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FAIR AND ECONOMICALLY SUSTAINABLE CHARGES FOR THE USE OF MOTORWAY INFRASTRUCTURE

Claus Doll

Institut für Wirtschaftspolitik und Wirtschaftsforschung

6.10.2003

Keywords: Cost Allocation, Charges, Motorway Infrastructure, Game Theory

JEL-Classification: e.g. C7, H2, H4, M4, R4

Contact Details:
Institut für Wirtschaftspolitik und Wirtschaftsforschung (IWW)
Universität Karlsruhe (TH)
Kollegium am Schloss, Bau IV

Tel: +49 (0)721 608-6042
Fax: +49 (0)721 607376
E-mail: doll@iww.uni-karlsruhe.de

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Abstract:

Recent studies in different countries have shown, that there is no agreement on a common procedure for a fair allocation of infrastructure costs. Thus, this paper follows the goal of advancing the discussion on the allocation of joint costs. We consider the case of road capital and running costs or German motorways, for which a sophisticated cost model had recently been developed and applied for estimating HGV toll levels. The cost model ensures the sustainability of road user charges by considering the full economic costs of providing, maintaining and operating the infrastructure, rather than considering short-run variable or marginal costs.

1 Aim and Structure of the Paper

In this paper the goal of advancing the discussion on the fair allocation of joint costs among users is followed. We consider the case of road capital and running costs or German motorways, for which a sophisticated cost model had recently been developed and applied for estimating HGV toll levels. The cost model ensures the sustainability of road user charges by considering the full economic costs of providing, maintaining and operating the infrastructure, rather than considering short-run variable or marginal costs.

The paper is organised as follows: Section 2 gives a very brief overview on recent road accounting and cost allocation studies and compares their results. Section 3 then goes into the economic theory of games and proposes some procedures, which are applicable in the case of road cost allocation. Section 4 discusses and describes the cost model used for the game-theoretic concepts. The Sections 5 finally presents some results and Section 6 gives some concluding remarks.

2 Traditional Cost Allocation Approaches

In recent years a number of governmental bodies in Europe and the United States have published road accounting studies. Among other goals, they follow the aim of calculating or verifying currently existing road user charges or taxes on fuels and on motor vehicles. For Germany the most relevant studies in this field are the report on proposed HGV charges for the planned motorway toll system by Prognos and IWW (University of Karlsruhe) to the Federal Ministry for Transport, Building and Housing (BMVBW) (Rommerskirchen et al. 2002) and the study on road and railway infrastructure costs by the German Institute for Economic Research (DIW), commissioned by the German Automobile Club (ADAC e.V.) and the Federal Agency for Freight Transport, Logistics and Disposal (BGL e.V.) (Link et al. 2000) In Austria the tariffs for the electronic motorway toll system from 2004 on was determined by a study conducted for the motorway financing society (ASFINAG (Herry et al. 2002) and in Switzerland there is an annual update made of the Swiss road accounts by the National Statistical Office (BFS) (Gritti and Schweizer 2001). On the European level the research project UNITE has carried out road cost accounts using the DIW model for 18 European countries (Link et al. 2002) and finally the Federal Highway Cost Allocation Study 1997 carried out for the U.S. department of Transportation, is to be mentioned. Table 1 compares the results per vehicle kilometre found by some of these studies, although a direct comparison is difficult due to the different classifications of motor vehicles used in the studies.

With the exception of the U.S. Highway cost allocation study and the reports for Austria, all of the studies mentioned above use some kind of equivalency factor system for allocating cost. However, there are big differences in the consideration of capacity demand by different vehicles and by the differentiation of construction elements. In the U.S. study, for some construction elements an incremental approach is applied, which allocates the additional costs caused by a particular vehicle type to this and all “more demanding” vehicles. A complete different approach is followed by the Austrian studies, which determine cost shares by applying regression models of construction and maintenance costs over various indicators of vehicle movements. This “econometric approach” results in relatively high cost shares, which are similar to thise found by the DIW model.

Table 1: Results of recent road cost allocation studies

Institute	Prognos IWW	DIW	DIW (UNITE)	Herry WKR	Herry
Area / country	D (BAB)	D (BAB)	D (BAB)	AT (A+S)	AT (ASFINAG)
Year	2003	1997	2005	2000	2004
Motorcycles	1,4	0,6	0,6		
Passenger cars	2,1	1,2	1,3	2,8	3,1
Buses	10,0	4,3	4,5	19,2	
Vans (< 3.5t)		2,2	2,3		
	2,6	3,7	5,6	9,0	13,7
HGV 3,5-12t, 2 axles	12,5			9,0	
HGV 12-18t, 2 axles		10,7		10,3	
HGV 18-28t, 3 axles		12,1	12,5	15,7	18,6
HGV > 28t, >=4 axles		12,4		28,6	31,3
Other use vehicles	3,8	7,5	7,7		
All vehicles	3,4	2,3	2,7	5,7	6,2
Share of HGV-costs	45,5%	57,3%	56,8%	56,1%	57,2%
Traffic volume HGV	17,0%	14,0%	137,0%	14,4%	13,9%

Source: Figures calculated from the sources stated in the table

The values presented in Table 1 show, that up to now there is no consensus on the correct allocation of road infrastructure costs. All approaches, including the Austrian allocation method, contain considerable elements of arbitrariness and thus will always be subject to criticism and mistrust by specific stakeholder groups or lobbyists. Thus, in the following sections we seek for more sophisticated methods of cost sharing, which are applicable in the case of road transport.

