Chair of Energy Economics and Public Sector Management
Dresden University of Technology
Faculty of Business Management and Economics

Nodal Pricing in the German Electricity Sector – A Welfare Economics Analysis, with Particular Reference to Implementing Offshore Wind Capacities

Authors: Florian Leuthold, Ina Rumiantseva, Hannes Weigt, Till Jeske and Christian von Hirschhausen

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Abstract

This paper compares the results of different pricing systems in the German electricity sector. In particular, we compare a competitive nodal pricing approach to a cost minimization scenario under a uniform price. The model also simulates the effects of increasing offshore wind energy in the North Sea, from the current 0 GW to 8 and 13 GW, respectively. Our model of the German electricity system includes 425 lines and 310 nodes of the 380-kV and the 220-kV grid. Power flows are calculated based on the DC Load Flow Model using a slightly modified version of the traditional approach (Schweppe et al., 1988, Stigler and Todem, 2005). Demand is proxied by linear demand functions that are regionally differentiated. Our results show that the nodal pricing regime is more efficient than uniform pricing and that offshore wind input leads to a significant welfare gain. The model also indicates that 8 GW offshore wind would be accommodated by the current network without significant investments.

JEL-code: L94, L51, D61

Key words: electricity, nodal pricing, welfare, Germany, wind energy

Abbreviations

AC	alternating current
CAISO	California Independent System Operator
CLP	competitive locational price
DC	direct current
DCLF	DC Load Flow Model
DENA	Deutsche Energie-Agentur (“German Energy Agency”)
HV	high voltage
ISO	independent system operator
kV	kilovolts
kW	kilowatts
kWa	kilowatt years
LBMP	location-based marginal pricing
LMP	locational marginal price
MC	marginal cost
MCP	market clearing price
MW	megawatts
MWh	megawatt hours
OC	opportunity cost
P	real power
Q	reactive power

Nomenclature

Symbols:

A	surface area [m^2]
a	prohibitive price [€/MWh]
B	line series susceptance [1/Ω]
b	slope
C	total costs of production [€]
d_n	demand at node n [MWh]
d_n^{ref}	reference demand at node n [MWh]
d^*	equilibrium demand [MWh]
d_n^*	equilibrium demand at node n [MWh]
G	line series conductance [1/Ω]
g_n	generation at node n [MW]
g_n^t	generation of plants of type t at node n [MW]
$g_n^{t,max}$	maximum generation capacity of plants of type t at node n [MW]
I_{max}	maximum allowable current line flow [A]
L_{jk}	losses of real power [MW]
l	length of a line [m]
P_{jk}	real power flow between two nodes [MW]
P_i	real power flow at line i [MW]
P_i^{max}	transmission capacity constraint at line i [MW]
p_{ref}	reference price [€/MWh]
p^*	equilibrium price [€/MWh]
p_n^*	nodal price at node n [€/MWh]
R_i	line resistance [Ω]
$V_{j,k}$	voltage magnitude at a node [volts]
W	welfare [€]
X_i	line reactance [Ω]
X_m	line reactance for m circuits [Ω/km]
Z	line impedance [Ω]
$\delta_{j,k}$	voltage angle at a node [rad]
ρ	specific electrical resistance (material characteristic) [Ωm]
ε	demand elasticity at reference demand
Θ_{jk}	voltage angle difference [rad]

Indices:

i	line between node j and node k	ref	reference
j, k	node within the network	t	type of generation plant
m	number of circuits		
max	maximum		
n	total number of nodes within the network		

1 Introduction

Nodal pricing has emerged as a powerful and efficient tool of transmission pricing, both in theory and – more recently – also in practice. Lessons from North America, Australia, and even in the UK and Scotland have proven nodal pricing to be workable without serious technical problems. Also the European Commission is currently considering nodal pricing in the context of the creation of the European single electricity market. Therefore, it seems that the continental European countries, too, should take nodal pricing more seriously.

Germany is currently undergoing substantial structural changes in the regulation of its electricity industry. It has not only finally implemented a sectoral regulator as required by the Acceleration Directive 2054/05/EC. Germany is currently also pondering more efficient network pricing both within the country, and in connection to its neighboring countries. In addition, Germany has pushed strongly for the development of renewable energies and their integration into the existing network. This is particularly the case for wind energy, where Germany has installed 17 GW onshore wind capacities; additional 5 to 15 GW are expected to be constructed offshore. At present, the technicalities of the integration of the offshore wind energy are being studied by industry and government, there is also a debate about the economic costs of integrating large-scale wind power into the German Electricity (DENA 2005a).

In this paper, we develop a model for nodal pricing in the Germany electricity sector. We carry out a welfare-economics analysis, paying particular attention to the effects of implementing large-scale wind power offshore. Our hypothesis is that nodal pricing is a more efficient method for allocation scarce capacities than the uniform pricing which is currently applied in Germany. By using a simplified version of the German high-voltage grid, we can simulate different pricing methods, as well as run scenarios about the deployment of large-scale wind power capacities in the North Sea. The objective of the paper is to come up with an economic *and* technical analysis of the issue, whereas technical aspects have dominated the debate so far.

The paper is structured in the following way: The next section provides an overview of the literature on nodal pricing, as well on a recent study of integrating wind energy in the German electricity grid. Section 3 describes the model that we use and the data. We use DC Load Flow Model according to Schweppe et al (1988) and Todem and Stigler (2005). We use a data set on the power generation in Germany (traditional and wind power), as well as the high voltage electricity (380 kV and 220 kV) networks. Section 4 gives an overview of the scenarios that we compare: Nodal pricing is compared to the cost minimization scenario under uniform pricing. Thereafter, different levels of offshore wind generation are introduced (8 GW and 13 GW). Section 5 then provides the results of the scenarios and their interpretation. We find that nodal pricing is superior to the cost minimization scenario.

Furthermore, we derive that wind capacities of about 8 GW can be accommodated by the existing grid, when using nodal pricing. In the scenario 13 GW offshore, significant network extensions become necessary. We conclude that nodal pricing is clearly superior in all scenarios and that it should be used for pricing electricity in Germany, whatever the size development of offshore wind energy will be.

2 Literature Review

2.1 DENA grid study

A recent study from the German Energy Agency (DENA 2005a) indicates high additional costs caused by the integration of additional wind plants into the existing grid by the year 2015. Particularly, the grid extensions due to emerging network bottlenecks would be cost-intensive. For the further development of renewable energy in Germany an efficient integration of onshore and offshore wind energy into the existing power system is important. Several capital-intensive investments would have to be made in order to keep the grid system reliable.

The study develops strategies for the increased use of renewable energies and their effects on the grid until 2015. The study focuses on the integration of the approximately 37 GW wind capacity – on- and offshore – into the electricity grid since, on a mid-term basis, wind has the highest potential of increasing the share of renewable energies in power generation. The DENA grid study is based on the current German uniform pricing model. The major results of the study are (DENA, 2005b, pp. 4-15):

- Approximately 400 km of the existing 380 kV grid has to be upgraded; approximately 850 km new construction is needed.
- Reliable energy supplies on today's standards can be guaranteed if certain technical measures are implemented.
- Approximately 20 to 40 million tons CO_2 emissions can be avoided until 2015 according to the structure of the power plans in operation.
- The additional costs for the expansion of wind energy will cost private households between 0.39 and 0.49 Cent € per kWh in 2015.

2.2 Pricing mechanisms

Competitive markets for electricity determine either a uniform marginal price, a set of nodal or locational marginal prices (LMP), or only a few zonal marginal prices. Although theory proves LMPs to be the most efficient, critics find the large number of LMPs – compared to one uniform or several zonal prices - confusing. They claim a uniform- or zonal-based model to be more transparent. The following section briefly describes the present pricing mechanism in Germany and the theoretical concepts of uniform, zonal and basic prices.

2.2.1 Present situation in Germany: cost minimization under uniform pricing

Electricity pricing in Germany relies on bilateral contracts and is based on a mixed price calculation containing a fixed component for network access and a variable demand charge. The latter is paid per unit of energy actually purchased. By paying a fixed network access charge, the customer rents a particular band which will be reserved for his energy delivery. This payment covers costs from losses, ancillary services, voltage transformation, and access to networks at lower voltage levels.

