree price discrimination leads to optimal deviations of marginal prices from marginal costs except for the user group of the highest demand. See Goldman, Leland and Sibley [10].

Notice, that we choose the price p_i indirectly by choosing x_i. Assuming an interior solution of (17) we can calculate from the first order conditions[8] :

$$[p(x_i) - c_i] + p'(x_i)(x_i - \bar{x}_{-i}) = 0 \tag{18}$$

and

$$-x_i - R_c(c_i) = 0 \tag{19}$$

The second condition determines the efficient amount of cost reducing investment conditional on services supplied. The first condition indicates an Cournot monopolist trying to equate marginal revenue to marginal costs under an regulatory constraint.
The properties of the MYC-mechanism can be summarized as follows:

proposition 1 *The modified yardstick competition MYC leads to a social optimal allocation, i.e. the Nash equilibrium $\{p^{MYC}, c^{MYC}\}$ is equal to $\{p^*, c^*\}$, where $\{p^*, c^*\}$ solves program (5). The access fee is equal to fix costs $R(c^*)$ and, as a result, profits are zero. The Nash equilibrium is symmetric and unique.*
Proof: See appendix

This properties extend also to the case of third degree price discrimination if a local monopolist i sets spatially differentiated prices. Suppose there are m different consumer groups with different variable uni costs $c_{ik}, \forall k \in \{1, 2, \cdots, m\}$. The monopolist i maximizes profits

$$\max_{p_{ik}, \pi_{ik}, c_{ik}} [p_{ik} x_k(p_{ik}) + \pi_{ik} - c_{ik} x_{ik} - R(c_{i1}, c_{i2}, \cdots, c_{im})] \tag{20}$$

where $R(.)$ is convex cost function of cost reducing investments. The regulatory constrain extends to

$$\bar{K}_{-i} = \frac{1}{n-1} \sum_{j \neq i} K(x_j, c_{j1}, c_{j2}, \cdots, c_{jm}) = \frac{1}{n-1} \sum_{j \neq i} [\sum_k c_{jk} x_{jk} + R(.)], \tag{21}$$

and average output to consumer group k

$$\bar{x}_{-ik} = \frac{1}{n-1} \sum_{j \neq i} x_{jk}, \quad \forall i \in \{1, 2, \cdots, n\} \tag{22}$$

The yardstick competition is established by introducing the regulatory constraint

$$\sum_k [p_{ik} \bar{x}_{-ik} + \pi_{ik}] \leq \bar{K}_{-i}, \quad \forall i \in \{1, 2, \cdots, n\} \tag{23}$$

This constraint is a straightforward extension of (16) for the case of spatial price discrimination. It is also a straightforward exercise to derive from the first order condition the Nash equilibrium. We, therefore, confine ourselves to summarize the main results:

[8]Sufficient conditions for a global maximum are specified in the appendix (proof to proposition 1).

corollary 1 *The extended yardstick competition mechanism for spatial price discrimination leads to a social optimal allocation. The necessary information set of the regulatory authority requires the observability of total costs, all prices and all service units delivered to m consumer groups.*
Proof: Similar to the proof of proposition (1) and, hence, omitted.

Second degree price discrimination requires a non-linear expense function that depends on the quantity sold to each consumer. The construction of such a tariff system selecting consumers according to demand relevant characteristics is well understood from the literature.[9] It can be derived for a profit maximizing monopolist and for a social surplus maximizing firm as well. It is well known that the optimal expense function is such that marginal expenses, i.e. unit prices, are above marginal costs except for the consumer group with the highest demand. But this property can not be achieved under yardstick competition YC (see (11)) due to the restriction that unit prices are to equal marginal costs.
The modified yardstick competition allows for second degree price discrimination without more information on the part of the regulator. The information set is identical to the case of spatial price discrimination. The regulator needs to observe the tariff system, the service output sold to the different consumer groups and total costs of each firm.
In the following we show how to establish yardstick competition within a simple model for two consumer groups.[10]

$$x_l(p) < x_h(p), \quad \forall x \tag{24}$$

where the subscripts l and h stand for "low" and "high" respectively. Customers maximize consumer surplus

$$S_i(p_i, \pi_i) = \int_{p_i}^{\bar{p}_i} p_i(v_i) dv_i - \pi_i, \; i = \{l, h\} \tag{25}$$

where $\bar{p}_i$ is defined implicitly by $x_i(\bar{p}_i) = 0$ and p_i, π_i is part of a sorting two part tariff system

$$TPS = \{p_i, \pi_i, i = l, h\} \tag{26}$$

Note, that this system is different to the more general one that defines non-linear expense functions. In our case of two groups of customers this would amount to the task to find a sorting and optimal menu of price-quantity combinations $M = \{T_i, x_i \, i = l, h\}$ where T_i are total expenses for x_i. Ti depends on x_i (non-linear expense function). It is well known that M is more effective than the menu of sorting two-part tariffs, i.e. leads to more aggregate consumer surplus.[11] However, M cannot be implemented by the modified yardstick competition system. A regulatory