3 Cost Allocation and Game Theory

3.1 Basic Concepts

As has been shown in Section 2, in the transport sector costs are frequently allocated to vehicle types using vehicle-kilometres or axle kilometres, weighted by some equivalence factor. This approach assumes, that costs caused by a specific vehicle class are strictly proportional to its annual mileage. This assumption does not hold true in the case of infrastrucucture costs. The high share of fixed or blockwise fixed costs and the considerable non-linearity of the road track cost function $C(Q_i,A_{ij})$ (where the Q_i denote the annual vehicle kilometres performed by vehicle type *i* and A_{ij} denotes characteristic *j* of vehicle type *i*) leads to a dependency of the costs allocated to a particular vehicle class *i* to the traffic volume of all other vhicle classes j=1..n. This property calls for a more advanced coast allocation principle.

Procedures to allocate common or non-separable costs among a group of agents can be found in cooperative game theory. The main idea of this economic discipline is to analyse the profit potential of some agents when they act together on the basis of binding contracts or agreements. The achievable gains or the minimal costs of each coalition of agents (or players) are written into the so-called “characteristic function” v(S, where S denotes a subset of the set of all players N={i=1..n)}. Starting from the characteristic function cooperative game theory sug-gests the following central cost sharing procedures:

› The Core
› The Nucleolus
› The Shapley Value and

› Aumann-Shapley prices.

All these concepts have in common, that all costs are allocated among the players (pareto-efficiency) and that no player will have more costs allocated than he would have to bear when he would act on its own (individual rationality). The latter concept constitutes of a minimum stability of the allocation.

The Core is a concept that delivers an entire set of cost allocation vectors that satisfy the pareto-efficiency condition and that does not allocate more costs to any coalition S than that coalition would bear when acting alone (X(S) <= vIS)). The core represents the most stable set of cost allocations, but it has the disadvantage that it might either be empty or very large. This is not satisfying for a cost allocation scheme, but nevertheless the core concept plays an important role as it allows to judge other allocation schemes for their stability properties.

The Nucleolus constitutes of a cost allocation vector X=x1..xn where the least outcome of any coalition of players is maximised. The outcome (or excess) of a coalition is defined as the dif-ference of the costs which it would have to bear when acting on its own (v(S)) minus the sum of the costs allocated to all members of S (X(S)). The Nucleolus is a specifically equitable alloca-tion procedure and it is located within the core if the core exists. The concept can easily be ap-plied to games with very many players of a countable number of classes and is therefore appli-cable for price setting. Formally, the Nucleolus and the least core (leave away restriction 1) are computed by solving the following linear programme:

$$\min_{S \subseteq N}\left(v(S) - X(S)\right) \rightarrow \max!$$

$$\text{Subject to}: \quad I: X(S) \leq_L X'(S) \mid \min_{S \subseteq N}\left(v(S) - X'(S)\right) = \min_{S \subseteq N}\left(v(S) - X(S)\right) \qquad (3.1)$$

$$\text{and} \quad II: X(N) = v(N)$$

with:

N:	Set of all players i=*1..n*
S:	Coalition, *S*⊆*N*
X(S):	Costs allocated to all members of coalition *S*
v(S):	Costs jointly caused by the Members of coalition *S*

In Equation 3.1, side condition (I) ensures the uniqueness of the Nucleolus by defining an unique order of the excess vector even if the excesses of two elements are identical. Side condition (II) ensures the desired property of pareto-efficiency, i.e. the allocation of total costs among all players.

The Shapley value is determined by a single formula rather than by a optimisation procedure. It can be approached by an axiomatic approach or alternatively by probability considerations. The axiomatic approach demands that the cost allocation vector X=x1..xn is (1) pareto-efficient, (2) allocates equal costs to players which cause equal costs to any coalition which they join (symmetry) and (3) that it does not matter whether the costs are separately allocated by cost elements (where each costs of its own characteristic function v1...vm) or whether the allocation uses the joint characteristic function v=v1+...+vm (additivity). The probabilistic approach defines the allocated costs as the average marginal costs of each player i considering each coalition S and the probability of its formation. The formula resulting from both approaches is given by the simple expression in Equation 3.2:

$$X_i = \sum_{S \subseteq N - \{i\}} \frac{|S|!\left(|N| - |S| - 1\right)!}{|N|!}\left(v(S) - v(S - \{i\})\right) \tag{3.2}$$