Identical consumers regarding peak load and amount of demand pay the same amount of money for energy transmission, independent from the location and the point in time of their consumption – uniform pricing of transmission. In case of a competitive market, there is also a uniform price of generation (equaling marginal cost) as energy is delivered for a fixed charge independent from load constraints. Hence, the assumption of a fixed transmission charge and a competitive market for production lead to uniform prices at each node.

Uniform pricing has been applied in Finland (since 1998), Sweden (since 1996), Alberta (since 2001), and Ontario (since 2002) and was in operation in the former England/ Wales-Pool (1990-2005), PJM (1997-1998) and in the first phase of the New England market from 1999 to 2003 (Ding and Fuller, 2005). Uniform pricing works efficiently only in the absence of congestion. In the case of congestion an uplift payment is required. This payment covers all costs from congestion but does not send adequate market signals as provided by nodal prices (see Krause, 2005). Therefore uniform pricing is not able to ensure an optimal allocation of energy and transmission capacities in a situation of congestion as seen e.g. in the case of New England (see Hogan, 1999). Xingwang et al (2003) sum this problem up as the incapability of uniform pricing to achieve harmony between market liquidity and efficient pricing.

2.2.2 Zonal pricing

One attempt to solve incentive problems of the uniform pricing approach was to introduce zonal pricing, which is currently applied in Norway (since 1991), Australia (since 1998), and Denmark (since 2000). The California ISO used nodal pricing from 1998 to 2002 (Ding and Fuller, 2005). According to this approach, the market is divided into several zones depending on their respective congestion costs. Higher prices are paid in zones where demand exceeds system capacity of transmission. The price of the respective reference node is applied to the whole zone. Zones are usually pre-defined and fix.[1]

Proponents of zonal pricing claim that it would balance well equity concerns and efficiency goals and is less complex and therefore more transparent to market participants (see Alaywan and Wu, 2004, p. 1). On the other hand, the zonal approach is criticized for its potential of market power abuse during periods of high demand and resulting congestion (see e.g. Borenstein et al, 2000). Johnsen et al (1999), however, could not find clear empirical evidence in a study on Norway.

Hogan (1999) rejects the model of zonal prices for a number of reasons. He refers to zonal pricing as "[…] an effort to treat fundamentally different locations as though they where the same […]" (p. 1). It would create more administrative rules, poorer incentives for investments, demands to pay generators not to generate power, and finally it is much more complicate to define zonal than nodal prices.[2] Contrarily, Krause (2005, p. 34) claims the zonal pricing system working fairly in Australia and Norway (see also Johnsen et al, 1999, p. 1). However, according to Alaywan and Wu (2004, p. 1), the zonal market design of California was considered having contributed to the energy crisis in 2000 and 2001.

However, regarding the evolution of market structures worldwide, nodal pricing seems to become increasingly the benchmark of congestion management due to its simplicity, effectiveness in practice and conformity with economic theory and physical laws.

2.2.3 Nodal pricing

Nodal pricing[3] is a method of determining market clearing prices for a number of locations on the transmission grid, called nodes. Each node represents a physical location on the transmission system including generators and loads. The price at each node reflects the locational value of energy, which includes the cost of energy and the cost of delivering it (i.e. losses and congestion). Nodal prices are determined by calculating the incremental cost of serving one additional MW of load at each respective location subject to system constraints (e.g. transmission limits, maximal generation capacity). Differences of prices between nodes reflect the costs of transmission.[4]

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[1] Johnsen et al. (1999, p. 3) state that the distinction between a nodal or zonal system in Norway is – for the reason of varying zones – less clearly defined. However, Norway's system is usually referred to as zonal. In Norway, zones may vary depending on the actual situation in the grid regarding congestion. Consequently, if the system is unconstrained there is only one zone (and the same price as under uniform or unconstrained nodal pricing), which was the case for 43.8% of the hours in 1998 (see Johnsen et al, 1999, p. 34). There were maximal six zones due to congestion (0.4% in 1998).

[2] Hogan cites the 1997 PJM attempt to install zonal pricing as an example, where the system collapsed as soon as constraints occurred. Generators rather run than respect transmission constraints – just responding to (distorted) signals from zonal pricing.

[3] There are at least three alternative denominations of "nodal prices": "Locational Marginal Price/ LMP" (PJM), "Location-Based Marginal Pricing/ LBMP" (NYISO) and "Competitive Locational Prices/ CLP" (Stoft, 2002).

[4] Congestion occurs if both of the following two conditions are fulfilled (Stoft, 2002, p. 392):

In order to optimize dispatch in a nodal pricing system, a classic supply and demand equilibrium price, has to be developed: The marginal generator is determined by matching offers from generators to bids from loads at each node. This process is carried out for a specific time interval (e.g. every 15 minutes) at each input and exit node on the transmission grid. The prices take into account the losses and constraints in the system, and generators are dispatched by the system operator, not only in ascending order of offers (or descending order of bids), but in accordance with the required security of the system. This results in a spot market with bid-based, security-constrained, economic dispatch with nodal prices as proposed by Hogan (2003, p. 2).

Nodal prices reflect the actual situation in the grid more transparently than uniform prices and represent adequate allocation signals. Nodal prices are one of several important considerations in analyzing where to site additional generation, transmission and load. The implementation of efficient congestion management methods on the basis of nodal pricing is crucial to cope with scarce transmission capacities and to ensure security of supply. In combination with further political measures there might be saved costly investments in transmission lines (see Bower, 2004).

Nodal pricing was first implemented in New Zealand (1997), followed by some US markets (e.g. PJM 1998, New York 1998, New England 2003).[5] On 1 April 2005, the British Electricity Trading and Transmission Arrangements ("BETTA") were introduced in UK extending the earlier "New Electricity Trading Arrangements" for England and Wales (NETA) to Scotland. With BETTA, nodal pricing was introduced for the British grid on the basis of marginal transmission investment requirements (Tornquist, 2005). The California ISO currently redesigns the procedures by which it performs forward scheduling and congestion management and plans to introduce nodal pricing by 2007 (CAISO, 2005).

2.2.4 Empirical studies on nodal pricing

Empirical analyses using the nodal pricing concept have been provided, e.g. for England/Wales, Austria, Italy and, most recently, for California. Green (2004) developed a thirteen node model of the transmission system in England and Wales incorporating losses and transmission constraints. The study analyzes the impact of different transmission pricing schemes (LMP, zonal and uniform pricing). Green shows that the introduction of the LMP concept would raise welfare by 1.5% compared to the uniform model on behalf of the larger consumer welfare (+2.6%) while generator profits would decrease by 1.1%. To strengthen these results, Green applies different values for demand

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1. The marginal costs of production differ between nodes.
2. Overall demand exceeds supply ability of the "cheapest" generator due to limited production or constrained line capacity. A line constraint can be caused when a particular branch of a network reaches its thermal limit or when a potential overload will occur due to a contingent event on another part of the network (e.g. generator black out). The latter is referred to as a security constraint.

[5] According to Ding and Fuller (2005), nodal pricing was introduced even earlier in some Latin American states (Chile 1982, Argentina 1992, Peru 1993, Bolivia 1994).

elasticity (-0.1, -0.25, -0.4) and shows that the increase of welfare is higher with a larger absolute elasticity value.

For the Austrian high voltage grid, Stigler and Todem (2005) have analyzed the economic impact of a nodal price based congestion management. Against the background of scarce transmission capacities in the East of Austria, the authors developed an optimization model with 165 nodes applicable to the bilateral Austrian electricity market.[6] On the basis of January 2004 data, they show in which places congestion occurs and which prices would be optimal. Stigler and Todem suggest a division of the network into two pricing zones according to their congestion situation. The most efficient solution to overcome the congestion problem would be to build an additional 380-kV line – the so called 'Steiermark'-line.

Ding and Fuller (2005) provide interesting results regarding the distribution of economic surplus under nodal, uniform and zonal pricing. They show for the Italian 400 kV grid that there is no loss in (total) social surplus using uniform or zonal pricing with a nodal pricing dispatch compared to a full nodal price system (dispatch and pricing nodal-based). The authors therefore calculated optimal dispatch on the basis of an optimal power-flow model, respecting transmission constraints and losses while defining uniform (respectively zonal) prices for financial settlements. The results, however, show that the distribution of economic surplus between supply and demand sides will vary depending on the pricing model. More importantly, the authors reveal perverse incentives for generators that are dispatched at different levels than uniform or zonal prices would suggest. "Constrained-on" generators may receive a smaller surplus than under nodal pricing settlement, even though the extra generation is needed (and vice versa for "constrained-off" generators). However, as economic data of the study where not completely realistic, the authors did not draw firm conclusions.