[9] See Goldman, M., Leland, H. and S. Sibley [10].
[10] This goes without loss of generality. The same results apply for the case of more than two groups or continuous characteristics of consumers. One simply has to add additional incentive compatibility constraints. For further details see Wolfstetter [22] p. 27 ff. Two types of consumers exist, "low" and "high". To assure that the optimal tariff system implies a sorting solution we assume (single crossing property
[11] See Wolfstetter [22], p. 33.

constraint that directly restricts total revenue, i.e. $T_{i1}+T_{i2} \leq \bar{K}_{-i}$, cannot implement the first best allocation $\{x^*, c^*\}$. A firm's revenue has to be expressed as a linear combination of prices and quantities (see (31)). In turn, this requires that the regulator knows the demand functions of the two consumer types. Prices p_i are the first derivative of the nonlinear expense function $T(x_i)$. The first derivatives of $T(x)$ at x_l and x_h are $p_l(x_l)$ and $p_h(x_h)$, respectively. The calculation of this values requires the regulatory authority to know the (inverse) demand functions of both consumer groups l and h. Hence, given the information set ^{MYC} the regulatory authority can only implement yardstick competition by the two-part tariffs system TPS.

It remains to introduce the remaining constraints for price discrimination. The incentive compatibility constraints are

$$S_l(p_l, \pi_l) - S_l(p_h, \pi_h) \geq 0 \quad (27)$$

$$S_h(p_h, \pi_h) - S_h(p_l, \pi_l) \geq 0 \quad (28)$$

The necessary participation constraints are:

$$S_l(p_l, \pi_l) \geq S_l(0, 0) = 0 \quad (29)$$

$$S_h(p_h, \pi_h) \geq S_h(0, 0) = 0 \quad (30)$$

From (27) and (28) follows[12] that $S_2(p_2, \pi_2) > 0$.

The suitable MCY for second degree price discrimination is

$$[p_l \bar{x}_{-il} + \pi_l] + [p_h \bar{x}_{-ih} + \pi_h] \leq \bar{K}_{-i}, \quad \forall i \in \{1, 2, \cdots, n\} \quad (31)$$

The local monopolist maximizes profits subject to the incentive compatibility constraints, the participation constraint and the MCY-constraint.

corollary 2 *The MYC for second degree price discrimination secures the social optimal allocation as Nash-equilibrium.*

Proof: See appendix.

Second degree price discrimination is optimal if the monopolist is regulated according to modified yardstick competition. We have shown that profit maximizing under the MYC-mechanism leads to an optimal allocation. Allowing for second degree price discrimination simply leads to include some further constraints and some additional variables into the relevant two optimization programs (5) and (17). But adding variables and introducing additonal constraints does not eliminate the basic equivalence of both programs.

3 Yardstick competition under regulatory lags

Both yardstick competition mechanisms are introduced within a static model where the final allocation is immediately reached as a Nash-equilibrium. However, one has to bear in mind that the competition forces do not unfold within a market process but as a result of the interference of a regulatory body. This virtual competition is

[12]This is a well known property. Hence, a proof is omitted. See e.g. Wolfstetter [22].

organized and institutionalized and requires a variety of efforts on the part of the regulator, e.g. the gathering of information, the calculation of costs, the econometric analysis in the case of heterogenous firms and the monitoring of firms. These activities require time and, hence, may lead to regulatory lags of certain extent.
Thus, it is interesting to introduce regulation lags and study how these effect the properties of the two mechanisms (YC and MYC).
Suppose, first, that the yardstick competition mechanism of Shleifer is lagged in that price caps today are derived from average marginal costs of the last period. Formally:

$$p_i^t = \frac{1}{n-1}\sum_{j\neq i} c_j^{t-1}, \quad i \in \{1, 2, \cdots, n\} \tag{32}$$

We also assume that access fees are capped by average fix costs of the last period, i.e.

$$\pi_i^t = \frac{1}{n-1}\sum_{j\neq i} F(c_j^{t-1}), \quad i \in \{1, 2, \cdots, n\} \tag{33}$$

Similar, we can introduce the lag-structure into the modified yardstick competition.

$$\bar{K}_{-i}^{t-1} = \frac{1}{n-1}\sum_{j\neq i} K(x_j^{t-1}, c_j^{t-1}) \tag{34}$$

$$= \frac{1}{n-1}\sum_{j\neq i}[c_j^{t-1}x_j^{t-1} + R(c_j^{t-1})], \quad \forall i \in \{1, 2, \cdots, n\}$$

and average output

$$\bar{x}_{-i}^{t-1} = \frac{1}{n-1}\sum_{j\neq i} x_j^{t-1}, \quad \forall i \in \{1, 2, \cdots, n\} \tag{35}$$

The respective regulation constraint[13] is

$$p_i^t \bar{x}_{-i}^{t-1} + \pi_i^t \leq \bar{K}_{-i}^{t-1}, \quad \forall i \in \{1, 2, \cdots, n\} \tag{36}$$

