A Merchant Mechanism for Electricity Transmission Expansion

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Abstract

We propose a merchant mechanism to expand electricity transmission based on long-term financial transmission rights (FTRs). Due to network loop flows, a change in network capacity might imply negative externalities on existing transmission property rights. The system operator thus needs a protocol for awarding incremental FTRs that maximize investors' preferences, and preserves certain currently unallocated FTRs (or proxy awards) so as to maintain revenue adequacy. In this paper we define a proxy award as the best use of the current network along the same direction as the incremental awards. We then develop a bi-level programming model for allocation of long-term FTRs according to this rule and apply it to different network topologies. We find that simultaneous feasibility for a transmission expansion project crucially depends on the investor-preference and the proxy-preference parameters. Likewise, for a given amount of pre-existing FTRs the larger the current capacity the greater the need to reserve some FTRs for possible negative externalities generated by the expansion changes.

1 Introduction

The analysis of incentives for electricity transmission expansion is not easy. Beyond economies of scale and cost sub-additivity externalities in electricity transmission are mainly due to "loop flows" that come up from complex network interactions.[1] The effects of loop flows imply that transmission opportunity costs are a function of the marginal costs of energy at each location. Power costs and transmission costs depend on each other since they are simultaneously settled in

[1] See Joskow and Tirole (2000), and Léautier (2001).

electricity dispatch. Loop flows imply that certain transmission investments might have negative externalities on the capacity of other (perhaps distant) transmission links (see Bushnell and Stoft, 1997). Moreover, the addition of new transmission capacity can sometimes paradoxically decrease the total capacity of the network (Hogan, 2002a).

The welfare effects of an increment in transmission capacity are analyzed by Léautier (2001). The welfare outcome of an expansion in the transmission grid depends on the weight in the welfare function of the generators' profits relative to the consumers' utility weight. Incumbent generators are not in general the best agents to carry out transmission expansion projects. Even though an increase in transmission capacity might allow them to engross their revenues due to increased access to new markets and higher transmission charges, such gains are usually overcome by the loss of their local market power.

The literature on incentives for long-term expansion of the transmission network is scarce. The economic analysis of electricity markets has been reduced to short-run issues, and has typically assumed that transmission capacity is fixed (see Joskow and Tirole, 2003). However, transmission capacity is random in nature, and it jointly depends on generation investment.

The way to solve transmission congestion in the short run is well known. In a power flow model, the price of transmission congestion is determined by the difference in nodal prices (see Hogan, 1992, 2002b). Yet, there is no consensus with respect to the method to attract investment to finance the long-term expansion of the transmission network, so as to reconcile the dual opposite incentives to congest the network in the short run, and to expand it in the long run. Incentive structures proposed to promote transmission investment range from a "merchant" mechanism, based on long-term financial transmission right (LTFTR) auctions (as in Hogan, 2002a), to regulatory mechanisms that charge the transmission firm the social cost of transmission congestion (see Léautier 2000, Vogelsang, 2001, and Joskow and Tirole, 2002).

In practice, regulation has been used in England, Wales and Norway to promote transmission expansion, while a combination of planning and auctions of long-term transmission rights has been tried in the Northeast of the U.S. A mixture of regulatory mechanisms and merchant incentives is alternatively used in the Australian market.

In this paper we develop a merchant model to attract investment to *small-scale* electricity transmission projects based on LTFTR auctions. Locational prices give market players incentives to initiate transmission investments. FTRs provide transmission property rights, since they hedge the market player against future price differences. Our model further develops basic conditions under which FTRs and locational pricing provide incentives for long-term investment in the transmission network.

In meshed networks, a change in network capacity might imply negative externalities on transmission property rights. Then, in the process of allocation of incremental FTRs, the system operator has to reserve certain unallocated FTRs so that the revenue adequacy of the transmission system is preserved. In order to deal with this issue, we develop a bi-level programming model for allocation of long-term FTRs and apply it to different network topologies.

The structure of the paper is as follows. In section 2 we carry out an analytical review on the relevant literature on electricity transmission expansion. In section 3 we develop our model. We first introduce FTRs and the feasibility rule, and then address the rationale for FTR allocation and efficient investments. We develop general optimality conditions as well. In section 4, we carry out applications of our model to a radial line, and to a three-node network. In section V we provide concluding comments.

2 Literature Review

There exist some hypotheses on structures for transmission investment: the market-power hypothesis, the incentive-regulation hypothesis, and the long-run financial-transmission-right hypothesis. The first approach seeks to derive optimal transmission expansion from the power-market structure of power generators, and takes into account the conjectures of each generator regarding other generators' marginal costs due to the expansion (Sheffrin and Wolak, 2001, Wolak, 2000, and The California ISO and London Economics International, 2003). The generators' bidding behaviors are estimated before and after a transmission upgrade, and a real-option analysis is used to derive the net present value of transmission and generation projects together with the computation of their joint probability.

The model shows that there are few benefits of transmission expansion until added capacity surpasses a certain threshold that, in turn, is determined by the possibility of induced congestion by the strategic behavior of generators with market power. The generation market structure then determines when transmission expansion yield benefits. Additionally, many small upgrades of the transmission grid result to be preferable to large greenfield projects when cost uncertainty is added to the model.

The contribution of this method is that it models the existing interdependence of transmission investment and generation investment within a transportation model with no network loop flows. However, as pointed out by Hogan (2002b), the use of a transportation model in the electricity sector is inadequate since it does not deal with discontinuities in transmission capacity implied by the multidimensional character of a meshed network.

The second method for transmission expansion is a regulatory alternative that relies on a "Transco" that simultaneously runs system operation and owns the transmission network. The Transco is regulated through benchmark regulation or price regulation so as to provide it with incentives to invest in the development of the grid, while avoiding congestion. Léautier (2000), Grande and Wangesteen (2000), and Harvard Electricity Policy Group (2002) discuss mechanisms that compare the Transco performance with a measure of welfare loss due to its activities. Joskow and Tirole (2002) propose a surplus-based mechanism to reward the Transco according to the redispatch costs avoided by the expansion, so that the Transco faces the complete social cost of transmission congestion.

Another regulatory alternative is a two-part tariff cap proposed by Vogelsang 2001 that solves the opposite incentives to congest the existing transmission grid in the short run, and to expand it in the long run. Incentives for investment in expansion of the network are achieved through the rebalancing of the fixed part and the variable part of the tariff. This method tries to deepen into the analysis of the cost and production functions for transmission services, which are not very well understood in the economics literature. Nonetheless, to achieve this goal Vogelsang needs to define an output (or throughput) for the Transco. As argued in the FTR literature (Bushnell and Stoft (1997), Hogan, (2002a), Hogan, (2002b)), this task is very difficult since the physical flow through a meshed transmission network cannot be traced.

The third approach is a "merchant" one based on LTFTR auctions by an independent system operator (ISO). This method deals with loop-flow externalities in that, to proceed with line expansions, the investor pays for the negative externalities it generates. To restore feasibility, the investor has to buy back sufficient transmission rights from those who hold them initially, or the ISO has to retain some transmission rights (*proxy awards*) during the LTFTR auction to assure that the expansion project does not violate the property rights of the original FTR holders. This is the core of an LTFTR auction (see Hogan, 2002 a).