The California ISO has – in the run-up to the planned implementation of its Market Redesign and technology Upgrade (MRTU)[7]- provided several studies on locational marginal pricing. The most recent one (August 2005) uses schedules and market bids of previous years, conditions of the future MRTU structure and the ISO's full network model in an Alternating Current (AC) Optimal Power Flow (OPF) simulation to estimate prices that may occur in the ISO's real-time market if it were based on locational marginal prices (CAISO 2005, p. 1). Prices were calculated and given as average per zone (total of 29 zones). The resulting LMPs are generally moderate, apart from some exceptions: less than 1% of the nodal prices exceeded \$100/MWh, and 91% of the nodal prices were below \$65. Furthermore, prices within one zone were generally very similar while significant zonal price

[6] Other electricity markets using the nodal price approach are usually centrally organized (PJM Interconnection, NYISO and New Zealand).

[7] The MRTU proposes a forward and real-time congestion management procedure that adjusts generation, load, import, and export schedules to clear congestion using an Alternating Current (AC) Optimal Power Flow algorithm (OPF) and a Full Network Model (FNM) that includes all buses and transmission constraints within the CAISO Control Area. (CAISO, 2005, p. 4)

variations last only a few hours per year. In conclusion, it was found that LMP pricing would produce stable and predictable prices. This result may refute concerns regarding the potential for high LMPs in certain constrained areas of the grid, where the cost of delivering energy to customers is increased due to frequent, severe congestion.

3 Model and Data

3.1 Optimization problem

A standard DC load flow model was used to simulate the German high voltage transmission system. The grid comprises 310 nodes (plus 19 auxilliary nodes) and two voltage levels (380 and 220 kV). A time static approach was chosen. Thus, different scenarios were computed separately, analyzed, and, subsequently, compared to each other. The period of time referred to is one hour. We do not consider a transmission reliability margin [(N-1)-constraint]. The model stresses transmission lines up to 100% of their thermal limit. This must be taken into account while analyzing results.

3.1.1 Cost minimization under uniform pricing

In both the nodal and the uniform pricing model social welfare is the objective value to maximize. The welfare equals total consumers' benefit minus costs of generation, what is identical to the sum of producers' and consumers' surplus (Figure 1).[8] The model determines optimal dispatch quantities of generation and loads as well as the voltage angles at each node/bus while respecting the physical laws[9] of power flow, particularly Kirchhoff's laws, capacity constraints of lines and generators, and demand characteristics.[10] In the case of a uniform price, price and demand per node are fixed. This is an admissible simplification for the static approach. In order to maximize welfare, the cost minimal dispatch has to be found. Hence, it becomes a cost minimization problem

[8] See Appendix A.

[9] Physical facts implicate that the maximum energy flow is limited. In case the so-called thermal limit is passed, undisturbed operation is not longer guaranteed. Moreover, current does not leave a node arbitrarily through any lines. Different outgoing lines act as current divider. That means that current flows leave a node reciprocally proportional to the impedances of the respective lines. The effect is that one cannot inject more energy at this node once on of the outgoing lines is congested even if other lines are still able to work with higher load. This is, particularly, decisive in highly meshed networks. For further information see relevant technical literature, e.g. Lunze (1987), Stoft (2002).

[10] To put it in a nutshell, all of the technical specifics are not primarily of economic relevance. However, they make up the framework for an economic consideration of electricity networks and must form constraints in the economic optimization problem. For deeper matter compare Koettnitz and Pundt (1967), Koettnitz et al (1986), Lunze (1987) and Stoft (2002).

$$\max\ W(d_n^{ref}) = \sum_n \left(\int_0^{d_n^{ref}} p_{ref}(d_n^{ref})\, d \cdot d_n^{ref} - \int_0^{d_n^{ref}} c(d_n^{ref})\, d \cdot d_n^{ref} \right) \tag{1}$$

s.t.	$\lvert P_i \rvert \leq P_i^{\max}$	line flow constraint	(2)
	$\sum_n g_n = \sum_n d_n + L$	energy balance constraint	(3)
	$\sum_{n,t} g_n^t \leq \sum_{n,t} g_n^{t,\max}$	generation constraint (per type of plant)	(4)

Total costs comprise only marginal costs of production at the power plants. Other costs as e.g. those arising from network operation and maintenance are neglected.

3.1.2 Nodal Pricing

In the case of nodal prices, welfare is maximized by finding the optimal demand for each node (Figure 1). Hence, the following set of equations has to be solved:[11]

$$\max\ W(d_n^{*}) = \sum_n \left(\int_0^{d_n^{*}} p^{*}(d_n^{*})\, d \cdot d_n^{*} - \int_0^{d_n^{*}} c(d_n^{*})\, d \cdot d_n^{*} \right) \tag{5}$$

s.t.	$\lvert P_i \rvert \leq P_i^{\max}$	line flow constraint	(6)
	$\sum_n g_n = \sum_n d_n + L$	energy balance constraint	(7)
	$\sum_{n,t} g_n^t \leq \sum_{n,t} g_n^{t,\max}$	generation constraint (per type of plant)	(8)

[11] Constraints to be obtained are the same as above. Compare also Hsu (1997) and Green (2004).

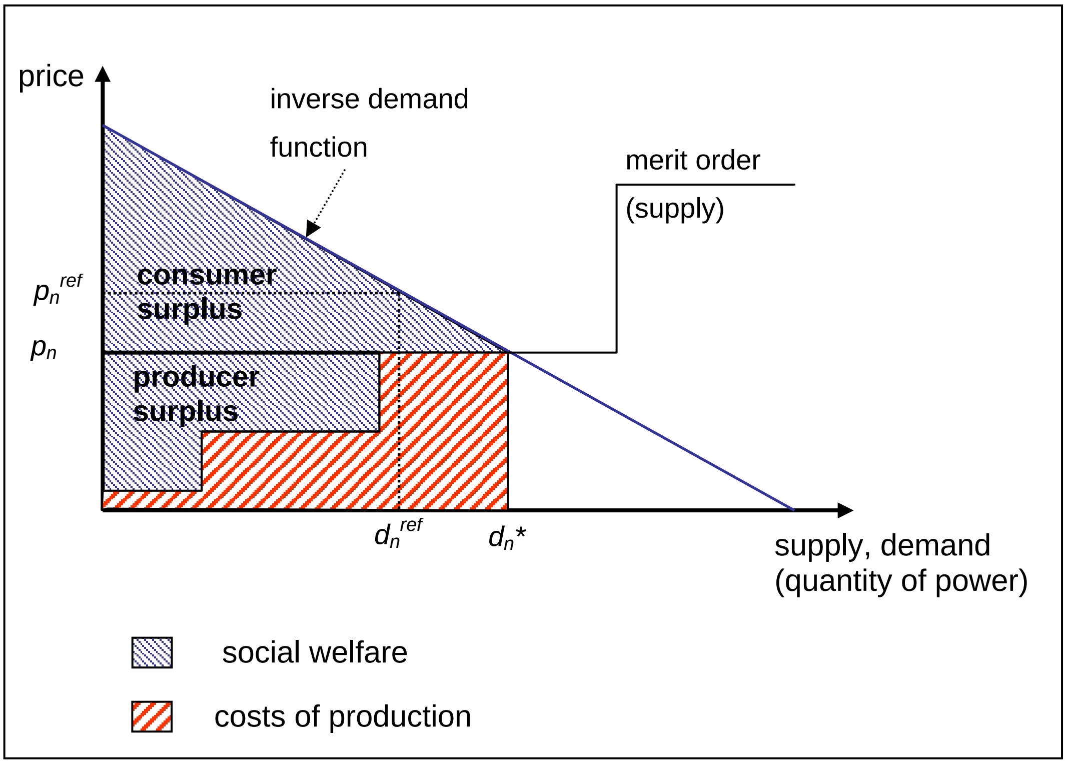

Figure 1: ***Social welfare and market clearing price***

Having derived the optimal dispatch for every node d_n^*, the corresponding market clearing nodal price p_n is given by the inverse demand function:[12]

$$p_n = p_n^{ref} + \frac{1}{\varepsilon} \cdot p_n^{ref} \cdot \left(\frac{d_n^*}{d_n^{ref}} - 1 \right) \tag{9}$$

3.2 The DC Load Flow Model

In general, Schweppe et al (1988) show that the DC Load Flow Model (DCLF) can be used as an instrument for an economic analysis of electricity networks. They apply it to their nodal price approach for electricity pricing. As calculations in electricity networks are sophisticated due to the occurrence of reactive power and the flow characteristic of electricity in highly meshed HV networks, simplifications are necessary. The DCLF helps to simplify the modeling of such networks in case of symmetrical steady states. The DCLF focuses on real power flows. It is, in particular, applicable for economic purposes as the transport of real power is the main task of electricity networks (Todem et al,

[12] Derivation of this equation is presented in Appendix A.