The main question to be answered is how firm behave in the presence of these two regulatory lags. Two cases can be distinguished: Myopic firm maximizing current profits and firms with a long run planning horizon maximizing it's present value. In the latter case, the dynamic nature needs the introduction of dynamic game theory. However, we first want to analyze the dynamic properties of the two mechanisms when firms are myopic. Latter, we will show that the sequence of Nash-equilibria of one period games played by myopic firms is subgame perfect within a repeated game.
If yardstick competition according to (32) and (33) is applied, the firm i will maximize profits for all periods t i.e.

$$\max_{c_i^t}[p_i^t x(p_i^t) + \pi_i^t - c_i^t x_i(p_i^t) - R(c_i^t)], \quad \forall i \in \{1, 2, \cdots, n\}, \quad \forall t \in [1, 2, \cdots, \infty) \tag{37}$$

[13]This constraint resembles the Vogelsang-Finsinger subsidy. Instead of a access fee F-V consider a subsidy and instead of the average total costs firm's own total costs are inserted. For a concise summary Armstrong and Sappington [1].

The sequence of symmetric Nash-equilibrium is characterized by

$$p^{t,YC} = c^{t-1,YC} \quad \text{and} \quad -x(p^{t,YC}) - F_c(c^{t,YC}) = 0, \quad \forall t \in [1, 2, \cdots, \infty) \tag{38}$$

which leads to the non-linear difference equation

$$-x(c^{t-1,YC}) - F_c(c^{t,YC}) = 0 \tag{39}$$

Utilizing this difference equation we can state the following result:

proposition 2 *The Yardstick competition process YC* $\{p_t^{YC}, c_t^{YC}\}$ *converges to the social optimal solution* $\{p^*, c^*\}$ *as unique steady state.*
Proof. See Appendix

The result is not astonishing if one recalls the assumption made to guarantee the existence of a unique optimal solution summarized graphically in picture 1. However, it also shows that it requires a bit time to reach the optimal solution. Hence, if one compares the properties of the two competition mechanisms one has to take into account the dynamic adjustment process a a further criterion of assessment.
Strictly speaking, the dynamic process is only defined for adjustments guaranteeing non-negative profits in each period. For instance, if the regulation device is introduced into a market without regulation where all firms held local monopoly power profits are positive until the process reaches the steady state. Hence, the regulation process is economically viable. But assume that yardstick competition has been introduced and the optimal solution is reached. If demand shrinks for some reasons an adjustment process begins towards the respective steady state. Within this process profits are negative in each period. Since in the steady state firms end up with zero profit there is no opportunity to cover these losses later. As a result, all firms would leave the market. This problem is well known[14] and is a disadvantage of dynamic regulation schemes with a lagged adjustments. There have been some proposals to alleviate or to eliminate this problem. The remedy amounts to introducing a cost-plus scheme., i.e. to allow profits in the steady state. In turn, these adders lead to a opportunity to exploit the regulation mechanism strategically[15] However, in the case of yardstick competition this disadvantage cannot occur since the profit constraint of firm i does not depend on K_i.
Nevertheless, in the following we do not want to elaborate on the optimal profit allowances in the face of uncertain demand but stick to the basic model. Hence, we analyze the adjustment process of MYC that is introduced in a unregulated market[16].
The dynamic properties of the modified yardstick competition follow from the behavior of firms maximizing their profits (37) under the regulatory constraint (36). The sequence of Nash-Cournot-equilibria are characterized[17] by the first-order conditions

$$[p(x^{t,MYC}) - \hat{c}(x^{t,MYC})] + p'(x^{t,MYC})(x^{t,MYC} - x^{t-1,MYC}) = 0 \tag{40}$$

[14] See Armstrong and Sappingtion [1]

[15] See [1].

[16] One can also assume that the origin of the MYC-process is characterized by a cost-plus-regulated monopolist. The crucial point is that the allocation of these monopolies is closer to the Cournot-solution than firms within a MYC-scheme.

[17] Notice, that we have solved the problem by maximizing with respect to x^t, c^t, π^t. The corresponding price p^t follows from the inverse demand function.

where $\hat{c}$ is defined by the first order condition

$$-x^{t,MYC} - F_c(\hat{c}) = 0 \tag{41}$$

The dynamic process of the MYC-mechanism takes place through the non-linear difference equation (40). It can be shown that the steady state is optimal and asymptotically stable.

proposition 3 *The Modified Yardstick Competition process MYC* $\{p_t^{MYC}, c_t^{MYC}\}$ *converges to the social optimal solution* $\{p^*, c^*\}$ *as unique steady state.*
Proof. See Appendix

Introducing a regulatory lag leads to a dynamic structure of the regulation process. As a result, the strategic behavior of firms has to be taken into account. In our model, we have to analyze the nature of subgame perfect equilibria. These equilibria are the result of strategy formation of non-myopic firms[18]. Firms take into account how their decisions in period t affect the Nash-Cournot-equilibrium in period $t+1$, and so forth. As a result, the sequence of one-period strategies of myopic firms is not the same as the sequence of subgame perfect strategies.
However, in the regulatory framework of Yardstick Competition both sequences coincide.

proposition 4 *The sequence of Nash-Cournot-equilibria that result from the profit maximizing behavior of myopic firms is also subgame perfect, i.e. is a closed loop equilibrium of the multi-stage game.*
Proof: See Appendix.