Joskow and Tirole (2003) criticize the LTFTR approach. They argue that the efficiency results of the *short-run* version of the FTR model rely on perfect-competition assumptions, which are not real for transmission networks. Moreover, defining an operational FTR auction is technically difficult[2] and, according to these authors, the FTR analysis is static (a contradiction with the dynamics of transmission investment). Joskow and Tirole analyze the implications of eliminating the perfect competition assumptions of the FTR model.

First, market power and vertical integration might impede the success of FTR auctions. Prices will not reflect the marginal cost of production in regions with transmission constraints. Generators in constrained regions will then withdraw capacity in order to increase their prices, and will overestimate the cost-saving gains from investments in transmission.[3]

Second, lumpiness in transmission investment makes the total value paid to investors through FTRs less than the social surplus created. The large and lumpy nature of major transmission upgrades requires long-term contracts before making the investment, or temporal property rights for the incremental investment.

Third, contingencies in electricity transmission impede the merchant approach to really solve the loop-flow problem. Moreover, existing transmission capacity and incremental capacity are stochastic. Even in a radial line, realized capacity could be less than expected capacity and the revenue-adequacy condition would not be met. Even more, the initial feasible FTR set can depend on random exogenous variables.

Fourth, an expansion in transmission capacity might negatively affect social welfare (as shown by Bushnell and Stoft, 1997).

Fifth, a moral hazard "in teams" problem arises due to the separation of transmission ownership and system operation in the FTR model. For instance, an outage can be claimed to be the consequence of poor maintenance (by the transmission owner) or of negligent dispatch (by the system operator).[4] Additionally, there is no perfect coordination of interdependent investments in

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[2] No restructured electricity sector in the world has adopted a pure merchant approach towards transmission expansion. Australia has implemented a mixture of regulated and merchant approaches (see Littlechild, 2003). Pope (2002), and Harvey (2002) propose LTFTR auctions for the New York ISO to provide a hedge against congestion costs. Gribik et al (2002) propose an auction method based on the physical characteristics (capacity and admittance) of a transmission network.

[3] Generators can exert local power when the transmission network is congested. (See Bushnell, 1999, Bushnell and Stoft, 1997, Joskow and Tirole, 2000, Oren,1997, Joskow and Schmalensee, 1983, Chao and Peck, 1997, Gilbert, Neuhoff, and Newbury, 2002, Cardell, Hitt, and Hogan, 1997, Borenstein, Bushnell, and Stoft, 1998, Wolfram, 1998, and Bushnell and Wolak, 1999).

[4] An example is the power outage of August 15, 2003, in the Northeast of the US, which affected six control areas (Ontario, Quebec, Midwest, PJM, New England, and New York) and more than 20 million consumers. A 9-second transmision grid technical and operational problem seems to have caused a cascade effect, which shut down 61,000 MW generation capacity. By the time this paper was written, there was not a clear

generation and transmission, and stochastic changes in supply and demand conditions imply uncertain nodal prices. Likewise, there is no equal access to investment opportunities since only the incumbent can efficiently carry out deepening transmission investments.

Hogan (2003) responds to the above criticisms. He argues that LTFTRs only grant efficient outcomes under lack of market power, and non-lumpy marginal expansions of the transmission network. He then thinks that regulation has an important role in fostering large and lumpy projects, and in mitigating market power abuses.

As argued by Pérez-Arriaga et al (1995), revenues from nodal prices only recover 25% of total costs. LTFTRs should then be complemented with a fix-price structure or, as in Rubio-Odériz and Pérez-Arriaga (2000) a *complementary charge* that allows the recovery of fixed costs.[5] This fact is recognized by Hogan (1999) who believes that complete reliance on market incentives for transmission investment is undesirable. Rather, Hogan (2003) thinks that merchant and regulated transmission investments might be combined so that regulated transmission investment is limited to projects where investment is large relative to market size, and lumpy so that it only makes sense as a single project as opposed as to many incremental small projects.

Hogan also responds to contingency concerns.[6] On one hand, only those contingencies outside the control of the system operator could lead to revenue inadequacy of FTRs, but such cases are rare and do not represent the most important contingency conditions. On the other hand, most of remaining contingencies are foreseen in a security-constrained dispatch of a meshed network with loops and parallel paths. If one of “n” transmission facilities is lost, the remaining power flows would still be feasible in an “n-1” contingency constrained dispatch.

Hogan (2003) also assumes that agency problems and information asymmetries are part of an institutional structure of the electricity industry where the ISO is separated from transmission ownership and where market players are decentralized. However, he thinks that the main issue on transmission investment is the decision of the boundary between merchant and regulated transmission expansion projects. It is not clear to him how asymmetric information might affect such a boundary.

Hogan (2002a) finally analyzes the implications of loop flows on transmission investment raised by Bushnell and Stoft (1997). He analytically provides some general axioms to properly define LTFTRs so as to deal with negative externalities implied by loop flows. We next present a model that develops the general analytical framework proposed by Hogan (2002a).

3 The Model

Assume an institutional structure where there are various established agents (generators, Gridcos, marketers, etc.) interested in the transmission grid expansion. Agents do not have market power in their respective market. Also assume that transmission projects are incrementally small relative to the total network so that the probability of lumpiness in transmission investment is small, and

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explanation for the cause of this outage, and there were ”finger pointings” among system operators of diferrent areas, and transmission providers.

[5] In the US, transmission fixed costs are recovered through a regulated fixed charge, even in those systems that are based on nodal pricing, and FTRs. This charge is usually regulated through cost of service.

[6] See Hogan (2002a), Hogan (2002b), and Hogan (2003).

that economic dispatch is carried out under security constraints to take care of possible contingencies (*n-1* criterion).

Under an initial condition of non-fully allocation of FTRs in the grid, the auctioning of incremental LTFTRs should satisfy the following basic criteria in order to deal with possible negative externalities associated with the expansion

(1) An LTFTR increment must keep being simultaneously feasible (*feasibility rule*).

(2) An LTFTR increment remains simultaneously feasible given that certain currently unallocated rights (or *proxy awards*) are preserved.

(3) Investors should maximize their objective function (*maximum value*).

(4) The LTFTR awarding process should apply both for decreases and increases in the grid capacity (*symmetry*).

As shown by Bushnell and Stoft (1996), and Bushnell and Stoft (1997), under these conditions allocation of new PTP-FTR obligations will not reduce social welfare. Hogan explains however that defining proxy awards is a difficult task. We next address these issues in a formal way in the context of an auction model designed to attract investment for transmission expansion.

3.1 The Power Flow Model and Proxy Awards

Consider the following economic dispatch model:[7]

$$\underset{Y,u\in U}{Max}\, B(d_P - g_P) \qquad (1)$$

$$s.t.$$

$$Y = Y_P = d_P - g_P, \qquad (2)$$

$$L(Y,u) + t^T Y = 0 \qquad (3)$$

$$K(Y,u) \leq 0 \qquad (4)$$

where d_p and g_p are load and generation at the different locations. The variable Y represents the real power bus net loads, including the swing bus S ($Y^T = (Y_s, \overline{Y}^T)$). $B(d_P\text{-}g_P)$ is the net benefit function,[8] and t is a unity column vector, $t^T = (1,1,...,1)$. All other parameters are represented in the control variable u. The objective (1) includes the maximization of benefit to loads and the minimization of generation costs. Equation (2) denotes the net load as the difference between load and generation. Equation (3) is a loss balance constraint where $L(Y,u)$ is a vector which denotes the losses in the network. In equation (4) $K(Y,u)$, is a vector of power flows in the lines, which are subject to transmission capacity limits. The corresponding multipliers or shadow prices for the constraints are (P, l_{ref}, l_{tran}) for net loads, reference bus energy and transmission constraints, respectively. When security constraints are taken into account (*n-1* criterion) this is a large-scale

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[7] Hogan (2002b) shows that the economic dispatch model can be extended to a market equilibrium model where the ISO produces transmission services, power dispatch, and spot-market coordination, while consumers have a concave utility function that depends on net loads, and on the level of consumption of other goods.