Aumann-Shapley prices are the result of extending the Shapley-value to games with infinitely many players. Like this, Aumann-Shapley prices can be approached by several ways: An axiomatic system, a probabilistic approach (= the mixing value) and by the limit which the formula of the Shapley value takes when N approaches infinity. All these approaches result in a single value, which is computed by the average marginal costs of player i concerning all levels of total demand, but a constant mix of demand concerning all players i=1..n. In contrast to the original Shapley value, which is very sensitive to the definition of players, Aumann-Shapley prices re-main relatively stable to the regrouping of players or to small changes in the characteristic function. However, as can be seen in the following definition function, they require a at least continuous cost functions. Fixed cost element cannot be allocated by this cost sharing mechanism. When C(q1...qn) describes the cost function of the n types of players, Aumann-Shapley prices are computed as follows:

$$p_i = \int_0^1 \frac{\partial f\left(t\mu_1 \ldots t\mu_n\right)}{\partial \mu_i} dt \tag{3.3}$$

with:

p_i:	Costs allocated to a non-atomic player *i*
f():	Cost function (characteristic function)
t:	Model variable
μ_i:	Total demand by player group *i*

These cost allocation methods and variations of them had been applied to allocate costs in various sectors of economy - mostly to allocate investment of service related costs. Littlechild and Thompson (1977) have estimated aircraft landing fees at Birmingham airport applying the Nucleolus and the Shapley value, Billera and Heath (1978) have determined optimal telephone billing rates and Castarno-Pardo and Garcia-Diaz (1995) have allocated road pavement costs using Aumann-Shapley prices. However, to our knowledge this approach has never been used before to allocate external costs of transport. Accordingly, the application of these techniques to the problem of allocating the costs of railway noise among several train classes is new and should bring some further light into the discussion of pricing for transport externalities.

3.2 Applicability for allocating road track costs

As is argued in Section 3.1, the Core-concept is not appropriate to the question of finding a fiar and equitable vector allocating joint costs among various agents as it is either not unique or my by empty. The uniqueness property is fulfilled by the Nucleolus, which is one of the core elements in case the core is non-empty, but it is very difficult to compute for many players. Further, one could say that the uniqueness of the Nucleolus is arbitrarily selected as it may depend on the order numbers of the players. Therefore, Holler and Illing (2001) have labelled the Nucleolus a “set-concept” comparable to the Core. For these reasons we concentrate on the Shapley-like concepts, which are the Shapley-value and the Aumann-Shapley-solution.

For real world problems the definition of what is a player is decisive for both, the applicability of certain concepts of game theory and for the output of the cost allocation itself. Both can be demonstrated for the Shapley-value. This solution concepts consists of the property to allocate fixed or blockwise fixed costs equally to all players for who they occur. This means, that the number of payers determines the share of costs borne by each of them. If we consider now the classification of road vehicles into two groups, e.g. passenger vehicles and lorries, each group gets allocated 50% of the fixed costs associated with road construction. If we now subdivide passenger vehicles into cars and buses, each group, including the lorries, which are nto subject to a reorganisation of the players, gets allocated only 33% of the fixed costs.

Vice versa, starting from this undesirable property we define the consistency condition as follows: A cost allocation scheme is consistent with respect to the grouping of players, if the allocation to a player i is not affected by the regrouping of other players $j \neq j$. This property should be fulfilled by cost allocation schemes in order to obtain results, which are robust with respect to the quality of input information. In case the goal of cost allocation is to derive user prices the players in the game are to be defined as the units, which are to be priced. More generally, the players should be atomic units, which can not be further subdivided. If the game is specified according to this principle, the Shapley-value is consistent according to the above definition.

If we consider that, in the case of road traffic, the single units (the atomic players) are the vehicle kilometres or vehicle movements on the considered network section, the problem we face gets obvious: The complexity of the Shapley-value, e.g. the number of possible coalitions to be analysed, is 2^n, (where n denotes the number of players). For each of those coalitions, the characteristic function needs to be calculated in average $1+n/2$ times. While the problem for 10 players (=1024 coalitions), the problem s well computable even by an Microsoft Excel spreadsheet model, a number of 25 players, which corresponds to the number of vehicle groups considered by the U.S. Highway Cost Allocation Study, yields 33’554’432 coalitions. Depending on the complexity of the cost estimation algorithm, this dimension may already entail considerable non-acceptable computing times.

But how to proceed with the allocation of costs at roads, which may well have more than 50’000 vehicle passages per day (in both directions)? To make the problem more simple, Littlechild and Thompson (1974) have introduced the class of airport games, which are characterised by a strict order of the users, concerning their requirements towards the construction standard of infrastructure assets. In this case the Shapley-value coincides with the incremental cost procedure applied by the U.S. FHCAS. Due to the additivity property of the Shapley value this order can be different for various parts of the infrastructure, but the class of airport games does exclude cases, where several players demand for different dimensions of the asset and where total costs are not simply a linear function of these dimensions. An example can be given in the case of road transport: Heavy axles demand for additional thickness of the different layers (in particular of the main course), while cars due to their high traffic volume and safety requirements determine the width of the pavement. Total costs then are proportional to the product out of with and thickness So we can not say that a road, which was built for trucks, is adequate for passenger cars. Consequently, airport games are not applicable to the present case of allocating road track costs.