Firms cannot gain from looking ahead. The tariff choice in period t cannot affect the strategy equilibrium in period $t+1$ since the regulatory constraint (36) for period $t+1$ does not depend on these instruments. The regulatory constraint of period $t+1$ does only depend on the choice of the other $n-1$ firms. Hence, the yardstick competition mechanism is not vulnerable against strategic behavior.
As we know from repeated games of infinite length other subgame perfect equilibria do exist[19]. Notice however, that the yardstick competition mechanism is not captured by the model structure of repeated games due to the dependence of payoffs in period t on actions taken in period $t-1$. To answer the question whether a trigger strategy exists within the YC- and the MYC-mecanism entails comparing the present value of profits under tacit collusion of all firms with the extra profits in the case of defection in period t and the present value of the following decreasing profits from $t+1$ until the steady state is reached. The main difference to the analysis of repeated games is that both equilibrium strategies cannot keep profits constant. In the steady state profits are zero. This analysis is important and on the agenda of further research.

[18]In fact, the concept of subgame perfectness was introduced by Selten [15] within a traditional oligopoly model. The dynamic structure was the result of demand slackness.

[19]See Fudenberg and Tirole [8]. It is well known (Folk theorem) that trigger strategies can sustain cooperative solutions.

4 A linear model

So far, the analysis has shown that both competition mechanisms lead firms into a efficient steady state as an unique equilibrium. It remains to investigate the welfare properties of both regulation systems during the transition period.
Unfortunately, the comparison of both systems along the transition path depends on the functional forms and on parameter values of the relevant difference equations. We, hence, confine the following analysis to a linear model structure. This allows us to keep the formal structure simple.
To begin with, we introduce the linear inverse demand function

$$p(x) = B - bx; \quad p'(x) = -b \tag{42}$$

Accordingly, optimal marginal costs can be generated by

$$R_c(c) = -A + ac, \quad c \in [0, A/a] \tag{43}$$

To guarantee th existence of a unique interior social optimum we have to recall conditions (7) and (8):

$$-p(x) > 1/R_{cc}(c) \;\rightarrow\; b > 1/a \quad \text{and} \quad p(0) > \hat{c}(0) \;\rightarrow\; B > A/a \tag{44}$$

where $\hat{c}$ is defined in (19).
If one inserts these functions into the difference equation (39) and recall the lag structure of the regulatory authority, i.e. $p^{t,YC} = c^{t-1,YC}$ then it is easy to arrive at the following linear difference equation

$$p^{t,YC} = \frac{A - B/b}{a} + \frac{1}{ba} p^{t-1,YC} \tag{45}$$

Similar, one can calculate the respective linear difference equation for the Modified Yardstick competition. If one inserts the functional forms (42)and (43) into (40) we obtain

$$p^{t,MYC} = \frac{Ab - B}{2ba - 1} + \frac{ba}{2ba - 1} p^{t-1,MYC} \tag{46}$$

To compare the two difference equations we depict the dynamic structure in a phase diagram:

picture 2 (comparing price trajectories)

From the picture we immediately can derive that the price of the YC-mechanism converges quicker to the optimal steady state than the price of the modified yardstick mechanism. Notice that this does apply to all admissable parameter values sustaining an interior optimal solution.

proposition 5 *The price path of YC is lower than that of MYC, .i.e.* $p^{t,YC} < p^{t,MYC}, \forall t$.
Proof: See Appendix.

The speed of price adjustment is an indicator of the welfare properties of both regulatory mechanisms. Obviously the YC-mechanism is more effective as the MYC-mechanism in terms of welfare. This can be seen from the following picture.
This can be explained with the help of the following picture:

picture 3 (welfare effects)

Let us start from the unregulated Cournot solution[20] $\{p^M, c^M\}$. If the two mechanisms are implemented, prices in period 1 decrease to c^M in the case of the YC-mechanism and to $p^{MYC,1}$ for the modified yardstick competition. Notice, that[21] $p^{MYC,1} > c^M$. From (39) we know how marginal costs are adjusted to the ouptput offered within the YC-mechanism. From (41) we can calculate the optimal marginal costs for firms under the MYC-mechanism. If we plot prices, output and marginal costs for both mechanisms in the picture we can identify total surplus for both regulatory systems. The difference for the first period is indicated by the hatched area. Yardstick competition as introduced by Shleifer realizes welfare gains more quickly than the MYC-mechanism.
However, to reach a final assessment of both mechanisms one has to compare these welfare effects during the transition periods with total welfare reached in the steady state. For instance, if a regulatory authority introduces the MYC-mechanism instead of yardstick competition as introduced by Shleifer to allow for second degree price discrimination she must take into account the slower increase of total welfare and balance these losses to future welfare gains due to discriminatory tariff structure.

[20]In the following we neglect the Index i.

[21]This follows from the comparison of the relevant difference equations (See picture 2). At the outset the price is the unregulated Cournot price p^M. In the first period $p^{1,YC}$ is equal to c^M by definition. From picture 2 we can read off that $p^{MYC,1} > c^M$.