[8] Function B is typically a measure of welfare, such as the difference between consumer surplus and generation costs (see Hogan, 2002b)

problem, and it prices anticipated contingencies through the security-constrained economic dispatch.

The locational prices P are the marginal generation cost or the marginal benefit of demand, which in turn equals the reference price of energy plus the marginal cost of losses and congestion. With the optimal solution (d^*, g^*, Y^*, u^*) and the associated shadow prices, we have the vector of locational prices as:

$$P^T = \nabla C(g^*) = \nabla B(d^*) = l_{ref} t^T + l_{ref} \nabla L_Y(Y^*, u^*) + l_{tran}^T \nabla K_Y(Y^*, u^*) \tag{5}$$

If losses[9] are not considered, only the energy price at the reference bus and the marginal cost of congestion contribute to set the locational price.

FTR obligations[10] hedge market players against differences in locational prices caused by transmission congestion.[11] FTRs are provided by an ISO, and are assumed to redistribute the congestion rents. The pay-off from these rights is given by:

$$FTR = (P_j - P_i)Q_{ij} \tag{6}$$

where P_j is the price at location j, P_i is the price at location i, and Q_{ij} is the directed quantity from point i to point j specified in the FTR. The FTR payoffs can take, negative, positive or zero values.

A set of FTRs is said to be simultaneously feasible if the associated set of net loads is simultaneously feasible, that is if the net loads satisfy the energy balance and transmission capacity constraints as well as the power flow equations given by:

$$\begin{aligned} &Y = \sum_k t_k^f \\ &L(Y,u) + t^t Y = 0, \\ &K(Y,u) \le 0 \end{aligned} \tag{7}$$

where $\sum_k t_k^f$ is the set of point-to-point obligations.[12]

If the set of FTRs is simultaneous feasible, then the FTRs satisfy the *revenue adequacy* condition in the sense that equilibrium payments collected by the ISO through economic dispatch will be greater than or equal to payments required under the FTR forward obligations.[13]

Assume now investments in new transmission capacity. The associated set of new FTRs for transmission expansion has to satisfy the simultaneous feasibility rule too. That is, the new and old FTRs have to be simultaneously feasible after the system expansion so that aggregate welfare is not reduced. Assume that T is the current partial allocation of long-term FTRs, then by assumption it is feasible ($K(T,u) \le 0$). Let a be the scalar amount of incremental FTR awards, and

[9] In the PJM (Pennsylvania, New Jersey and Maryland) market design, the locational prices are defined without respect to losses (DC-network), while in New York the locational prices are calculated based on an AC-network with marginal losses.
[10] FTRs could be options with a payoff equal to max($(l_j - l_i) P_{ij}$,0).
[11] See Hogan (1992).
[12] The set of point-to-point obligations can be decomposed into a set of balanced and unbalanced (injection or withdrawal of energy) obligations (see Hogan 2002b).
[13] Revenue adequacy is the financial counterpart of the physical concept of availability of transmission capacity (see Hogan, 2002a).

$\hat{t}$ the scalar amount of proxy awards. Furthermore let d be directional vector[14] such that ad is the MW amount of incremental FTR awards**,** and $\hat{t}d$ is the MW amount of proxy awards between different locations. Any incremental FTR award ad should comply with feasibility rule in the expanded grid. Hence we must have $K^{+}(T+ad,u)\leq 0$.

When certain currently unallocated rights (proxy awards) $\hat{t}d$ in the existing grid must be preserved, combined with existing rights they sum up to $T+\hat{t}d$.[15] Then the expanded grid K^{+} should also satisfy simultaneous feasibility so that $K^{+}(T+ad,u)\leq 0$ and $K^{+}(T+\hat{t}d+ad,u)\leq 0$ for incremental awards ad.

A question then arises regarding the way to best define proxy awards. One possibility is to define them as the "best use" of the current network along the same direction as the incremental awards.[16] This includes both positive and negative incremental FTR awards. The best use in a three-node network may be thought of as a single incremental FTR in one direction or a combination of incremental FTRs defined by the directional vector d, which the investor has preference for. Hogan (2002a) proposes two ways of defining "best use":

Preset proxy preferences (p)

$$\hat{y} = T + \hat{t}d,$$

$$\hat{t} \in \arg\max_{t} \left\{ tpd \mid K(T+td) \leq 0 \right\}$$

or, (8)

Investor preferences $(b(ad))$

$$\hat{y} = T + \hat{t}d,$$

$$\hat{t} \in \arg \min_{\substack{t \\ K(T+td)\leq 0}} \left\{ \max_{a\geq 0} \left\{ b(ad) \mid K^{+}(T+td+ad) \leq 0 \right\} \right\}$$

In the preset proxy formulation the objective is to maximize the value (defined by prices p) of the proxy awards given the pre-existing FTRs, and the power flow constraints in the pre-expansion network. In the investor preference formulation the objective is to maximize the investor's value (defined by the bid functions for different directions, $b(ad)$) of incremental FTR awards given the proxy and pre-existing FTRs and the power flow constraints in the expanded network, while simultaneously calculating the minimum proxy scalar amount that satisfies the power flow constraints in the pre-expansion network.

We will use as a proxy protocol the first definition. We next analyze the way to use this protocol to carry out an auction of LTFTRs that stimulates investment in transmission.

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[14] Each element in the directional vector represents an FTR between two locations and the directional vector may have many elements representing combinations of FTRs.

[15] Proxy awards are then currently unallocated FTRs in the pre-existing network that basically facilitate the allocation of incremental FTRs and help to preserve revenue adequacy by reserving capacity for hedges in the expanded network

[16] Another possibility would be to define every possible use of the current grid as a proxy award. However, this would imply that any investment beyond a radial line would be precluded, and that incremental award of FTRs might require adding capacity to every link on every path of a meshed network.

3.2 The Auction Model

Assume the preset proxy rule is used to derive prices that maximize the investor preference $b(ad)$ for an award of a MWs of FTRs in direction d. We then have the following auction maximization problem:

$$
\begin{aligned}
&\underset{a,\hat{t},d}{Max}\ \ b(ad) \\
&s.t. \\
&K^{+}(T+ad) \le 0, \\
&K^{+}(T+\hat{t}d+ad) \le 0, \\
&\hat{t} \in \operatorname*{argmax}_{t}\{\ tpd \mid K(T+td) \le 0\}, \\
&\|d\| = 1, \\
&a \ge 0.
\end{aligned}
\qquad (9)
$$

In this model, the investor's preference is maximized subject to the simultaneous feasibility conditions, and the best use protocol. We add a constraint on the norm of the directional vector to preclude the trivial case $d = 0$. We want to explore if such an auction can produce acceptable proxy and incremental awards. We next analyze this issue within a framework that ignores losses, and utilizes a DC-load approximation.