Fragnelli et al. (2000) have extended the class of airport games to so-called infrastructure games, which consist of two parts: construction games and maintenance games. While construction games are identical tot he airport games, maintenance games are described by a matrix of additional maintenance costs, which each user group causes on a ligher-level infrastructure. In the case of railway infrastructure cost allocation, where this model had been applied, this makes sense as e.g. regional trains also cause damages to high speed tracks and their specific facilities, although they do not need them. But also here a strict order of players concerning their cost causation is required and thus also the infrastructure games according to Fragnelli et al. (2000) do not apply to the road case.

A pragmatic solution for making the Shapley-value computable for a big number of players is to slightly change the number of players as follows: First, we determine the maximum number of players to be considered. All these players are equal in size, i.e. they consist of the same amount of traffic volume and each player is assigned to the characteristics a specific vehicle class. In other words, we press the original demand structure into a grid square and consider the contents of each grid a player in the new game. We only must ensure that each class or players is represented by at least one element of the grid. Clearly, the presentation of the prevailing demand structure improves with an increasing number of grid elements.

As many of the players, and consequently many of the coalitions to be analysed, are identical, the number of computation steps can be drastically reduced. The resulting procedure is called the "extended Shapley-value.

$$\phi_j = \left(\prod_{i \in M} \binom{\mu_i}{k_i^S} \frac{\kappa^S! - (q - \kappa^S - 1)!}{q!} \left(\mu_i - k_i^S\right) \left[v\left(\frac{n}{q} \left(\begin{pmatrix} k_1^S \\ \vdots \\ k_m^S \end{pmatrix} + \varsigma_j \right) \right) - v\left(\frac{n}{q} \begin{pmatrix} k_1^S \\ \vdots \\ k_m^S \end{pmatrix} \right) \right] \right) \tag{3.4}$$

with:

Φ_j:	Total amount allocated to player goup *j*.
n:	Number of players in the original game
m:	Number of player groups in the extended game
q:	Number of grid elements in the extended game
M:	Set of all player groups in the extended game (M={i=1...m})
μ_i:	Total number of grid elements allocated to player group *i*
S:	Index for a coalition of payer groups, represented by the vector k_i^S
k_i^S:	Number of grid elements of player group *i* in coalition *S*
κ^S:	Total number of grid elements in coalition *S*
ς_j:	Unity vector with $\varsigma_j(j)=1$ and $\varsigma_j(i \neq j)=0$ for each $i \in M$

If there is no modification to the original demand structure required to fit to the grid structure, the procedure leads to an exact computation of the Shapley value with much less computational effort. However, concerning usual traffic pattern on motorways we observe, that some vehicle classes do have a share of less than 1%, while passenger cars usually constitute of 70% to 90% of daily vehicle kilometres. On the one hand this uneven player structure favours the application of the extended Shapley concept as the huge players reduce the number of combinations of coalitions, as many of them are equal. On the other hand, the good representation of small players requires a high number of grid elements, which exponentially increases the complexity of the procedure. Table 2 presents the complexity issue for a very simple game structure.

Another approach towards the allocation of joint costs is the application of the Aumann-Shapley concept. However, in its elementary definition according to Equation 3.3 it puts some quite important restrictions on the cost function *f*: First, it must be continuous across the domain of all demand values $t\mu$ and second, $f(0)=0$ must be fulfilled. Together with some other conditions game theory defines the space of functions *pNA*, which must contain *f*. Aumann and Shapley show that only in this case that their so-called “diagonal formula” leads to a solution according to their axiomatic principles stated in section 3.1.

Table 2: Complexity of the extended Shapley procedure

Size of players i					Number of grid elements q:			
µ1	µ2	µ3	µ4	µ5	10	20	50	100
0,35	0,35	0,30			64	448	5.184	40.176
0,60	0,30	0,10			56	273	2.976	20.801
0,90	0,05	0,05			10	76	414	3.276
0,25	0,25	0,25	0,25		81	1.296	28.561	456.976
0,50	0,30	0,15	0,05		48	616	9.984	151.776
0,80	0,10	0,05	0,05		18	204	2.214	32.076
0,20	0,20	0,20	0,20	0,20	243	3.125	161.051	4.084.101
0,80	0,10	0,05	0,03	0,03	18	102	2.952	48.114
Original Shapley value:					1.024	1.048.576	1,13E+15	1,27E+30

Source: Own calculations

Mertens (1988a and 1988b) has proposed an extension of the diagonal formula due to Aumann and Shapley by exchanging the order of differentiation and integration in Equation 3.3. By this modification we achieve that blockwise fixed costs to not matter any more. The only remaining requirements to the cost function now is $f(0)=0$ and f must be differentiable around 0 and the level of total demand μ. Both conditions can easily be fulfilled by setting $f(|\mu|<k)=0$ (k is a small real constant) and by slightly in- or decreasing μ in case the second condition is violated.