5 Summary and Outlook

In this paper we have introduced a modified yardstick competition mechanism which allows for price discrimination. In a static model we have shown that the MYC-mechanism replicates the social optimum as a Nash-equilibrium. In addition, the mechanism is based on a information set that seems to be more accessible for the regulatory authority than that of the YC-mechanism. Specifically, the regulatory authority needs not to separate total costs into fixed (capital) costs and variable costs. If the local monopolies are multiproduct firms the regulatory agency does not need to know the true allocation of costs to customer groups. The knowledge of total costs and output sold to customers is sufficient to implement the mechanism. Firms are not required to link prices to variable costs. Price discrimination is possible and, hence, leads to increased total welfare.

If one takes regulatory lags into account, then in turns out, that the MYC-mechanism is approaching the optimal steady state slower than yardstick competition as introduced by Shleifer. As a result, a total assessment of both systems has to take into account the welfare effects during the transition path with total welfare in the steady state.

It remains to discuss, how the MYC-mechanism can be implemented when firms are heterogenous. Shleifer has proposed[22] a "reduced-form" regulation. In the case of yardstick competition the regulatory authority runs a regression of variable costs c against observable exogenous characteristics, say θ. The main clue of the regression is that it only includes the variables of all j firms except i if the price caps for firm i are calculated.

If we introduce the MYC-mechanism, the "reduced-form" approach of Shleifer has to be modified as follows. First, regressions must be run between total costs and the observable characteristics. Again, total costs for firm i allowing for local characteristics are calculated without utilizing the total costs of "i" reported to the regulatory authority. Total costs acknowledged for firm i are[23] are $\hat{K}_i = \hat{\alpha} + \hat{\beta}\theta_i$ where $\hat{\alpha}, \hat{\beta}$ are estimated parameter of the reduced form. Secondly, regressions between output and the characteristics have to be conducted. Again, the relevant output in the regulatory constraint (16) is determined by the predictor $\hat{x}_i = \hat{\delta} + \hat{\gamma}\theta_i$. Again, $\hat{\delta}, \hat{\gamma}$ are the respective estimates. Of course, this approach can also be extended to the case of multiproduct firms either under second degree or third degree price discrimination. We have to add regression equations which refer to the output of each consumer group.

The modified yardstick mechanism (MYC) has the clear advantage of reduced information requirements, and thus a potential improvement over the current applications. Further research should address the comparison with traditional yardstick competition in more depth, specifically with regard to investment behavior and tacit collusion, as well as technical implementation issues.

[22] See Shleifer [16] p. 324.

[23] We restrict our discussion to the case where a linear specification are sufficiently good predictors.

6 Appendix

6.1 proof to proposition 1

We first show that the optimal solution $\{x^*, c^*\}$ is a symmetric Nash-Cournot-equilibrium. Recall the first order conditions (18) and (19) and insert $\{x_i^{MYC} = x^*, c_i^{MYC} = c^*, \forall i\}$. Notice thereby, $\bar{x}_{-i} = x^*$. This yields the first order conditions for an social optimum (6). Hence, $\{x^*, c^*\}$ is a Nash-Cournot-equilibrium. Since an interior optimal solution exists per assumption a symmetric Nash-Cournot-equilibrium exists as well. The uniqueness of the optimal solution implies also that only one symmetric Nash-Cournot-equilibrium exists. Since the prices p_i^* equal constant marginal costs c_i^*it follows from (31) that the access fee π_i covers the fixed costs $F(c_i^*)$ and that profits are nil.
It remains to show that no asymmetric Nash-Cournot-equilibrium exists. To do so, we first have to assure, that the first order conditions (18) and (19) are necessary and sufficient for solving (17). According to a theorem of Arrow [24] the Kuhn-Tucker-conditions are necessary and sufficient if both, the profit function and the regulation constraint are quasi-concave. Together with the assumptions with regard to the cost function the assumption $p''(x) \leq 0$ with regard to the inverse demand function is necessary to assure the quasi-concavity of the constraint (16).
From (19) we can find the optimal response-function $\hat{c}(x_i)$ which solves $-x_i - R(c) = 0$. Inserting this response function into (18) and multiplying by $n-1$ yields

$$p'(x_i)[(n-1)x_i - \sum_{j \neq i} x_j] + (n-1)[p(x_i) - \hat{c}(x_i)] = 0 \tag{47}$$

which yields after some arrangements

$$p'(x_i)[x_i - \bar{X}] + \frac{n-1}{n}[p(x_i) - \hat{c}(x_i)] = 0 \tag{48}$$

where $\bar{X} = \frac{1}{n}\sum_{i=1}^{n} x_i$ is the average value of all outputs x_i. (48) is the implizit form of an inclusive reaction function[25] defining x_i as function of $\bar{X}$.
If we now assume, per absurdum, an asymmetric Nash-Cournot equilibrium there must be at least one firm, say firm k, choosing output $x_k > \bar{X}$ and at least one firm, say firm m, choosing $x_m < \bar{X}$. From (48) it follows that $p(x_k) - \hat{c}(x_k) > 0$ and $p(x_m) - \hat{c}(x_m) < 0$ respectively. But this cannot be, since $p(x_k) - \hat{c}(x_k) > 0$ and $p(x_m) - \hat{c}(x_m) < 0$ imply that $x_k < x_m$ (see figure 1 in section 2). Hence, no asymmetric Nash-Cournot-equilibrium exists.