2005, p. 5). Hence, real power is the main commodity that customers demand and that generates benefits.[13]

Overbye et al (2004, p. 2) emphasize three advantages of the DCLF compared to an AC model:

1. The problem becomes smaller (about half the size).
2. The solution is noniterative.
3. The network topology does not depend on the power flowing and has to be factored once only.

Furthermore, they argue that the DCLF is adequate for modeling LMPs albeit there are some buses at which the deviation is significantly high. The latter particularly occurs on lines with high reactive power and low real power flows (Overbye et al, 2004, p. 4).

Schweppe et al (1988, pp. 272-274) describe the way from a complete AC Load Flow to a DCLF. Therefore, a decoupled AC Load Flow model is generated assuming that real power (P) flows according to the differences of the voltage angles (Θ_{jk}) between two nodes. Reactive power flow (Q) is caused by differences in voltage magnitudes (V). Consequently, one can model the real power flow by only focusing on voltage angle differences. The paper of Stigler and Todem (2005, pp. 114-115) explains the basic equations that are described by Schweppe et al in detail:

$$P_{jk} = G_i \left|\underline{V}_j\right|^2 - G_i \left|\underline{V}_j\right| \left|\underline{V}_k\right| \cdot \cos\Theta_{jk} + B_i \left|\underline{V}_j\right| \left|\underline{V}_k\right| \cdot \sin\Theta_{jk} \tag{10}$$

$$\Theta_{\mathrm{jk}} = \left(\delta_j - \delta_k\right) \tag{11}$$

$$B_i = \frac{X_i}{X_i^2 + R_i^2} \tag{12}$$

$$G_i = \frac{R_i}{X_i^2 + R_i^2} \tag{13}$$

Equation (10) is the basis for all further calculations – both the lossless DC load flow and the transmission losses (Stigler and Todem, 2005, pp. 116-118). Moreover, two basic assumptions must be made (Schweppe, 1988, p. 314):

1. The voltage angle difference Θ_{jk} is very small.
2. The voltage magnitudes $\underline{V}$ are standardized to per unit calculation. Hence, they can be considered to be equally one at each node ($\underline{V}_j \approx \underline{V}_k$).

[13] Relevant reactive power issues such as the necessity or influence, respectively, of investments in compensation facilities can not be modeled by DC flows

The calculation of lossless real power flows is the first step along the way to use the DCLF in a dynamic economic model of an electricity network. In order to approximate the lossless line flows, suppose that:

$$\cos \Theta_{\mathrm{jk}} \approx 1 \tag{14}$$

$$\sin \Theta_{jk} \approx \Theta_{jk} \tag{15}$$

This yields a linear equation for the lossless line flows:

$$P_{jk} = B_i \cdot \Theta_{jk} \tag{16}$$

The second step is the estimation of losses occurring along the line. Losses are important as they cause the sum of generation not to equal the sum of demand. Thus, transmission lines are stressed by demand plus losses. In order to approximate the losses on a line, equation (14) must be complemented by the second order term of the Taylor series approximation:

$$\cos \Theta_{\mathrm{jk}} \approx 1 - \frac{\Theta_{jk}^2}{2} \tag{17}$$

Then, using further assumptions and mathematical rearrangements[14], transmission losses can be calculated by:

$$L_{jk} = R_i \cdot P_{jk}^2 \tag{18}$$

Both equations (16) and (18) provide the required relationship between demand and generation as well as the resulting real power flow. One can now implement the model to observe changes in line flows caused by changes in demand or generation, respectively. Combining this with a set of economic information such as demand and supply functions for each node will enable us to assign a specific price for each node of the network.

3.3 Data

The subsequent chapter describes the empirical data used in the model. The nodes are taken as the substations from the German integrated network (VGE, 2000, UCTE, 2004). Only substations of the high and extra high voltage level were taken into consideration under the assumption that the entire

[14] See Appendix B.

electricity transportation for all voltage levels takes place through high voltage transmission. Hence, 291 regular plus 19 auxiliary nodes within the 380 kV and the 220 kV levels were detected.[15]

A line's characteristic can be described by three main factors: maximum thermal limit, line resistance and line reactance. The maximum thermal limit is, basically, influenced by the type and the length of the line as well as by the voltage We assumed four cables[16] per wire for 380 kV circuits and two cables[17] per wire for the 220kV level (Pundt, 1983, p. 11 et seq., Pundt and Schegner, 1997, p. 38 et seq.). An adequate value for the apparent power *S* is 1500 MVA for the 380 kV level up to a length of 100 km, and, respectively, 400 MVA for a 220 kV level circuit up to a length of 90 km (Pundt, 1983, p. 11). The admissible apparent power decreases for a continuous line longer than the given lengths (ibid.).

Maximal current can be derived as follows:

$$I_{\max} = \frac{S}{\sqrt{3} * V} \tag{19}$$

In our model, the possible current doubles when using a double circuit line, and is three times larger for a triple circuit line. These maximal current values are necessary for the maximum power flow constraint in the model. Realistic values for the resistances and reactances of high voltage circuits are subject to empirical experiences. Pundt and Schegner (1997, p. 39) give a satisfactory approximation for reliable values within the German grid (Table 1).

Number of circuits	**Voltage level [kV]**	**Resistance [Ω/km]**	**Reactance [Ω/km]**
Double circuit	380	0.03	0.26
	220	0.078	0.29

Table 1: ***Values for reactance and resistance***

Source: Pundt and Schegner (1997, p. 39).

[15] Auxiliary nodes became necessary where lines split up without a node or where the course of a line is ambiguous. Lines of different voltage levels are listed separately, so the there may be more than one connection between two nodes – e.g. one 380 kV double circuit and one 220 kV double circuit. Our model embraces 425 electricity lines for Germany only. It does not include cross-border flows.

[16] 240/40 AlSt.

[17] 185/32 AlSt.

The evaluation of the capacity of German power plants was based on several sources, mainly the ‘Yearbook on European Energy and Raw-Materials Industry 2005’ (Table 2).[18] For power plants with the possibility to run with an alternative type of fuel, only the main type of fuel was regarded.

The data for wind energy converters were taken from the German Wind Energy Association’s report on installed wind energy capacity (DEWI, 2005). The total capacity amounts to nearly 17 GW. In 2005, over 17,000 wind energy converters were installed in Germany. To simplify the data integration, wind concentration zones were established comprising three to five zones per federal state. The cumulated installed capacity per federal state was divided by the number of wind concentration zones in the specific state and allocated to the concentration zones. The capacity of each concentration zone was allocated equally to surrounding nodes located a maximum of 50 km from the zone.[19]

Fuel	Installed capacity [GW]	Fuel	Installed capacity [GW]
Coal	31.222	Wind (onshore)	16.695
Lignite	20.982	Natural gas	18.146
Nuclear Power	20.680	Fuel oil	6.078
Pump water	5.950	***Total***	***103.058***

Table 2: German power plant capacities

Source: VGE (2004), own calculations.

3.3.1 Generation costs

The node specific generation costs are calculated on a marginal cost basis. There are several studies and approaches to estimate marginal costs of power generation (see EIA, 2004, p. 49, Pfaffenberger and Hille, 2004, DENA, 2005a, p. 278). In our study, the marginal costs are based on the costs of the fuel excluding operating and service costs. An exception is the wind power generation, which was priced at costs according to estimations in the DENA study (DENA, 2005b, p. 14). We refer to these costs as opportunity costs of wind as they arise, particularly, from balancing and response power costs. For all other power plants, we use the average marginal generation cost per plant type according to Schröter (2004, p. 7) as they seem to form a mean compared to the above mentioned studies (Table 3).

[18] See also http://www.energy-yearbook.de/.
[19] The simplification may lead to higher congestion at nodes near concentration zones than in reality.