In our case the cost function is not explicitly given as a mathematical expression and we thus deal with differential quotients rather than with derivatives. This gives us the possibility to introduce a “smoothing factor”, which determines the width of the differentiation interval. If we define the minimal required width e of the differential interval as the reverse value of the number of integration steps *m*, the smoothing factor *d* is a multiplier of *e*. As shown in Figure 1, the value limits to a 100% of cost allocation for increasing *m* (which implies decreasing *e*). Further, smoothing factors >>1 also the alternation in the degree of cost allocation can be reduced considerable.

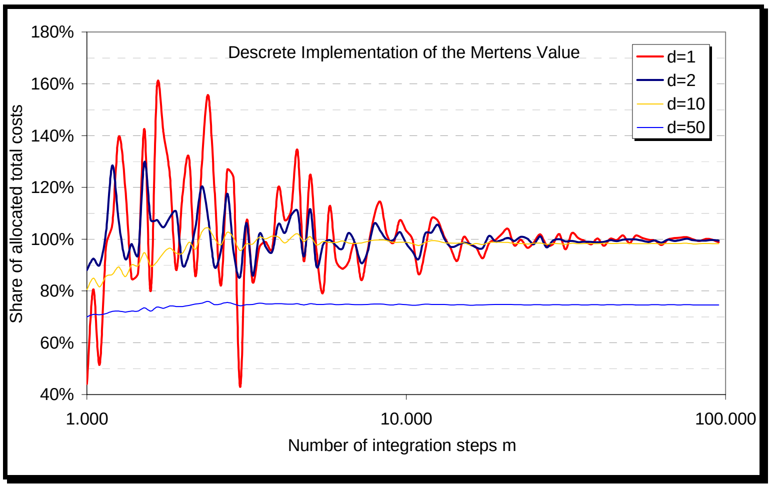

Figure 1: Limiting of the Mertens value

Source: Own calculations.

Comparing the complexity of the Mertens algorithm, which results already in a good level of cost allocation for 100’0000 integration steps, to the required number of iterations of the extended Shapley procedure, the Mertens value is clearly preferable. Considering the increase in complexity with additional player groups this judgement gets even more pronounced. While the extended Shapley procedure grows exponentially with the number of players, the Mertens value is linear to this respect.

4 the Cost Model

4.1 Basic concept

The cost model we use to generate the characteristic function is fairly simple. This property is important because it has to be computed several thousand, or even million times. The model is defined on the basis of the cost model for the German federal road network, which was developed by Prognos and IWW in 2002 for calculating the tariffs of the planned HGV motorway toll (Rommerskirchen et al. 2002). However, the model used here had to be extended and simplified in some dimensions.

It was extended to take into account the traffic volumes and the traffic mix when designing road structures in order to give a realistic estimate of the costs associated with each coalition of players. In this respect the model structure is organised along the main planning and maintenance steps of road investment procedures. On the other hand, the Prognos/IWW model had to be simplified with respect to the consideration of the stochastic depreciation of assets and the consideration of road quality data. The cost function used here is a simple time-based model of capital depreciation, which ignores the influence of traffic loads on depreciation periods.

Similar to the Prognos/IWW model, the cost function here considers depreciation, interest on capital and running costs for 18 different construction elements, which can be grouped into construction elements for line segments and construction elements for nodal points. For each of these elements the average life expectancy (=depreciation period) and the unit costs of construction are taken from the Prognos/IWW model. The original values of the unit costs had been provided by the German

Ministry for Transportation, Building and Housing (MBVBW) according to recent construction projects as well as from the forecast on rehabilitation requirements until 2010 (Maerschalk 2001). Here, we have averaged the values across federal states and road types in order to keep the model simple. Further, for each construction element a set of parameters is given, which determines the functional relationship between traffic volume, traffic mix and the dimensioning of the element. Table 3 presents the construction elements and their basic parameters.

Table 3: Construction elements of the cost model

Construction element	Used for: Lines.	Nodes	Depreciation period	Unit replacement costs
Land purchase	x	x	inf.	10,96 €/m²
Earthworks	x	x	90	70,84 €/m²
Frost preservation course	x	x	90	97,45 €/m³
Main course	x	x	45	97,45 €/m³
Binder Course	x	x	30	238,68 €/m³
Surface course	x	x	15	164,95 €/m³
Bridges	x	x	65	2181,30 €/m²
Tunnels	x		90	1693,19 €/m²
Other engineering works	x		50	100,00 €/m
Equipment	x	x	18	607,36 €/lane-m

Source: Figures based on Rommerskirchen (2002)

The basis for calculating capital costs is the estimation of gross asset values for each of the construction elements. Gross values are calculated by the replacement cost principle by multiplying the dimensions (m, lane-m, m² or m³) of each construction element with the unit replacement costs from Table 3. The dimensioning of each element is carried out separately for each coalition considered in the cost allocation procedure. The dimensioning process will be described in detail throughout this section.