6.2 proof of corollary 2

To proof the corollary we first have to determine the socially optimal two-part tariff system TPS^*. Since its properties are well known we keep the following characterization very brief[26]. The optimal tariff system maximizes total consumer surplus

$$S_l(p_l, \pi_l) + S_h(p_h, \pi_{2h}) \tag{49}$$

subject to the constraints (27) (with Lagrangean λ_1), (28)(with Lagrangean λ_2), (29)(with Lagrangean λ_3) and the profit constraint (with Lagrangean λ_4)

$$(p_l - c)x_l + \pi_l + (p_h - c)x_h + \pi_h - F(c) \geq 0. \tag{50}$$

[24]See Takayama [18].
[25]See e.g. Wolfstetter [22] p. 91.
[26]For further details see e.g. Wolfstetter ([22]) or Gravelle and Rees [11], 280-283.

Notice, that we have omitted the subscript i due to the assumption of identical firms. From the Kuhn-Tucker-conditions it is straightforward to show that

$$\lambda_4(p_l - c)x_l'(p_l) + \lambda_2(x_h(p_l) - x_l(p_l) = 0 \quad (51)$$
$$\lambda_4(p_h - c)x_h'(p_h) = 0 \quad (52)$$
$$-(x_l(p_l + x_h(p_h) - F_c(c) = 0 \quad (53)$$

where $\lambda_4 > 0$ and $\lambda_2 > 0$. These equations follow as a result of the single-crossing-property (24) and the assumption, that the Coase-tariff $TPS^{coase} = \{p_l = p_h = c^*; \pi_l = \pi_h = F(c^*)/2\}$ cannot be implemented, due to violating the participation constraint (29). In the case of a pooling tariff structure consumer surplus of the l-group would be negative. Hence, a sorting tariff structure is to be implemented. Prices $p_i, i = l, h$, are chosen such that consumers with low demand are priced above marginal costs and consumers with high demand pay marginal costs for each service unit. Moreover, we know that the surplus of consumers with low demand ist fully exploited and that profits are zero. In addition, it is also well known that $\lambda_1 = 0$ since the incentive compatibility constraint (27) ist not binding. Collecting all relevant equations, the optimal price discrimination TSP^* and the optimal choice of costs c^* are determined by the equations (28), (29), (50), (52) and (53). The corresponding two-part-tariff structure under yardstick competition follows from deriving the Nash-Cournot-equilibrium. Each firm i maximizes profits

$$(p_{il} - c_i)x_{il} + \pi_{il} + (p_{ih} - c_i)x_{ih} + \pi_{ih} - F(c_i) \quad (54)$$

subject to the constraints (27) (with Lagrangean λ_{i1}), (28)(with Lagrangean λ_{i2}), (29)(with Lagrangean λ_{i3}) and the regulation constraint (31) (with Lagrangean μ_i), $\forall i \in \{1, 2, \cdots, n\}$. The profit maximizing tariff ist characterized by the following first order conditions:

$$(p_{il} - c_i)x_{il}'(p_{il}) + x_{il}(p_{il})(1 - \lambda_{i1} - \lambda_{i3}) - \mu_i \bar{x}_{-il} + \lambda_{i2}x_{ih}(p_{il}) = 0 \quad (55)$$
$$1 - \mu_i - \lambda_{i1} + \lambda_{i2} - \lambda_{i3} = 0 \quad (56)$$
$$(p_{ih} - c_i)x_{ih}'(p_{ih}) + x_{ih}(p_{ih})(1 - \lambda_{i2}) - \mu_i \bar{x}_{-ih} + \lambda_{i1}x_{il}(p_{ih}) = 0 \quad (57)$$
$$1 - \mu_i + \lambda_{i1} - \lambda_{i2} = 0 \quad (58)$$
$$-(x_{il}(p_l) + x_{ih}(p_{ih})) - F_c(c_i) = 0 \quad (59)$$

Assuming the existence of a symmetric Nash-Cournot- equilibrium TPS^{MYC} and inserting (56) into (55) and (58) into (57), respectively, yields the equations

$$(p_l - c)x_l'(p_l) + \lambda_2(x_h(p_l) - x_l(p_l)) = 0 \quad (60)$$
$$(p_h - c)x_h'(p_h) + \lambda_1(x_l(p_h) - x_h(p_h)) = 0 \quad (61)$$

It remains to show that (60) and ((61) determine together with the constraints (27), (28), (29) and (31) the optimal solution TPS^*, i.e. $TPS^{MYC} = TPS^*$.
To begin with, notice that firms do not choose a unique two-part tariff. Assume, per absurdum, that $TPS^{MYC} = \{p_l = p_h; \pi_l = \pi_h\}$. Then, by (60) and (61) it follows that $\lambda_{i1} = \lambda_{i2} = 0$. Together with (53) we can infer that $TPS^{MYC} = TPS^{Coase}$ which, by assumption, cannot be implemented. Hence, $\lambda_{i1} \geq 0$ and/or $\lambda_{i2} \geq 0$, one of which with strong inequality. From (60) and (61) it follows that $\{p_l^{MYC} > p_h^{MYC}\}$.
Next, we show, that $\lambda_{i1} = 0$. It turns out that the constraint (27) is not binding. This can easily be seen if one utilize the Kuhn-Tucker condition $\lambda_{i2}(28 = 0)$ which implies that

$$\int_{p_h}^{\bar{p}_h} x_h(v)dv - \pi_h - \int_{p_l}^{\bar{p}_h} x_h(v)dv + \pi_l = 0 \quad (62)$$