Fuel	Marginal costs [€/MWh]	Fuel	Marginal costs [€/MWh]
Coal	18.00	Wind	4.05
Lignite	15.00	Natural gas	40.00
Nuclear power	10.00	Fuel oil	50.00
Pump water	13.33	Running water	0.00

Table 3: Marginal costs of power generation per fuel

Source: DENA (2005a) and Schröter (2004).

3.3.2 Demand

In order to derive node-specific demand, we assume a positive correlation between economic income and total electricity demand. We split the federal states into administrative local districts and identified their population figures (DESTATIS, 2005). Inhabitants per node were calculated distributing a district's population figure equally to all nodes of the district. In a second step, annual per capita energy consumption had to be determined for every node. Therefore, the annual average per capita energy consumption of Germany (DESTATIS, 2005) was multiplied by the ratios of Germany's total GDP and the federal states specific GDP (Statistik-Portal, 2005). This resulted in a weighted per capita consumption for every federal state. All nodes within one federal state were assumed to have the same per capita consumption. Multiplying the annual per capita consumption of a node by its population figure and dividing this by 8,760 finally gives the hourly node specific demand. Summing up the nodes' demand resulted in a total demand of 56.24 GWh.[20]

4 Scenarios, Results, and Interpretation

4.1 Scenarios

Four basic scenarios were considered (Table 4):

1. *Status quo*: no additional offshore wind energy plants using the cost minimization approach.
2. *Nodal prices without offshore wind:* no additional offshore wind energy plants using the nodal pricing model.

[20] A disadvantage of the received data is that the results are average values. This lowers the significance of the model because the variability of demand remains unconsidered. In order to solve this problem, the node specific demands will be modified in different scenarios and adjusted by system load data of the respective transmission system operators.

3. *Nodal prices plus 8 GW:* additional 8 GW offshore wind energy plants using the nodal pricing model.
4. *Nodal prices plus 13 GW:* additional 13 GW offshore wind energy plants and grid extension using the nodal pricing model.

Furthermore, for all of these scenarios, reference demand was varied. Average demand was assumed to equal 56.24 GW. According to VDN (2005), peak load in Germany was 77,200 MW in 2004, being almost 1.4 times the average load. For this reason, high demand was calculated multiplying average demand by 1.3. Low demand was assumed to be 0.7 times average demand.

The installed onshore wind energy capacity was allocated to different wind power generation zones, which were then assigned to certain nodes.[21] For the calculation of the load flow it was assumed that the feed-in of offshore and onshore generated electricity is at most equal to the aggregated installed capacity of the wind plants.

Scenario	Demand at loads	Price	Capacity of offshore wind energy plants	Grid capacity
1.) Status quo	low average high	fix	0 GW	existing lines
2.) Nodal prices without offshore wind	low average high	nodal	0 GW	existing lines
3.) Nodal prices plus 8 GW	low average high	nodal	8 GW	existing lines (full capacity)
4.) Nodal prices plus 13 GW	low average high	nodal	13 GW	grid extension

Table 4: Scenarios

In the subsequent sections, results will be discussed. Scenarios will be compared as following:

[21] In this study no geographical differences in the strength of wind (e.g. strong wind vs. light wind) were adopted. In case of distinguishing wind generated electricity by regions, a higher load on the transmission lines from North to South and from East to West would result (DENA, 2005a, p. 75 et sqq.).

- Status quo vs. nodal prices without offshore wind (scenario 1 vs. scenario 2)
- Nodal prices without wind vs. nodal prices plus 8 GW (scenario 2 vs. scenario 3)
- Nodal prices plus 8 GW vs. nodal prices plus 13 GW (scenario 3 vs. scenario 4)

4.2 Results and interpretation

4.2.1 Status quo vs. nodal pricing (scenario 1 vs. scenario 2)

In a first step, we analyzed whether nodal pricing was superior to cost minimization under uniform pricing with regard to the respective social welfares. To ensure comparability of these scenarios, the same input data were used. Within the nodal price scenario demand and price could vary, whereas, the cost minimization approach uses a given uniform price. Neither of these scenarios considers the integration of additional offshore wind energy. Thus, the impact on social welfare of introducing a competitive nodal pricing scheme in Germany compared to the current situation is obtained. The results refer to hourly values. Marginal cost bidding and a demand elasticity of -0.25 at the reference point are supposed. In order to define the reference price, the EEX average price[22] for the relevant period – same as for demand calculation – was estimated using the 200-day-line.

	Cost minimization			**Nodal pricing**				
Scenario	Welfare [Mio. €]	Demand [GWh]	Losses [MWh]	Welfare [Mio. €]	Welfare change	Demand [GWh]	Demand change	Losses [MWh]
Average	4.44	56.3	1544	4.48	+ 0.9%	62.0	+ 10.3%	1547
Low	3.19	39.4	1446	3.23	+ 1.3%	44.4	+ 12.9%	1365
High	5.67	73.1	1720	5.67	+ 0.6%	79.2	+ 8.4%	1890

Table 5: Results for cost minimization and nodal pricing

Welfare under nodal pricing exceeds welfare under cost minimization by 0.9% on average (Table 5). In addition, the price level decreases significantly. In case of average demand, the mean nodal price is 17.07 €/MWh which is about 60% of the reference price (Figure 2). Due to congested lines prices differ for each node, but even the highest prices are below the actual reference price. Additionally, the losses per demand are lower under nodal pricing.

Variations in welfare gain between the scenarios result from two facts. First, the introduction of nodal prices allows prices to vary from node to node implying that energy is allocated according to the willingness to pay at each node (pictured by the demand curve for each node). Second, onshore wind input causes low opportunity cost which we treat as marginal cost of wind supply. Therefore, the

[22] Note that the EEX volumes at the moment only account for approximately 10% of the entire market volume.

welfare spread between cost minimization and nodal pricing is largest during low load periods as the uniform price error is largest.

Altogether, the welfare under nodal pricing is greater for all three demand scenarios – low, average and high load. Note that off-shore wind has not been included, yet. The analysis has proven that nodal pricing creates greater social welfare than cost minimization under a uniform price.

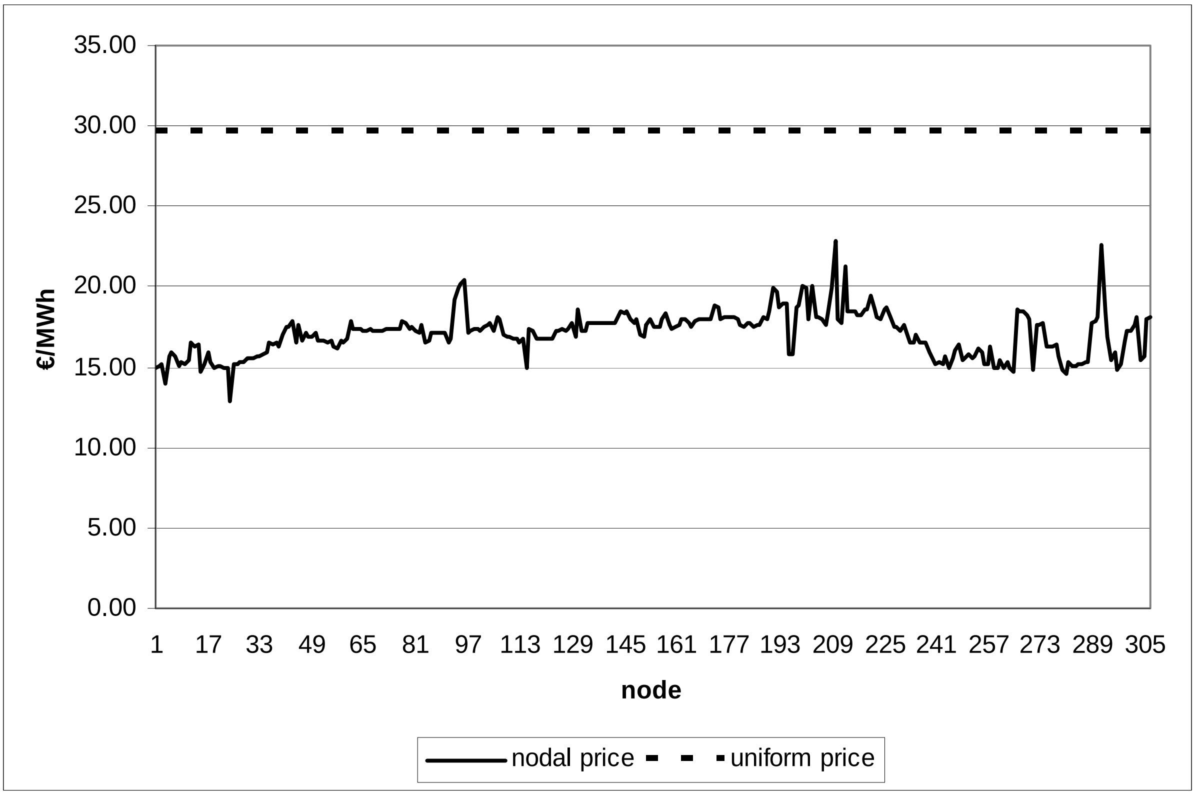

Figure 2: Nodal prices (average demand)