The auction model is a non-linear optimization problem of "bi-level" nature.[17] There are two optimization stages. Maximization is non-myopic since the result of the lower problem (first stage) depends on the direction chosen in the upper problem (second stage).[18] Bi-level problems are solved by first transforming the lower problem (i.e. the allocation of proxy awards) into to a set of Kuhn-Tucker equations that are subsequently substituted in the upper problem (i.e. the maximization of the investors' preference). The model can then be understood as a Stackelberg problem although it is not intending to optimize the same type of objective function at each stage.[19]

The Lagrangian (L) for the lower problem is:

$$L(\hat{t},d,l) = \hat{t}pd - l^{T}(K(T+\hat{t}d))$$

where l^{T} is the Lagrange multiplier vector associated with transmission capacity on the respective transmission lines before the expansion. It is the shadow price of the simultaneous feasibility restriction for proxy awards. The Kuhn-Tucker conditions are:

$$\frac{\partial L(\hat{t},d,l)}{\partial \hat{t}} = 0,\ \frac{\partial L(\hat{t},d,l)}{\partial l} \ge 0$$

$$l^{T}\frac{\partial L(\hat{t},d,l)}{\partial l} = 0,\ \ l \ge 0$$

The transformed problem is then written as:

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[17] See Shimizu et al. (1997).

[18] The model could also be interpreted as having multiple periods. Although we do not explicitly include in our model a discount factor, we assume that it is included in the investor's preference parameter b.

[19] Other examples in the economics literature where an upper level maximization takes the optimality conditions of another problem as constraints are given in Mirrlees (1971), Brito and Oakland (1977), and Rosellón (2000).

$$
\begin{aligned}
&\underset{a,\hat{t},d,l}{Max}\ \ b(ad) \\
&s.t. \\
&K^{+}(T+ad)\le 0, && (w) \\
&K^{+}(T+\hat{t}d+ad)\le 0, && (g) \\
&\frac{\partial L(\hat{t},d,l)}{\partial \hat{t}}=0, && (q) \\
&l^{T}\frac{\partial L(\hat{t},d,l)}{\partial l}=0, && (z) \\
&\frac{\partial L(\hat{t},d,l)}{\partial l}\ge 0, && (e) \\
&\|d\|=1, && (j) \\
&a\ge 0, && (k) \\
&l\ge 0 && (p)
\end{aligned}
\tag{10}
$$

where w, g, q, z, e, j, k and p are Lagrange multipliers associated with each constraint. More specifically, w is the shadow price of the simultaneous feasibility restriction for existing and incremental FTRs; g is the shadow price of the simultaneous feasibility restriction for existing FTRs, proxy awards and incremental FTRs; q, V, e are the shadow prices of the restriction on optimal proxy FTRs; j, k are the shadow prices of the non-negativity constraints for a and l, respectively; and p is the shadow price of the unit restriction on d.

The Lagrangian of the auction problem is:

$$
\begin{aligned}
L(a,\hat{t},d,l,\Omega) &= b(ad) - w^{T}(K^{+}(T+ad)) \\
&-g^{T}(K^{+}(T+\hat{t}d+ad)) - q^{T}\frac{\partial L(\hat{t},d,l)}{\partial \hat{t}} \\
&-z^{T}\left(l^{T}\frac{\partial L(\hat{t},d,l)}{\partial l}\right) + e^{T}\frac{\partial L(\hat{t},d,l)}{\partial l} \\
&+j^{T}(1-\|d\|) + k^{T}a + p^{T}l
\end{aligned}
\tag{11}
$$

where $\Omega = (w, g, q, z, e, j, k, p)$ denotes the vector of Lagrange multipliers. Kuhn-Tucker conditions for the upper problem are:

$$
\begin{aligned}
\frac{\partial L(a,\hat{t},d,l,\Omega)}{\partial a} &= \frac{\partial b(ad)}{\partial a} - \left[\frac{\partial K^{+}(T+ad)}{\partial a}\right]^{T} w \\
&-\left[\frac{\partial K^{+}(T+\hat{t}d+ad)}{\partial a}\right]^{T} g + k = 0
\end{aligned}
\tag{12}
$$

$$
\begin{aligned}
\frac{\partial L(a,\hat{t},d,l,\Omega)}{\partial \hat{t}} &= -\left[\frac{\partial K^{+}(T+\hat{t}d+ad)}{\partial \hat{t}}\right]^{T} g \\
&-\left[\frac{\partial^{2} L(\hat{t},d,l)}{\partial \hat{t}\,\partial l}\right]^{T} l\,z + \left[\frac{\partial L^{2}(\hat{t},d,l)}{\partial \hat{t}\,\partial l}\right]^{T} e = 0
\end{aligned}
\tag{13}
$$

$$\frac{\partial L(a,\hat{t},d,l,\Omega)}{\partial d} = \frac{\partial b(ad)}{\partial d} - \left[\frac{\partial K^{+}(T+ad)}{\partial d}\right]^{T} w \tag{14}$$

$$-\left[\frac{\partial K^{+}(T+\hat{t}d+ad)}{\partial d}\right]^{T} g - \left[\frac{\partial^{2} L(\hat{t},d,l)}{\partial d \partial \hat{t}}\right]^{T} q$$

$$-\left[\frac{\partial^{2} L(\hat{t},d,l)}{\partial d \partial l}\right]^{T} l z + \left[\frac{\partial^{2} L(\hat{t},d,l)}{\partial d \partial l}\right]^{T} e - \left[\frac{\partial \|d\|}{\partial d}\right]^{T} j = 0$$

$$\frac{\partial L(a,\hat{t},d,l,\Omega)}{\partial l} = -\frac{\partial^{2} L(\hat{t},d,l)}{\partial l \partial \hat{t}} q - \frac{\partial L(\hat{t},d,l)}{\partial l} z + p = 0, \tag{15}$$

$$\frac{\partial L(a,\hat{t},d,l,\Omega)}{\partial w} = -K^{+}(T+ad) \geq 0, \tag{16}$$

$$\frac{\partial L(a,\hat{t},d,l,\Omega)}{dg} = -K^{+}(T+\hat{t}d+ad) \geq 0 \tag{17}$$

$$\frac{\partial L(a,\hat{t},d,l,\Omega)}{\partial q} = -\frac{\partial L(\hat{t},d,l)}{\partial \hat{t}} = 0, \tag{18}$$

$$\frac{\partial L(a,\hat{t},d,l,\Omega)}{\partial z} = -l^{T} \frac{\partial L(\hat{t},d,l)}{\partial l} = 0, \tag{19}$$

$$\frac{\partial L(a,\hat{t},d,l,\Omega)}{\partial e} = \frac{\partial L(\hat{t},d,l)}{\partial l} \geq 0, \tag{20}$$

$$\frac{\partial L(a,\hat{t},d,l,\Omega)}{\partial j} = 1 - \|d\| = 0, \tag{21}$$

$$\frac{\partial L(a,\hat{t},d,l,\Omega)}{\partial k} = a \geq 0, \tag{22}$$

$$\frac{\partial L(a,\hat{t},d,l,\Omega)}{\partial p} = l \geq 0, \tag{23}$$

$$w^{T} \frac{\partial L(a,\hat{t},d,l,\Omega)}{\partial w} = 0, \ w \geq 0, \tag{24}$$

$$g^{T} \frac{\partial L(a,\hat{t},d,l,\Omega)}{\partial g} = 0, \ g \geq 0, \tag{25}$$