After some rearrangements we end up with

$$\int_{p_h}^{p_l} x_h(v)dv = \pi_h - \pi_l \tag{63}$$

Similar, it follows from (27)

$$\int_{p_h}^{p_l} x_l(v)dv \leq \pi_h - \pi_l \tag{64}$$

Comparing (63) (64) and recalling the single-crossing condition (24) renders (27) as strict inequality. Hence, by the respective Kuhn-Tucker-condition it follows that $\lambda_{i1} = 0$.
Collecting all relevant equations, the Tariff structure and the production costs c under MYC are determined by the equations (28), (29), (31), (61) and (59). These equations are identical to those determining the optimal solution $\{TPS^*, c^*\}$. Hence, the solutions coincide.

6.3 proof of proposition 2

To proof as asymptotical stability of the difference equation (39) we differentiate with respect to $c^{t-1,YC}$ which yields:

$$\frac{dc^{t,YC}}{dc^{t-1,YC}} = \frac{-x'(c^{t-1,YC})}{R_{cc}(c^{t,YC})} > 0 \tag{65}$$

Recalling the second order condition (7) we have $p'(x) > 1/R_{cc}(c) \;\rightarrow\; 1 > -x'(c)/R_{cc}(c)$ where $p = c$ due to the regulation constraint. Hence, the positive slope in (65) is less than 1.
As a next step set $c^{t-1,YC} = 0$. Due to (8) we can derive from $-x^{max} - R_{cc}(c^{t,YC}) = 0$ that $c^{t,YC} > 0$, i.e. the intercept of the difference equation in a $c^{t,YC} - c^{t-1,YC}$-phase diagram is positive. Since the slope is less than 1, a asymptotically stable equilibrium exists.

6.4 proof of proposition 3

To proof the asymptotic stability we show that the slope of first order difference equation (40) is positive and less than 1. Due to the assumed existence of an unique optimal solution the steady state solution is unique as well. To show this, insert the steady state solution$x^{t,MYC} = x^{t,MYC} = \bar{x}^{MYC}$ into (40). This yields $p(\bar{x}^{MYC}) = \hat{c}(\bar{x}^{MYC})$ which is the optimal solution. To prove stability, we have to differentiate (40) with respect to $x^{t-1,MYC}$, taking $x^{t,MYC}$ aus a function of $x^{t-1,MYC}$. This yields

$$\frac{dx^{t,MYC}}{dx^{t-1,MYC}} = \frac{p'(x^{t,MYC})}{[p''(x^{t,MYC})(x^{t,MYC} - x^{t-1,MYC}) + 2p'(x^{t,MYC}) - \hat{c}'(x^{t,MYC})]} > 0 \tag{66}$$

We can see that the bracketed term in the denominator is negative for all $x^{t,MYC} \geq x^{t-1,MYC}$ and given the assumptions with respect to the second order condition of the optimal solution (see picture 1). It is also a straightforward exercise to see that the expression on the right hand side is less than 1. From (40) it follows that $x^{t,MYC}(0) > 0$. Hence, the process is asymptotically stable and converges to the steady state.

6.5 proof of proposition 4

We confine the proof to the MYC mechanism. The proof for YC is a straightforward application of the latter.
To derive a subgame perfect equilibrium we will utilize the principle of optimality. Consider firm i in period t. Firm i wants to maxmize the present value of profits given the history of the previos period $t-1$, i.e. the vector $\mathbf{z}^{t-1} = \{\mathbf{x}^{t-1}, \pi^{t-1}, \mathbf{c}^{t-1}\}$, where the boldfaces indicate n-dimensional vectors of the respective variables for n firms. If a subgame perfect equilibrium exists then each firm solves the following maximization program given the equilibrium strategies of the other $n-1$ firms (principle of optimality[27])

$$
\begin{aligned}
J_{it}(\mathbf{z}^{t-1}) &= \\
& max_{\{x_i^t, \pi_i^t, c_i^t\}}[(p(x_i^t) - c_i^t)x_i^t + \pi_i^t - F(c_i^t) + \\
& \delta J_{i,t+1}(x_i^t, \mathbf{x}_{-i}^{MYC,t}, \pi_i^t, \pi_{-i}^{MYC,t}, c_i^t, \mathbf{c}_{-i}^{MYC,t})]
\end{aligned} \tag{67}
$$

for all $i = \{1, 2, \cdots, n\}$ and $t = [1, 2, \cdots, \infty)$. δ is a constant discount factor and $\{\mathbf{x}_{-i}^{MYC,t}, \pi_{-i}^{MYC,t}, \mathbf{c}_i^{MYC,t}\}$ are the respective subgame perfect equilibrium values of the $n-1$ firms except i. The maximization program for each firm has to take into account the relevant regulatory constraints, i.e.