4.2.2 Nodal prices plus 8 GW (scenario 3)

In a next step, additional offshore wind energy plants were integrated into the existing grid. The aim was to find out how much offshore wind energy could be fed into the grid at most without any extension of lines. Consequently, they were not considered in the model, which is based on marginal costs. Offshore wind capacities in the North Sea were supposed to feed in at three injection nodes along the coastline (Brunsbüttel, Emden, Wilhelmshaven). Operating the lines at their capacity limit and differing between the three possible scenarios, the model provides three values for input of power (Table 6) with a maximum of 7.9 GW in the high load case.

	Wind offshore [GW]	Wind onshore [GW]
Average load	7.8	16.7
Low load	7.1	16.7
High load	7.9	16.7

Table 6: Maximum possible input without extension in lines

The constant input of wind onshore power is the result of a German law guaranting fixed selling prices for power of windmills. For this reason, the maximum possible input of wind onshore power has to be taken as input. As for the wind offshore power, the amount of input is capped due to the available line capacities depending on demand level. The opportunity costs of generating one unit of wind energy were set to 4.05 €/MWh according to the findings of the DENA grid study (DENA, 2005b).

Compared to the nodal pricing model without offshore wind parks, an additional welfare gain of 1% occurs. The average nodal price decreases about 10% to 15.4 €/MWh. Particularly the nodes in Northern Germany benefit from the additional wind energy (Figure 3). Overall, the Southern part of Germany is nearly unaffected by offshore wind generation.

As handled in the model, the German grid was loaded with an excessively high offshore energy input. On the one hand this gives the maximum possible value without grid extensions; on the other hand it helps to find out which lines may cause congestions due to offshore wind energy. In the average load scenario, eight lines, of which six were located in North-West Germany, were congested. These lines are the first ones to be considered for a grid upgrade if additional offshore energy is installed.

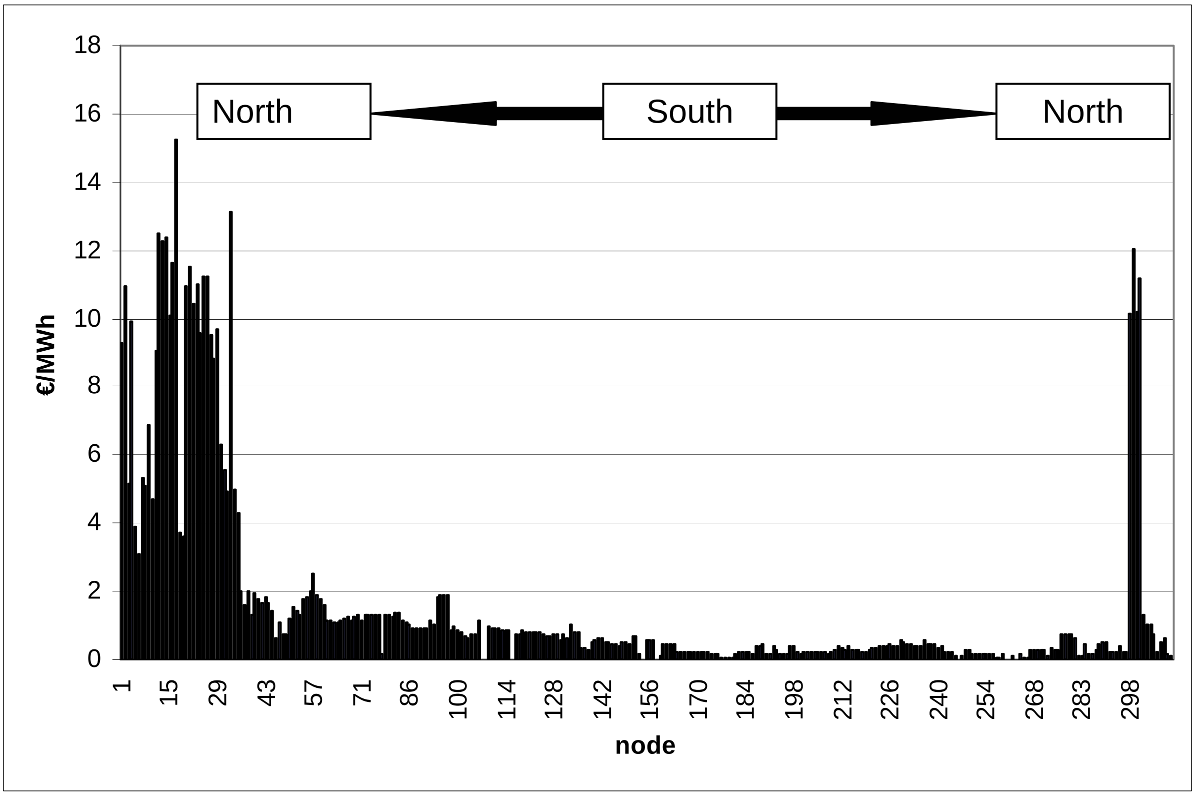

Figure 3: Nodal price difference ("without offshore" minus "plus 8 GW")

4.2.3 Nodal prices plus 13 GW (scenario 4)

One scenario of energy policy makers in Germany is to install offshore wind energy plants with up to 15 GW capacity. [23] Hence, a scenario considering additional 13 GW offshore wind energy plant was run. In our model, this would require an extension of the grid by four lines and an upgrade of two lines.[24] Fix costs from an expansion of plant and grid capacity were neglected and offshore energy was supposed to be fed into nodes at the coast. This study implemented a goal of 13 GW by building and upgrading electricity lines in the north of Germany. Some of them have already been planned (VEG, 2001).

Although, our grid extension is very ambitious (Table 7), the maximum offshore capacity could not be raised higher than 13.3 GW (Table 8). This is caused by additional congestions in North Germany. The results show that a grid extension has to be planned very carefully to maximize the additional benefits. Regularly, new congestions appear in the downstream grid limiting the effect of the extension.

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[23] The DENA grid study (DENA, 2005a) even proposes offshore wind capacities of 20 GW until 2020.
[24] These lines are planned to construct according to VGE (2000).

	From node	To node	Type of line
1.	Emden	Diele	double 380 kV
2.	Wilhelmshaven	Conneforde	double 380 kV
3.	Diele	Cloppenburg	double 380 kV
4.	Cloppenburg	St. Hülfe	double 380 kV
5.	Emden	Conneforde	upgrade to 380kV
6.	Cloppenburg	Conneforde	upgrade to 380kV

Table 7: Grid extension

	Offshore wind [GW]
Average load	11.1
Low load	12.6
High load	13.3

Table 8: Maximum wind offshore energy with extended grid

The additional welfare gain compared to the grid without extension is about 0.8%. The average price decreases about 2.5% to 15.06 €/MWh and again, mainly nodes in Northern Germany benefit from the grid extension. Surprisingly, some nodes have prices below their costs of generation. This is caused by the welfare maximizing attempt. If additional demand at one node enables supply at a node with a higher willingness to pay, the welfare gain from this node can be higher than the loss from the first node. This phenomenon, only occuring in few cases in Northern Germany, enables a higher flow of cheap wind energy to the South. In addition, some nodes have to pay a higher price caused by congestions (Figure 4).