$$q^{T} \frac{\partial L(a,\hat{t},d,l,\Omega)}{\partial q} = 0, \tag{26}$$

$$z^{T} \frac{\partial L(a,\hat{t},d,l,\Omega)}{\partial z} = 0, \tag{27}$$

$$e^{T} \frac{\partial L(a,\hat{t},d,l,\Omega)}{\partial e} = 0, \ e \geq 0, \tag{28}$$

$$j^{T} \frac{\partial L(a,\hat{t},d,l,\Omega)}{\partial j} = 0, \tag{29}$$

$$k^{T} a = 0, \ k \geq 0, \tag{30}$$

$$p^{T} l = 0, p \geq 0 \tag{31}$$

The constraint $\frac{\partial L(\hat{t},d,l)}{\partial l}=0$ is redundant when the preset proxy preference (p) is non-zero, since it is a sub-gradient of the constraint $l^T\frac{\partial L(\hat{t},d,l)}{\partial l}=0$, and e is therefore zero when p is non-zero. We show in a later example that q and j are zero because the associated constraints are redundant. The binding constraint in the lower level problem is $l^T\frac{\partial L(\hat{t},d,l)}{\partial l}=0$, since some transmission constraints are fully utilized by proxy awards.

This is a non-linear problem, and its solution depends on the initial value of the bid parameter (b), the current partial allocation (T), and the topology of the network prior to and after the expansion.[20] A general solution method utilizing Kuhn-Tucker conditions would be through checking which of the constraints are binding.[21] One way to identify the active inequality constraints is the active set method.[22] In this paper we solve the problem in detail for different network topologies, including a radial line and a three-node network.

4 Simulations

4.1 Radial line

Let us first analyze a radial transmission line that is expanded as in Figure 1.

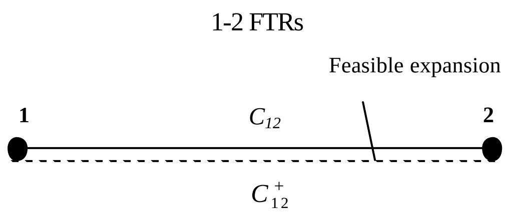

Figure 1. An expanded line and its feasible expansion.

The corresponding optimization problem is:

---

[20] According to Shimizu et al (1997), the necessary optimality conditions for this problem are satisfied. The objective function and the constraints are differentiable functions in the region bounded by the constraints. A local optimal solution and Kuhn-Tucker vectors then exist.

[21] There are other methods available such as transformation methods (penalty and multiplier), and non-transformation methods (feasible and infeasible). See Shimuzu et al. (1997).

[22] This method considers a tentative list of constraints that are assumed to be binding. This is a working list, and consists of the indices of binding constraints at the current iteration. Because this list may not be the solution list, the list is modified either by adding another constraint to the list or by removing one from the list. Geometrically, the active set method tends to step around the boundary defined by the inequality constraints. (See Nash and Sofer, 1988).

$$\begin{aligned}
&\underset{a,\hat{t},d}{Max} \;\; b_{12} a d_{12} \\
&s.t. \\
&T_{12} + a d_{12} \le C_{12}^{+} \\
&T_{12} + \hat{t} d_{12} + a d_{12} \le C_{12}^{+} \\
&\hat{t}(d_{12}) \in \underset{t}{\operatorname{argmax}} \{ \; t p_{12} d_{12} \mid T_{12} + t d_{12} \le C_{12} \} \\
&\| d_{12} \| = 1, \\
&a \ge 0
\end{aligned} \tag{32}$$

where C_{12} is the transmission capacity of the network before the expansion, C_{12}^{+} is the transmission capacity of the network after the expansion, and b_{12} is the investor preference. The first order conditions of the lower maximization problem can then be added as constraints to the upper problem:

$$\begin{aligned}
&\underset{a,\hat{t},d_{12},l}{Max} \;\; b_{12} a d_{12} \\
&s.t. \\
&T_{12} + a d_{12} \le C_{12}^{+} \\
&T_{12} + \hat{t} d_{12} + a d_{12} \le C_{12}^{+} \\
&l\, d_{12} - p_{12} d_{12} = 0 \\
&l\,(C_{12} - T_{12} - \hat{t} d_{12}) = 0 \\
&T_{12} + \hat{t} d_{12} \le C_{12} \\
&d_{12}^{2} = 1 \\
&a,\, l \ge 0
\end{aligned} \tag{33}$$

Since the grid is being expanded, the constraint on simultaneous feasibility of incremental FTRs $T_{12} + a d_{12} \le C_{12}^{+}$ is non-binding. The solution to this problem provides the values for the decision variables, and shadow prices:[23] $d_{12} = 1$, because the network is being expanded. Additionally $g = b_{12}$ which implies that the higher the value of the investor-preference parameter b_{12} the more the investor values post-expansion transmission capacity (its marginal valuation of transmission capacity increases with the bid value).

Similarly, we get $l = p_{12}$ which implies that the higher the value of the preset proxy preference parameter p_{12} the higher marginal valuation of pre-expansion transmission capacity. Other results are $q = 0$, $z = g / p_{12} = b_{12} / p_{12}$ and $e = 0$. This was expected since only one restriction for the lower problem is binding because the two other are redundant. The value of the binding Lagrange multiplier equals the ratio between the investor's bid value and the preset proxy parameter.

It also follows that $j = 0$ which is to be expected because the directional vector d is non-zero. Furthermore, $\hat{t} = C_{12} - T_{12}$, which means that for given existing rights the higher the current capacity the larger the need for reserving some proxy FTRs for possible negative externalities

[23] The mathematical derivation of these values is presented in annex 1.

generated by the expansion. Proxy awards are auctioned as a hedge against externalities generated by the expanded network.

We finally get $a = C_{12}^{+} - T_{12} - \hat{t} = C_{12}^{+} - C_{12}$, which shows that the optimal amount of additional MWs of FTRs in direction d directly depends on the amount of capacity expansion. Transmission capacity is in fact fully utilized by proxy awards (in the pre-expansion network), and by incremental FTRs (in the expanded network). Likewise, the investor receives a reward equal to the MW amount of new transmission capacity that it creates.

4.2 Three-node Network with Two Links

We now consider a three-node network example from Bushnell and Stoft (1997) where there is an expansion of line 1-2. The network is illustrated in Figure 2 and the feasible expansion in Figure 3.

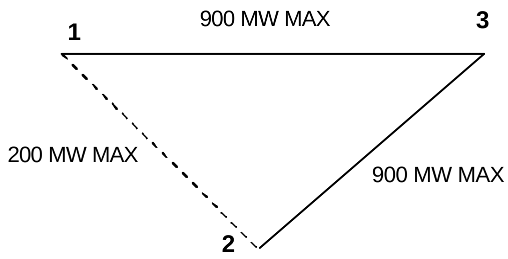

Figure 2. Three-node network with expansion of line 1-2.