When dimensioning the assets, e.g. setting their length (if not given by the network database), width and thickness (or more generally: the construction standard), we take the viewpoint of a traffic planner at the period of investment or renewal. Alternatively, we could have decided to put ourselves in the position of the decision maker at the current time period, which would be somewhat more compatible to the concept of replacement costs, but this approach would not be able to explain the causation of the costs associated with the actually existing infrastructure asset.

For the calculation of net asset values, information on the age of earthworks, the main course, the surface course and bridges is provided by the transport network database. The age of all assets, which are not directly referred to in database are set according to similar construction elements, of which data is existing. In particular it is assumed that the age of nodal points (including intersections and exit points) equals those of the line segments. In case the age of a particular asset exceeds its life expectancy it is assumed that it has been re-invested meanwhile and thus the age is reduced by the life expectancy.

The net values then are simply determined by a linear depreciation from the investment period to the ent of the life expectancy of the construction element. The use of variable life spans, which are depending on traffic load, would cause problems as at a particular time period an asset might be written down and thus would be re-invested for some coalitions, while it would be nearly written down for others. As this could lead to paradox effects, we have decided to keep the depreciation period of assets fix (compare the values given in Table 3).

Depreciation costs than are simply determined by the quotient of the gross capital value and the assets depreciation period. Interest costs are determined by multiplying the net asset value with a social interest rate. As the current model uses the replacement cost approach, and thus the replacement cost values already contain price inflation, we have to apply real interest values here. In accordance with Rommerskirchen (2002) we use a rate of 3.5%.

The cost model does not yet contain a sophisticated procedure for estimating running costs. Thus, we assume a value of 0.03 € per vehicle-km in order to meet the total running costs reported by the Prognos/IWW model.

4.2 Characteristics of traffic demand

The dimensions of the road elements are determined by three characteristics of the vehicle stream, which are:

- Passenger car equivalents (PE-weighted vehicle kilometres)
- Equivalent standard axle loads (ESAL-weighted vehicle kilometres) and
- Desired design speeds.

In the detailed annexes of the U.S. Federal Highway Cost Allocation Study 1997 (FHWA 1997) it is reported, that passenger car equivalents of goods are far from being a constant value. They depend on the vehicles' geometry, on the ration between vehicle weight and horsepower, on the gradient of the road and on traffic conditions. Out of the data found in

U.S. sources and in the German HBS manual on Road Design (FGSV 2001a) we have constructed a simple model of the form:

$$PE_i = \left(\frac{Q}{4000}\right)^{0.574} (1+S)^{45.71} \left(\frac{G_i}{12.5}\right)^{0.786} \left(\frac{L_i}{4}\right)^{113} \left(\frac{V_i}{120}\right)^{2.290} \tag{4.1}$$

with:

PE_i:	Passenger car equivalent of vehicle type i
Q:	Traffic volume in both directions; Q>4000
S:	Gradient of the road segment
$G:_i$	Weight / horsepower ratio of vehicle type i (Kg/kWh)
$L:_i$	Length of vehicle type *i* (m)
Vi:	Usual travel speed of vehicle type *i* (kph)

We assume, that the decisive parameter for the design of the horizontal road structure is the PE-weighted traffic volume in the peak hour at the end of the life expectancy of the main course. This can be easily calculated using vehicle demand growth rates and share of traffic in the peak period from the vehicle database in combination with data on the remaining life expectancy of the main course and the PE factors according to equation 4.1.

The standard axle factor of each vehicle type is calculated according to the scheme set out in the German RStO manual for road pavement design (FGSV 2001b) is calculated according to the 4th power rule. We consider the total weight of the vehicle and the load of the heaviest axle. This data is provided by the vehicle database. Further we assume that all other axles are loaded equally by the remaining vehicle weight. In the preliminary version of the model we assume, that all vehicles are fully loaded up to the total permissible gross weight. For purposes of road thickness design the ESAL-weighted traffic volumes during the entire life expectancy of the respective assets is required.

The desired design speed simply represents the highest usual travel speed among all vehicle classes, which is given by the vehicle database.

4.3 Dimensioning and Design principles

The width design of road segments distinguishes between a number of different design elements according to the German RAS-Q manual on the cross-sectional design of road profiles (FGSV 1996). The design elements and their width are presented Table 4 for a number of norm profiles. Considering the typical use of these norm profiles with respect to traffic volume, traffic mix and functional type of the road link given in compare FGSV 1988, we were able to create a simple design model for arbitrary traffic demand situations, which are characterised by the variables introduced above.