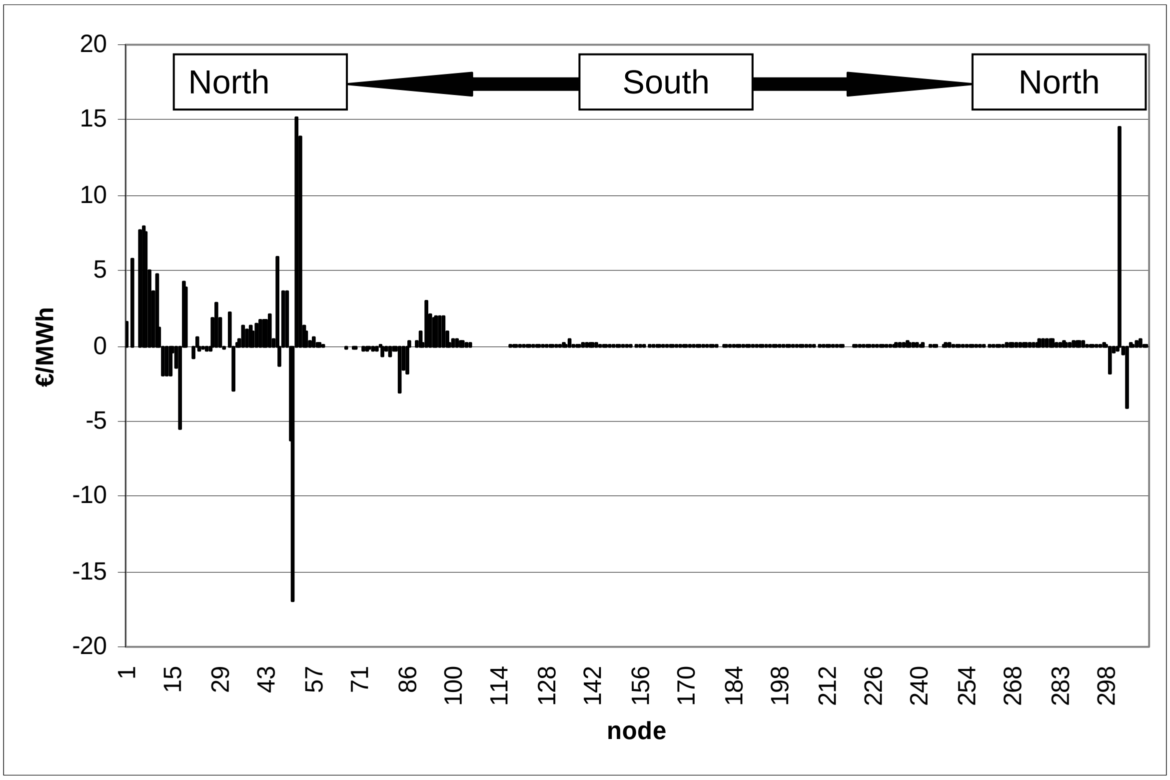

Figure 4: Nodal price difference ("plus 8 GW" minus "plus 13GW")

It must be considered that the network extension investments are not taken into account in the welfare analysis. Furthermore, grid constructions are likely to become political issues. Under these circumstances, the welfare gain from grid extensions must be regarded differently to the welfare gain from introducing nodal pricing. As only a limited amount of wind energy can be transported to demand centers in the South, in a nodal pricing regime only Northern Germany would participate in the welfare gain resulting from offshore wind. The analysis shows that the German electricity grid is not suited for a high amount of offshore capacity. The highly meshed network rather seems to be designed for decentralized input.[25]

[25] Note that, here, 'decentralized' refers to units significantly smaller than 8 GW.

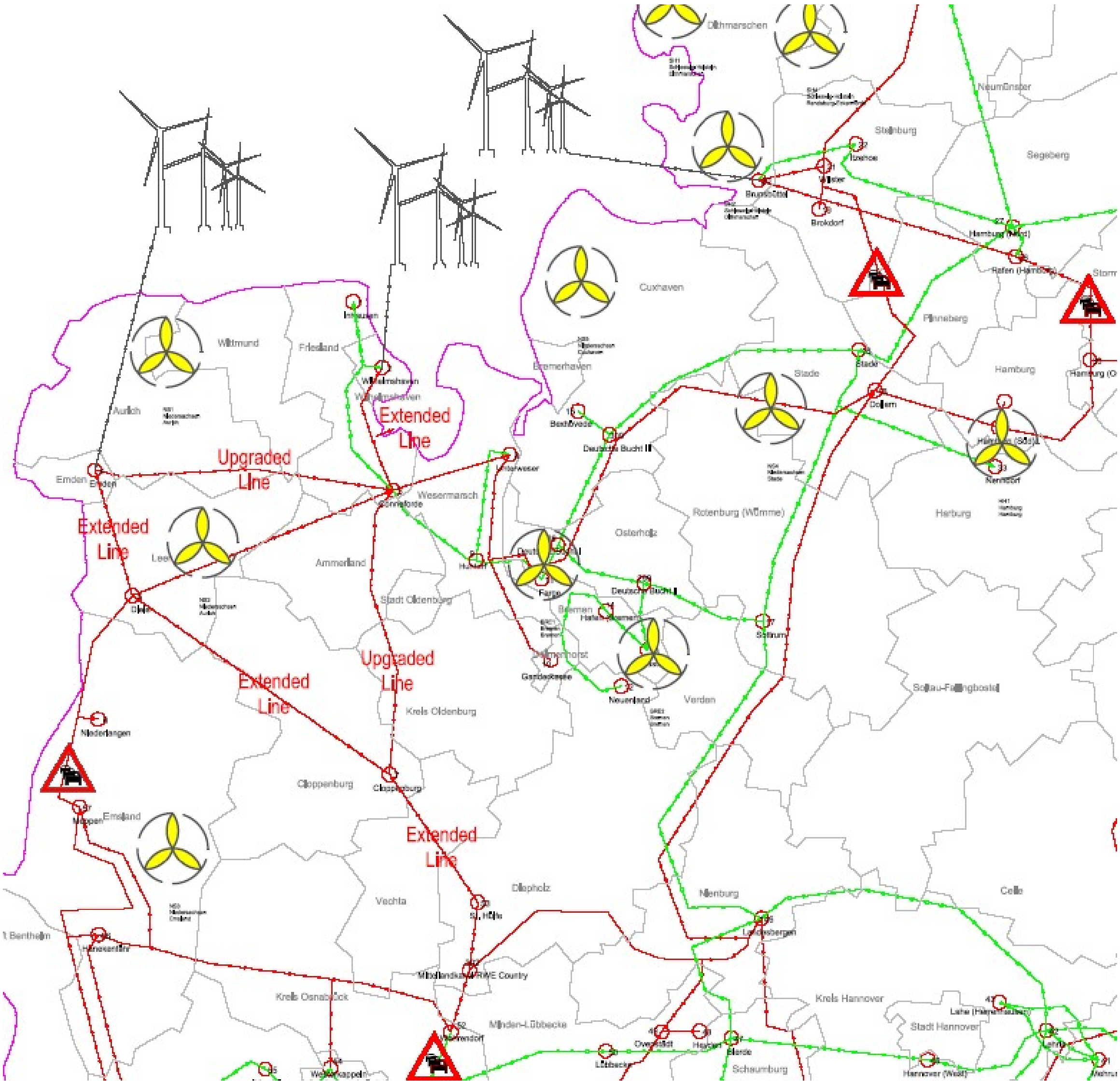

Figure 5: Congested lines around the North Sea 13 GW average

4.3 Comparison of all models

The analysis showed that the nodal model approach is superior to the cost minimization approach under uniform pricing. Furthermore, additional input of offshore wind energy increases welfare (Figure *6*).The latter is, particularly, caused by the comparably low opportunity costs of wind energy. Increasing these costs would lead to a decrease of welfare benefits. Eventually, the gain could turn into a loss if opportunity costs are higher than the marginal costs of nuclear and lignite energy. In this case, the political feed-in guarantee for wind energy would lead to a drawback of cheaper fossil energy and, thereby, cause welfare to decrease. On the other hand, if opportunity costs are lower than the value estimated in the DENA study, the welfare gain would become much more significant. However, while this is important for the offshore wind scenarios it does not influence the result on the superiority of the nodal pricing approach.