$$p(x_i^t)\bar{x}_{-i}^{t-1} + \pi_i^t \leq \bar{K}_{-i}^{t-1}, \quad \forall i \text{ and } t \tag{68}$$

and

$$p(x_i^{MYC,t+1})\bar{x}_{-i}^t + \pi_i^{MYC,t+1} \leq \bar{K}_{-i}^t \quad \forall i \text{ and } t \tag{69}$$

where $\bar{x}_{-i}^{t-1}$ and $\bar{K}^b t - 1_{-i}$ are defined in (14) and (13) respectively. $x_i^{MYC,t+1}$ and $\pi_i^{MYC,t+1}$ refer to the subgame perfect equilibrium in period $t+1$.
The optimal value $J_{it}(.)$ depends on the history bfz^{t-1} i.e. the MYC-equilibrium in period t depends on the vector $\mathbf{z}^{t-1}$. We therefore can write

$$
\begin{aligned}
J_{it}(\mathbf{z}^{t-1}) = [(p(x_i^{MYC,t}(\mathbf{z}^{t-1})) - c_i^{MYC,t}(\mathbf{z}^{t-1})x_i^t(\mathbf{z}^{t-1}) + \pi_i^t(\mathbf{z}^{t-1})) - F(c_i^t(\mathbf{z}^{t-1}) \\
+\delta J_{i,t+1}(\mathbf{x}^{MYC,t}, \pi^{MYC,t}, \mathbf{c}^{MYC,t})]
\end{aligned} \tag{70}
$$

and

$$p(x_i^{MYC,t})\bar{x}_{-i}^{t-1} + \pi_i^{MYC,t} \leq \bar{K}_{-i}^{t-1}, \quad \forall i \text{ and } t \tag{71}$$

If we look at the respective definition of $J_{i,t+1}$ (utilizing (67) for period $t+1$) we immediately see that $\mathbf{z}^{MYC,t+1}$ depends on the history $\mathbf{z}^t = \mathbf{z}^{MYC,t}$. Hence, we see that (70) and the relevant constraints (69) and (71) define a closed loop equlibrium.c
The next step to proof the proposition is to show that $J_{it}(.)$ is independent of $x_i^{t-1}, \pi_i^{t-1}, c_i^{t-1}$. First of all observe that (70) and the regulatory constraint (71) do not depend on π^{t-1}. Hence, π^{t-1} does not influence the MYC-equilibrium of period t. Thus, we can confine the analysis to the effects of x_i^{t-1}, c_i^{t-1}. To determine the effects of the latter two we first begin with x_i^{t-1}. We differentiate $J_{it}(.)$:

$$\left.\frac{dJ_{it}(.)}{dx_i^{t-1}}\right|_{\mathbf{h}_{-i}^{t-1}=const.} = \frac{\partial J_{it}(.)}{\partial x_i^t}\frac{\partial x_i^{MYC,t}}{\partial x_i^{t-1}} + \frac{\partial J_{it}(.)}{\partial c_i^t}\frac{\partial c_i^{MYC,t}}{\partial x_i^{t-1}} + \frac{\partial J_{it}(.)}{\partial x_i^{t-1}} \tag{72}$$

where $\mathbf{h}_{-i}^{t-1} = \{\mathbf{x}_{-i}^{t-1}, \mathbf{c}^{t-1}\}$, i.e. is the history of the c-variables and the x- variables except that of firm i.

[27] See Selten [15] and Fudenberg and Tirole [9] p. 130-134.

On the right hand side we easily can show that

$$\frac{\partial x_i^{MYC,t}}{\partial x_i^{t-1}} = \frac{\partial c_i^{MYC,t}}{\partial x_i^{t-1}} = \frac{\partial J_{it}(.)}{\partial x_i^{t-1}} = 0 \tag{73}$$

This follows from inspection of (67) and (71). Both equations do not depend on x_i^{t-1}. Notice also, $x_j^{MYC,t}$ and $c_j^{MYC,t}$, $j \neq i$ do not enter these equations. Hence, indirect effects through the effects of x_i^{t-1} on $x_j^{MYC,t}$ do not enter $J_{it}(.)$.
Similar, we can show that,

$$\frac{dJ_{it}(.)}{dc_i^{t-1}}\bigg|_{\mathbf{k}_{-i}^{t-1}=const.} = 0. \tag{74}$$

where $\mathbf{k}_{-i}^{t-1} = \{\mathbf{x}^{t-1}, \mathbf{c}_{-i}^{t-1}\}$, i.e. is the history of the x-variables and the c- variables except that of firm i. Collecting these results we return to (67) and solve for $\{x_i^t, \pi_i^t, c_i^t\}$. Also, since $J_{i,t+1}(.)$ does not depend on these variables rspectively the first order conditions are the same as for myopic firms, i.e. eqs. (40) and (41). Hence, the the sequence of Nash-equlibria of myopic firms are identical to the subgame perfect equilibrium defined in (67).

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