Table 4: Specification of Norm Profiles

Rule-cross section	Number of lanes	Width					
		lanes [m]	Edge trims [m]	Central reserves [m]	Emergency lanes [m]	Shoulder [m]	Separation reserves [m]
1	2	3	4	5	6	7	8
RQ 35,5	6	3,75 /3,50	0,75/0,50	3,50	2,50	1,50	3,00
RQ 33	6	3,50	0,50	3,00	2,00	1,50	3,00
RQ 29,5	4	3,75	0,75	3,50	2,50	1,50	3,00
RQ 26	4	3,50	0,50	3,00	2,00	1,50	3,00
RQ 20	4	3,25	0,50	2,00	-	1,50	1,75
RQ 15,5	2+1	3,75/3,25/3,50	0,25	-	-	2,50/1,50	1,75
RQ 10,5	2	3,50	0,25	-	-	1,50	1,75
RQ 9,5	2	3,00	0,25	-	-	1,50	1,75
RQ 7,5	2	2,75		-	-	1,50	1,25

Source: FGSV 1996

Starting from the single design elements presented in Table 4, we can define the width of the superstructure (consisting of the frost preservation course the main course, the binder course and the surface layer, bridges and tunnels), substructures (additinally consisting of all non-paved earthworks) and the total land to be purchased. Figure 2 presents the figures retrieved for an average traffic composition of 15% heavy vehicles and a design speed between 80 kph for low traffic volumes and 120 kph for high demand levels. Further to these default values, the design of the single profile elements are made variable with respect to the design speed and the vehicle geometry. Considering further the variability of the PE factors according to Equation 4.1, we receive a rather flexible model of structure road profile design.

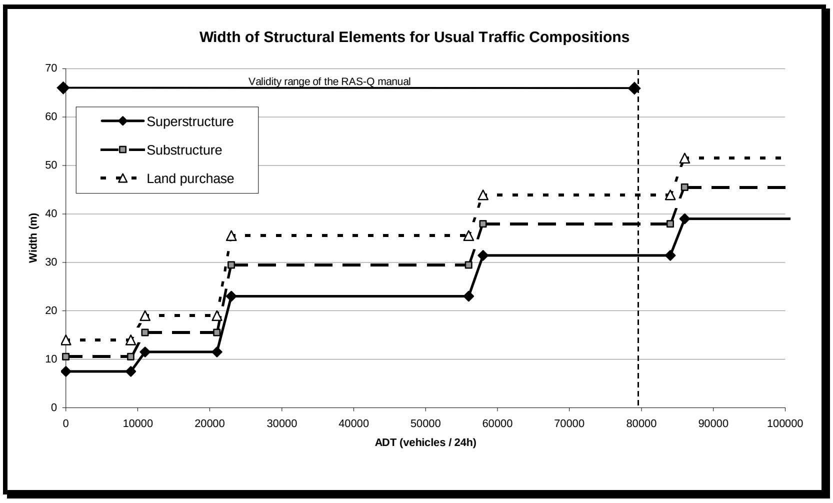

Figure 2: Road width design for a usual composition of the traffic flow

Source: Figures from FGSV (1996)

The design of the thickness of the several superstructure layers (frost preservation course, main course, binder course and surface course) is provided in relation to the RStO manual (FGSV 2001b). The input data provided is shown in Figure 3. Based on this data it was possible to estimate design models for each layer. The following functional form was chosen:

$$D_j = a_j + b_j ESAL_n^{c_j} \tag{4.1}$$

with:

D_j: Thickness of layer *j* (in m)
$ESAL_jL$ Standard axle loading during the depreciation period of *j*
a_j,b_j,c_j: Model parameters

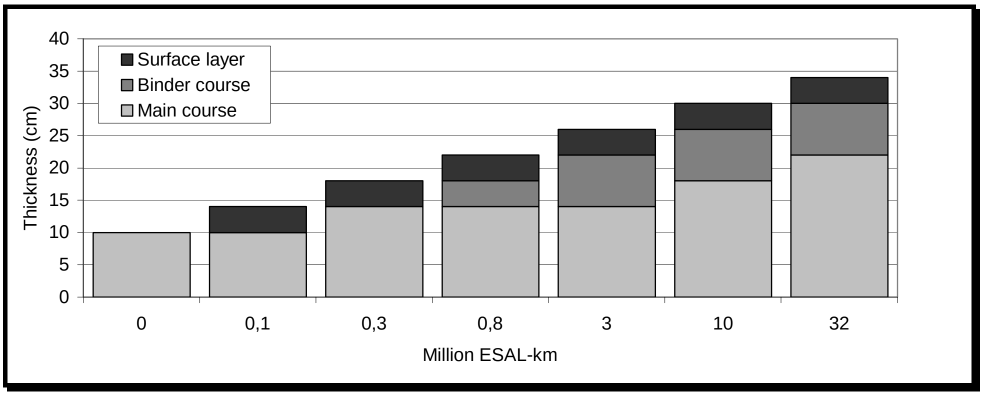

Figure 3: Thickness of superstructure layers for flexible pavements
Source: Figures from FGSV (2001b)

For other structural elements, which do not have a physical thickness, the thickness property is used to express the influence of traffic flow or regional parameters on purchase or construction prices.