The network expansion problem for identical links and FTRs between buses 1-3 and 2-3 is formulated as:

$$
\begin{aligned}
&\underset{a,\hat{t},d}{Max} \quad a(b_{13}d_{13} + b_{23}d_{23}) \\
&s.t. \\
&\frac{2}{3}(T_{13} + ad_{13}) + \frac{1}{3}(T_{23} + ad_{23}) \le C_{13} \\
&\frac{2}{3}(T_{13} + \hat{t}d_{13} + ad_{13}) + \frac{1}{3}(T_{23} + \hat{t}d_{23} + ad_{23}) \le C_{13} \\
&\frac{1}{3}(T_{13} + ad_{13}) + \frac{2}{3}(T_{23} + ad_{23}) \le C_{23} \\
&\frac{1}{3}(T_{13} + \hat{t}d_{13} + ad_{13}) + \frac{2}{3}(T_{23} + \hat{t}d_{23} + ad_{23}) \le C_{23} \\
&\frac{1}{3}(T_{13} + ad_{13}) - \frac{1}{3}(T_{23} + ad_{23}) \le C_{12} \\
&\frac{1}{3}(T_{13} + \hat{t}d_{13} + ad_{13}) - \frac{1}{3}(T_{23} + \hat{t}d_{23} + ad_{23}) \le C_{12}
\end{aligned} \tag{34}
$$

$$-\frac{1}{3}(T_{13}+ad_{13})+\frac{1}{3}(T_{23}+ad_{23})\leq C_{21}$$
$$-\frac{1}{3}(T_{13}+\hat{t}d_{13}+ad_{13})+\frac{1}{3}(T_{23}+\hat{t}d_{23}+ad_{23})\leq C_{21}$$

$$\hat{t}(d)\in \underset{t}{\operatorname{argmax}}\{t(p_{13}d_{13}+p_{23}d_{23})\}$$
$$(T_{13}+td_{13})\leq C_{13}$$
$$(T_{23}+td_{23})\leq C_{23}$$
$$\|d\|=1$$
$$a\geq 0$$

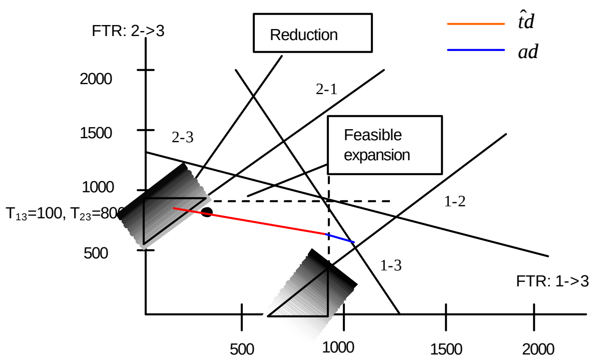

Figure 3. Feasible expansion of FTRs.

Annex 2 presents the calculations to obtain the power transfer distribution factors (PTDFs) for the post expansion network. In Figure 3 the pre-existing FTRs in the direction 2-3 do not use the full capacity of the pre-expansion network and become infeasible after inserting line 1-2. The preference is for FTRs in the direction 1-3 for transmission expansion. As seen from Figure 3 the maximum amount of proxy and incremental FTRs in the direction 1-3 that can be obtained is 1100, and corresponds to the point where the 1-3 and 1-2 transmission capacity constraint intersects.

In solving this problem, we get:[24]

$$d_{13}=\frac{(1/3g_1-1/3g_2)}{\left((2/3g_1+1/3g_2-z\,l)^2+(1/3g_1-1/3g_2)^2\right)^{1/2}}$$
$$d_{23}=\frac{-(2/3g_1+1/3g_2-z\,l)}{\left((2/3g_1+1/3g_2-z\,l)^2+(1/3g_1-1/3g_2)^2\right)^{1/2}}$$
$$a=\frac{C_{12}}{d_{13}}$$
$$\hat{t}=\frac{(C_{13}-T_{13})}{d_{13}}$$

[24] The detailed mathematical derivation of solutions to program 33 is presented in annex 1.

$$g_1 = \frac{(b_{13} + Bb_{23} + g_2(B/3 - 1/3))}{(2/3 + B/3)}$$

$$g_2 = \frac{1}{(1 - B - AB + A)}\left[b_{13}(1 + 3A - B - 2A - AB)\right.$$
$$\left. + b_{23}(B + 3AB - B^2 - 2A - AB)\right]$$
$$z\,l = (1 + A)g_1 - A(b_{13} + b_{23})$$

with

$$A = \frac{C_{12}}{(C_{13} - T_{13})}$$
$$B = \frac{1}{(1 + A)}\frac{(C_{13} - 2C_{12} - T_{23})}{(C_{13} - T_{13})}$$

where g_1 and g_2 are the Lagrange multipliers associated with transmission capacity on the lines 1-3 and 1-2, respectively, in the expanded network, and z is the multiplier associated with the Kuhn-Tucker condition regarding transmission capacity in the pre-expansion network for the line 1-3. This line has the Lagrange multiplier l associated with it before expansion. So as to characterize the solution to our model, we now calculate the Lagrange multipliers and decision variables for particular parameter values. In particular, we find the solution for the allocation presented in Figure 3. We assume the following bid values, preset proxy preferences and pre-existing amount of FTRs:

$b_{13} = 40,\ b_{23} = 10,$

$p_{13} = 60,\ p_{23} = 10,$

$T_{13} = 100,\ T_{23} = 800$

From these parameters we find that the marginal value of transmission capacity on line 1-3 and line 1-2 are $g_1 = 39.6$ and $g_2 = 33.6$, respectively. Thus the investor values transmission capacity on line 1-3 more than on line 1-2. We find that the product of the Kuhn-Tucker multiplier and the transmission capacity multiplier for the line 1-3 is $z\,l = 37$.

Likewise, the values of the decision variables are calculated as:

$d_{13} = 0.958,\ d_{23} = -0.287,$

$a = 208,\ \hat{t} = 835,$

The MW amount of awarded proxy FTRs in the direction 1-3 is $\hat{t}d_{13} = 800$, and the amount of awarded incremental FTRs is $ad_{13} = 200$. The amount of incremental 1-3 FTRs corresponds to the new transmission capacity on line 1-2 that the investor has created. There is also an allocation of proxy FTRs such that the full capacity of line 1-3 is utilized. Similarly the proxy awards in direction 2-3 is $\hat{t}d_{23} = -240$, and the amount of awarded incremental FTRs is $ad_{23} = -60$. The amount of incremental 2-3 FTRs is minimized and corresponds to 20% of the reduction (300) in pre-existing FTRs. The incremental 2-3 awards are mitigating FTRs, and are necessary to restore feasibility. The investor is responsible for additional counterflows so that it pays back for the negative externalities it creates. The solution is indicated by the large dot in Figure 3 and consists of both pre-existing and incremental FTR awards amounting to $T_{13} + ad_{13} = 300$ and $T_{23} + ad_{23} = 740$. The allocation of incremental FTRs is minimized because the model takes into account that one line is expanded, and some of the pre-existing FTRs become infeasible after the expansion.

This illustrates that the amount of incremental FTRs in the preference direction must be greater than zero such that feasibility is restored. Both the proxy and incremental FTRs exhaust transmission capacity in the pre-expansion and expanded grid, respectively. The proxy FTRs help allocating incremental FTRs by preserving capacity in the pre-expansion network, which results in an allocation of incremental FTRs amounting to the new transmission capacity created in 1-2 direction.[25] The proxy awards are transmission congestion hedges that can be auctioned to electricity market players in the expanded network.