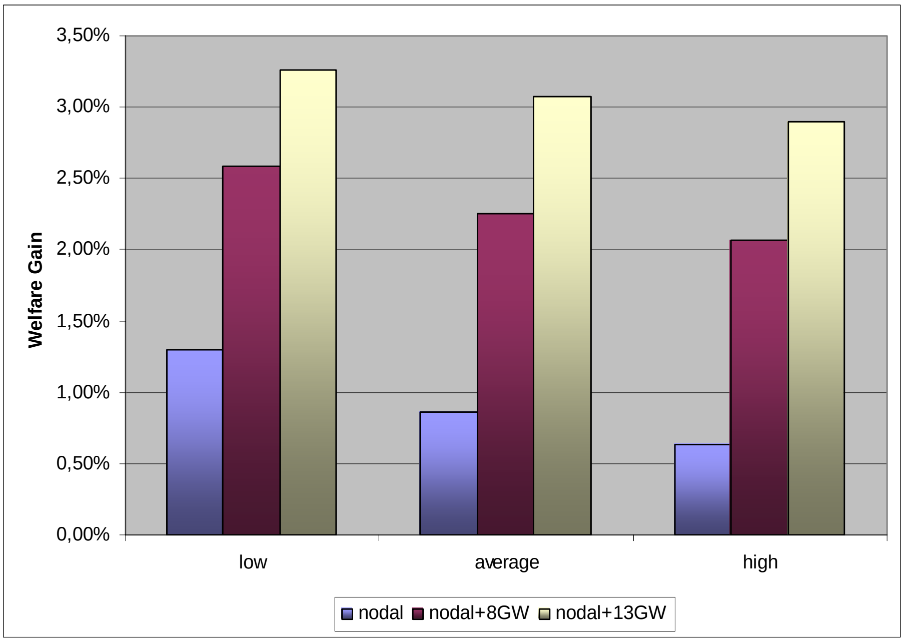

Figure 6: Summary of the nodal price scenarios

Another interesting circumstance is the impact of wind energy on grid losses. The dispatch of energy is more efficient under nodal pricing and thus the ratio of losses to demand is lower. Injection of additional offshore energy into the grid increases this ratio significantly (Table 9). An possible explanation consists in the fact that energy is injected only at three nodes in Northern Germany. This highly centralized energy input has to be transported to demand centers in Central and South Germany since not the entire wind energy can be consumed at Northern German nodes. The welfare maximizing approach takes this problem into account. Since lost energy has to be generated, it increases the total costs of generation. The calculated scenarios represent the optimal dispatch under the assumed conditions.

Demand case	Cost minimization	Nodal pricing without	Nodal pricing + 8GW	Nodal pricing + 13GW
Low	0.037	0.031	0.052	0.059
Average	0.027	0.025	0.046	0.050
High	0.024	0.024	0.039	0.043

Table 9: Ratio of losses against demand

Congestions occur in all of the nodal pricing scenarios. However, the number of congested lines increases strongly in the scenarios with additional offshore wind capacities. In short, the DENA study shows fewer congested lines. Our congestions (Figure *5*), furthermore, differ from the congestions pointed out by the DENA grid study (2005a). There is only one congested line in this model that corresponds to the DENA congestions. But, the congestions occur in similar regions. An important reason is that the DENA did assume offshore wind facilities in the Baltic Sea, too. This changes the stress on the lines and allows a better distribution of the energy throughout the grid. However, at this point of time, it seems that large wind parks are much more likely to be erected in the North Sea if they come at all.

5 Conclusions

This paper has analyzed the implication of additional wind supply into the German high-voltage grid, using a nodal pricing approach.[26] First of all, we show that a nodal pricing scheme is economically superior to cost minimization under a uniform price. Welfare increases between 0.6% and 1.3% within the nodal price approach. Note that this seems to be quite low but implies a large number in absolute terms. Demand increases significantly under nodal pricing, whereas a demand increase is not allowed by definition in the cost minimization scenario due to the uniform price.

Moreover, it has been illustrated that there is an additional welfare increase of about 1% on average in case of additional offshore wind input into the German power grid. However, the results show that there is a limit of wind energy distribution at about 8 GW without network extension. Beyond that we were forced to extent the modeled grid in order to supply further wind energy. Even though welfare increases again by 0.8% when escalating the wind supply after the grid extension, the downstream grid cannot carry the burden and congestions occur on lines leaving adjacent nodes. Consequently, prices at the inland nodes do not differ significantly as their demand is accommodated by the same generating pool as before.

Note as well that the 8 GW cap for the current grid already includes that only offshore wind facilities generate in North-West Germany, whereas existing large-scale plants would not be dispatched. However, these facilities might be required for balancing and response power. Problems may arise if the construction of offshore wind mills provides negative incentives for investments in other types of generation plants. This is not represented in our straightforward marginal consideration.

[26] Note that we did not consider the case without onshore wind energy availability. Control and backup power are regarded to exist to a sufficient degree and cause marginal costs that are, here, referred to as opportunity costs for wind.

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Appendix A: Inverse Demand, Nodal Price and Welfare

Assume a linear inverse demand function of the general form as given in (A.1) with the slope *b* being negative. Hence, the demand function is pictured in equation (A.2).

$$p(d) = a + b \cdot d \tag{A.1}$$

$$d(p) = -\frac{a}{b} + \frac{1}{b} \cdot p \tag{A.2}$$

Now calculating the demand elasticity according to (A.3), yields equation (A.4). This will subsequently lead to the calculation of the slope *b* (A.5).

$$\varepsilon = \frac{\delta d}{\delta p} \cdot \frac{p}{d} \tag{A.3}$$

$$\varepsilon = \frac{1}{b} \cdot \frac{p}{d} \tag{A.4}$$

In order to derive prohibitive price *a* and slope *b*, we assume that the demand elasticity ε equals -0.25 at the reference point. Prohibitive price *a* and slope *b* for each node can, then, be calculated on the basis of given reference price and demand – (A.5) and (A.6).

$$b = \frac{p^{ref}}{d^{ref}} \cdot \frac{1}{\varepsilon} \tag{A.5}$$

$$a = p^{ref} - b \cdot d^{ref} \tag{A.6}$$

Applying (A.6) and (A.5) yields, after simplifications, (A.7).

$$p = p^{ref} - \frac{p^{ref}}{d^{ref}} \cdot \frac{1}{\varepsilon} \cdot d^{ref} + \frac{p^{ref}}{d^{ref}} \cdot \frac{1}{\varepsilon} \cdot d$$

$$p = p^{ref} + \frac{1}{\varepsilon} \cdot p^{ref} \cdot \left(\frac{d}{d^{ref}} - 1 \right) \tag{A.7}$$

For the optimal demand at node *n* (d_n*), the node specific reference demand and price[27], we finally get the nodal price:

[27] Within the scope of this study, the reference price was assumed to be identical for all nodes.

$$p_n = p_n^{ref} + \frac{1}{\varepsilon} \cdot p_n^{ref} \cdot \left(\frac{d_n^*}{d_n^{ref}} - 1 \right) \tag{A.8}$$

Social welfare is calculated as the sum over the welfare of every node. Therefore, total costs C are subtracted from the benefit given demand d_n*. The benefit describes the value that the consumption of one unit of electrical power gives to the customer. It is pictured by the area below the inverse demand function. Hence, it can be derived by integrating the inverse demand function from zero to the optimal demand point.

$$\begin{aligned} W(d_n^*) &= \sum_n \left(\int_0^{d_n^*} p^*(d_n^*)\, d\, d_n^* - \int_0^{d_n^*} c(d_n^*)\, d\, d_n^* \right) \\ &= \sum_n \left[p_n^{ref} d_n^* (1 - \frac{1}{\varepsilon}) + \frac{1}{2} \left(\left(d_n^* \right)^2 \frac{1}{\varepsilon} \cdot \frac{p_n^{ref}}{d_n^{ref}} \right) \right] \text{- C} \end{aligned} \tag{A.9}$$

Appendix B: Assumptions for calculating transmission losses in the DCLF

According to Todem (2004, pp.130-131), transmission losses are made up by the sum of the load flows along a line. Those can flow in both directions, so that:

$$L_{jk} = P_{jk} + P_{kj} \tag{B.1}$$

Furthermore it can be stated that:

$$G_i = G_{jk} = G_{kj} \tag{B.2}$$

$$B_i = B_{jk} = B_{kj} \tag{B.3}$$

$$sin\Theta_{jk} = -\, sin\Theta_{kj} \tag{B.4}$$

$$cos\Theta_{jk} = cos\Theta_{kj} \tag{B.5}$$

Under the assumptions that

$$X_i \succ\succ R_i \tag{B.6}$$

one can insert equation (10) into (B.1) and then simplify the resulting equation by inserting equations (16), (17), (B.2), (B.3), (B.4) and (B.5). These steps lead to the equation for estimating transmission losses (18).