In the example provided by Bushnell and Stoft (1997), the investor with pre-existing FTRs chooses the most profitable incremental FTR based on optimizing its final benefit. The investor is then awarded a mitigating incremental 1-2 FTR with associated power flows corresponding to the difference between the ex-ante and ex-post optimal dispatches. The pre-existing FTRs correspond to the actual dispatch of the system and become infeasible after expanding line 1-2, and therefore a mitigating 1-2 FTR[26] is allocated so that feasibility is exactly restored (that is, the investor "pays back" for the negative externalities to other agents). There is no allocation of proxy awards because the pre-expansion network is fully allocated by FTRs before the expansion. The amount of incremental FTRs is minimized because they represent a negative value to the investor and decrease its revenues from the pre-existing FTRs.

Bushnell and Stoft (1997) demonstrate that the increase in social welfare will be at least as large as the ex-post value of new contracts, when the FTRs initially match dispatch in the aggregate and new FTRs are allocated according to the feasibility rule. In particular, if social welfare is decreased by a transmission expansion, the investor will have to take FTRs with a negative value. (If social welfare is increased there will be free riding). Some agents might still benefit from investments that reduce social welfare, whenever their own commercial interests improve to an extent that more than offsets the negative value of the new FTRs. This problem can be solved if it is required that FTRs are used by each agent as a perfect hedge for their net load. In such a case, FTRs allocated under the feasibility rule ensure that no one will benefit from an expansion that reduces welfare. Our mechanism implicitly achieves this last property but through the use of proxy awards.

5 Concluding remarks

We proposed a merchant mechanism to expand electricity transmission. Proxy awards (or reserved FTRs) are a fundamental part of this mechanism. We defined them according to the best use of the current network along the same direction of the incremental expansion. The incremental FTR awards are allocated according to the investor preferences, and depend on the initial partial allocation of FTRs and network topology before and after expansion.

Our examples showed that the internalization of possible negative externalities caused by potential expansion is possible according to the rule proposed by Hogan (2002a): allocation of

[25] Note that this result will depend on the network interactions. In some cases the amount of incremental FTRs in the preference direction will differ from the new capacity created on a specific line. However, it will always amount to the new capacity created as defined by the scalar amount of incremental FTRs times the directional vector.

[26] The incremental 1-2 FTR can be decomposed into a 1-3 FTR and a 3-2 FTR

FTRs before (proxy FTRs) and after (incremental FTRs) the expansion is in the same direction and according to the feasibility rule. Under these circumstances, the investor will have the proper incentives to invest in transmission expansion in its preference direction given by his bid parameters. Likewise the larger the existing current capacity the greater the number of FTRs that must be reserved in order to deal with potential negative externalities depending on post network topology.

Our mechanism of long term FTRs is basically a way to hedge consumers or generators from long-run nodal price fluctuations by providing them with the necessary property transmission rights. Although our model is specifically designed to deal with loop flows, and the security-constrained version of our model can take care of contingency concerns, our proposed mechanism is to be applied to small line increments in meshed transmission networks. LTFTRs are efficient under non-lumpy marginal expansions of the transmission network, and lack of market power. Regulation has then an important complementary role in fostering large and lumpy projects where investment is large relative to market size, and in mitigating market power. Since revenues from nodal prices only recover a small part of total costs, LTFTRs must be complemented with a regulated framework that allows the recovery of fixed costs. The challenge is to effectively combine merchant and regulated transmission investments or, as Hogan (2003) puts it, to establish a rule in practice for drawing a line between merchant and regulated investment.

6 Annexes

6.1 Annex 1

6.1.1 Solution to program 32

The Lagrangian of the problem is:

$$\begin{aligned} L(a,\hat{t},d_{12}, l ,\Omega) &= b_{12} a d_{12} + g\,(C_{12}^{+} - T_{12} - (a+\hat{t})d_{12}) \\ &-q\,(p_{12}d_{12} - l\,d_{12}) - z\,(\,l\,(C_{12} - T_{12} - \hat{t}d_{12})) \\ &+e(C_{12} - T_{12} - \hat{t}d_{12}) + j\,(1-d_{12}^{2}) + k\,a + p\,l \end{aligned} \tag{35}$$

where g, q, z, e, j, k, and p are the multipliers associated with the respective constraints.

At optimality the Kuhn-Tucker conditions are:

$$\frac{\partial L(a,\hat{t},d_{12}, l ,\Omega)}{\partial a} = b_{12}d_{12} - g d_{12} = 0, \tag{36}$$

$$\begin{aligned} \frac{\partial L(a,\hat{t},d_{12}, l ,\Omega)}{\partial d_{12}} &= a b_{12} - (\hat{t} + a)g - (p_{12} - l)q \\ &+ l\,z\,\hat{t} - e\hat{t} - 2d_{12}j = 0, \end{aligned} \tag{37}$$

$$\frac{\partial L(a,\hat{t},d_{12}, l ,\Omega)}{\partial \hat{t}} = -g d_{12} + l\,z d_{12} - e d_{12} = 0, \tag{38}$$

$$\frac{\partial L(a,\hat{t},d_{12}, l ,\Omega)}{\partial l} = -d_{12}q - (C_{12} - T_{12} - \hat{t}d_{12})z = 0, \tag{39}$$

$$\frac{\partial L(a,\hat{t},d_{12},l,\Omega)}{\partial g} = (C_{12}^{+} - T_{12} - (a+\hat{t})d_{12}) = 0, \tag{40}$$

$$\frac{\partial L(a,\hat{t},d_{12},l,\Omega)}{\partial q} = -(p_{12}d_{12} - l d_{12}) = 0, \tag{41}$$

$$\frac{\partial L(a,\hat{t},d_{12},l,\Omega)}{\partial z} = -l\,(C_{12} - T_{12} - \hat{t}d_{12}) = 0, \tag{42}$$

$$\frac{\partial L(a,\hat{t},d_{12},l,\Omega)}{\partial e} = (C_{12} - T_{12} - \hat{t}d_{12}) = 0, \tag{43}$$

$$\frac{\partial L(a,\hat{t},d_{12},l,\Omega)}{\partial j} = (1 - d_{12}^{2}) = 0, \tag{44}$$

$$\frac{\partial L(a,\hat{t},d_{12},l,\Omega)}{\partial k} = a > 0,\ k = 0, \tag{45}$$

$$\frac{\partial L(a,\hat{t},d_{12},l,\Omega)}{\partial p} = l > 0,\ p = 0, \tag{46}$$

$$g, z \geq 0. \tag{47}$$

Equation (44) gives $d_{12} = 1$. Equation (36) gives $g = b_{12}$. Equation (41) gives $l = p_{12}$, equation (38) $z = g / p_{12} = b_{12} / p_{12}$ (e is zero because the constraint is redundant), and equation (39) $q = 0$. From this it follows (equation (37)) that $j = 0$ Furthermore equation (42) gives $\hat{t} = C_{12} - T_{12}$. Equation (40) implies that $a = C_{12}^{+} - T_{12} - \hat{t} = C_{12}^{+} - C_{12}$.

6.1.2 Solution to program 33

The Lagrangian of the problem is:

$$\begin{aligned}
&L(a,\hat{t},d,l,m,\Omega) = \\
&a(b_{13}d_{13} + b_{23}d_{23}) + g_1(C_{13} - \frac{2}{3}(T_{13} + (\hat{t}+a)d_{13}) - \frac{1}{3}(T_{23} + (\hat{t}+a)d_{23})) \\
&+g_2(C_{12} - \frac{1}{3}(T_{13} + (\hat{t}+a)d_{13}) + \frac{1}{3}(T_{23} + (\hat{t}+a)d_{23})) \\
&-z\,(l\,(C_{13} - (T_{13} + \hat{t}d_{13})) \\
&+e(C_{13} - (T_{13} + \hat{t}d_{13})) \\
&+j\,(1 - d_{13}^{2} - d_{23}^{2}) + ka + pl
\end{aligned} \tag{48}$$

where g_1 and g_2 are the Lagrange multipliers associated with transmission capacity on the lines 1-3 and 1-2 in the reduced network, respectively. z is the multiplier associated with the Kuhn-Tucker condition of transmission capacity in the pre-expansion network for line 1-3. This line has the Lagrange multipliers l associated with it before expansion. e is the investor's marginal value of transmission capacity in the pre-expansion network when allocating incremental FTRs. The normalization condition has the multiplier j and the non-negativity conditions have the associated multipliers k and p. The first order conditions are:

$$\frac{\partial L(a,\hat{t},d,l,\Omega)}{\partial a} = (b_{13}d_{13} + b_{23}d_{23}) - (\frac{2}{3}d_{13} + \frac{1}{3}d_{23})g_1 \tag{49}$$

$$-(\frac{1}{3}d_{13} - \frac{1}{3}d_{23})g_2 = 0,$$

$$\frac{\partial L(a,\hat{t},d,l,\Omega)}{\partial d_{13}} = ab_{13} - \frac{2}{3}(\hat{t}+a)g_1 - \frac{1}{3}(\hat{t}+a)g_2 \tag{50}$$

$$+zl\hat{t} - e\hat{t} - 2jd_{13} = 0,$$

$$\frac{\partial L(a,\hat{t},d,l,\Omega)}{\partial d_{23}} = ab_{23} - \frac{1}{3}(\hat{t}+a)g_1 + \frac{1}{3}(\hat{t}+a)g_2 \tag{51}$$

$$-2jd_{23} = 0,$$

$$\frac{\partial L(a,\hat{t},d,l,\Omega)}{\partial \hat{t}} = -(\frac{2}{3}d_{13} + \frac{1}{3}d_{23})g_1 - (\frac{1}{3}d_{13} - \frac{1}{3}d_{23})g_2 \tag{52}$$

$$+d_{13}zl - d_{13}e = 0,$$

$$\frac{\partial L(a,\hat{t},d,l,\Omega)}{\partial l} = -z(C_{13} - T_{13} + \hat{t}d_{13}) = 0, \tag{53}$$

$$\frac{\partial L(a,\hat{t},d,l,\Omega)}{\partial g_1} = C_{13} - \frac{2}{3}(T_{13} + (\hat{t}+a)d_{13}) \tag{54}$$

$$-\frac{1}{3}(T_{23} + (\hat{t}+a)d_{23}) = 0,$$

$$\frac{\partial L(a,\hat{t},d,l,\Omega)}{\partial g_2} = C_{12} - \frac{1}{3}(T_{13} + (\hat{t}+a)d_{13}) \tag{55}$$

$$+\frac{1}{3}(T_{23} + (\hat{t}+a)d_{23}) = 0,$$

$$\frac{\partial L(a,\hat{t},d,l,\Omega)}{\partial z} = -l(C_{13} - T_{13} + \hat{t}d_{13}) = 0, \tag{56}$$

$$\frac{\partial L(a,\hat{t},d,l,\Omega)}{\partial e} = (C_{13} - T_{13} - \hat{t}d_{13}) = 0, \tag{57}$$

$$\frac{\partial L(a,\hat{t},d,l,\Omega)}{\partial j} = 1 - d_{13}^2 - d_{23}^2 = 0, \tag{58}$$

$$\frac{\partial L(a,\hat{t},d,l,\Omega)}{\partial k} = a > 0,\ k = 0, \tag{59}$$

$$\frac{\partial L(a,\hat{t},d,l,\Omega)}{\partial p} = l > 0,\ p = 0, \tag{60}$$

The solution for the first order conditions is given by:

$$d_{13} = \frac{(1/3g_1 - 1/3g_2)}{\left((2/3g_1 + 1/3g_2 - zl)^2 + (1/3g_1 - 1/3g_2)^2\right)^{1/2}}$$

$$d_{23} = \frac{-(2/3g_1 + 1/3g_2 - zl)}{\left((2/3g_1 + 1/3g_2 - zl)^2 + (1/3g_1 - 1/3g_2)^2\right)^{1/2}}$$

$$a = \frac{C_{12}}{d_{13}}$$

$$\hat{t} = \frac{(C_{13} - T_{13})}{d_{13}}$$

$$g_1 = \frac{(b_{13} + Bb_{23} + g_2(B/3 - 1/3))}{(2/3 + B/3)}$$

$$g_2 = \frac{1}{(1 - B - AB + A)}\left[b_{13}(1 + 3A - B - 2A - AB)\right.$$
$$\left. + b_{23}(B + 3AB - B^2 - 2A - AB)\right]$$

$$zl = (1 + A)g_1 - A(b_{13} + b_{23})$$

with

$$A = \frac{C_{12}}{(C_{13} - T_{13})}$$

$$B = \frac{1}{(1 + A)}\frac{(C_{13} - 2C_{12} - T_{23})}{(C_{13} - T_{13})}$$

6.2 Annex 2

This annex derives the power transfer distribution factors (PTDFs) for the three-node network with two parallel lines, and where all lines have identical reactance. The net injection (or net generation) of power at each bus is denoted P_i. We have the following relationship between the net injection, the power flows P_{ij} and phase angles $?_i$:

$$P_i = \sum_j P_{ij} = \sum_j \frac{1}{x_{ij}}(q_i - q_j)$$

where x_{ij} is the line inductive reactance in per unit.

We can write the power flow equations as:

$$\begin{bmatrix} P_1 \\ P_2 \\ P_3 \end{bmatrix} = \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix}$$

The matrix is called the susceptance matrix. The matrix is singular, but by declaring one of the buses to have a phase angle of zero and eliminating its row and column from the matrix, the reactance matrix can be obtained by inversion. The resulting equation then gives the bus angles as a function of the bus injection:

$$\begin{bmatrix} q_2 \\ q_3 \end{bmatrix} = \begin{bmatrix} 2/3 & 1/3 \\ 1/3 & 2/3 \end{bmatrix} \begin{bmatrix} P_2 \\ P_3 \end{bmatrix}$$

The *PTDF* is the fraction of the amount of a transaction from one node to another node that flows over a given line. $PTDF_{ij,mn}$ is the fraction of a transaction from node m to node n that flows over a transmission line connecting node i and node j. The equation for the *PTDF* is:

$$PTDF_{ij,mn} = \frac{x_{im} - x_{jm} - x_{in} + x_{jn}}{x_{ij}}$$

where x_{ij} is the reactance of the transmission line connecting node i and node j and x_{im} is the entry in the i^{th} row and the m^{th} column of the bus reactance matrix. Utilizing the formula for the specific example network gives:

$PTDF_{12,13} = 1/3$, $PTDF_{13,13} = 2/3$, $PTDF_{23,13} = 1/3$,

$PTDF_{12,23} = -1/3$, $PTDF_{13,23} = 1/3$, $PTDF_{23,23} = 2/3$

$PTDF_{21,13} = -1/3$, $PTDF_{21,23} = 1/3$

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