- In this sense, the “thickness” of land purchase expresses the difference in purchase prices per m² between different area types. For urban areas German data provided by the Federal Statistical Office (Statistisches Bundesamt) reveals, that land prices for transportation infrastructure are nearly five times higher than in rural areas.
- Concerning earthworks, the parameter expresses the difficulty of ground preparation, including the establishment of embankments and

drainage in relation to the terrain formation. However, this model part is not implemented yet.

- In the case of bridges the parameter expresses the increased strength required due to heavy axles. From German and Swiss experiences we know that approximately 85% of bridge costs are fixed. For the remaining 15% we use the parameters estimated for the the thickness of the main course as other information had not been evaluated up to now.

Besides Tunnels and other engineering works, nodal points consist of the same construction elements than node-free road segments. In German two manuals for the design of nodes exist: The RAS-K-1 manual for nodal points at grade (FGSV 1988a) and the RAL-K-2 manual for grade-separated nodes (FGSV 1976). From these manuals we retrieve the following design principles for the length of travel lanes at nodes:

Up to a volume of turning traffic in either direction of 750 PE per hour with a design speed of 90 kph we can apply junctions at grade. However, this value decreases significantly when the design speed increases above 100 kph. Crossings and exit points at grade may consist of special lanes for traffic turning left and / right. Their requirement and dimension depends on traffic volumes and design speed.

We assume that level-free crossing consist of direct ramps for turning right, indirect ramps for turning left and weaving lanes for traffic exiting and entering the main travel lanes. The total length of all ramps can be estimated as a function of the radius of the indirect ramps. This radius varies between 25m for a design speed of 30 kph (on the ramp) and 50m for a maximum design speed of 40 kph. By defining a simple linear function, which explains the design speed in the ramp by the design speed on the node-free road segment we finally retrieve a relationship between the general design speed and the length of the ramps.

The width and the thickness of the ramps are defined in accordance with the design principles for straight road segments. Specific, however, is the determination of the land purchase, which is not simply going along the ramps but cuvers the entire area, enclosed by the node. For level free crossings we assume that 70% of all ramps are bridges.

There are two decisive parameters influencing the costs of nodal points: The share of turning traffic and the average number of nodal points per network kilometre. In the first case we estimate, that from the traffic flow per direction of the node-free lanes 15% turn into each direction. I.e. 70% of traffic goes straight on. The traffic mix of turning traffic is assumed to be equal to that on the road segments. Concerning the second parameter we estimate on the basis of data from the German federal road network, there is a nodal point each 12 km.

5 Results

The following subsections contain some results, which demonstrate a number of basic properties of the two cost allocation procedures which have been identified as appropriate tools to allocate the costs of large games with non-continuous cost functions: The extended Shapley value and the discrete implementation of the Mertens value. To generate these results, the following vehicle flow database is used:

Table 5: Average daily traffic per network kilometre

Pass. car	Mot. cycle	Bus+ coach	LDV < 3.5t	LDV > 3.5t	Oth. use v.	HGV 2 ax.	HGV 3 ax.	HGV 4 ax.	HGV 5+ ax.
22300	1357	3435	4125	385	37	166	347	666	1823

The costs are computed for a straight and flat road segment of 1 km length with 100 m of bridges and a tunnel of 25 m length. The age of assets is set to 50% of their life expectancy. The density of nodal points is assumed to be one each 12 network kilometres.

5.1 Performance of the Extended Shapley Procedure

The first question we are interested in is, how the number of grid elements, i.e. the difference between prevailing and model-internal traffic volumes, impacts the result of the allocation process. Table 6 thus presents total and average costs by vehicle category, which are allocated using 30 to 90 grid elements. As expected, the costs allocated to the vehicle categories with low traffic volumes are widely over-estimated the smaller the number of grids is. Only for passenger cars the fluctuation of allocated costs remains relatively stable within a range of 10%. As average costs are derived from total costs using the adapted traffic volumes, they deviate much less than the total cost figures. Accordingly, we can conclude that the extended Shapley value is appropriate for approaching tariffs, even with a relatively small number of grid elements (The computing time of the algorithm is approximately one our per network link using 90 grid elements, and only one minute with 60 elements).

The two right columns of Table 6 present calculations with an adapted traffic volume configuration, which already fits into a 50-element grid.

This delivers an exact solution of the Shapley value and thus it allows to compare the results of the extended Shapley algorithm to those of the Mertens procedure. While especially for the heavy vehicles the average cost results are surprisingly similar, considerable differences must be constituted for passenger cars, and here in particular for motorcycles and buses.

Table 6: Results for various grid densities of the extended Shapley procedure

Vehicle-category	Original traffic mix – Extended Shapley procedure – Number of grid elements							Adapted traffic mix	
								Ext-S Grid	Mertens Inter.
	30	40	50	60	70	80	90	50	100000
	Total allocated annual costs (Euro / road-km)								
Pass. car	209,1	205,1	221,3	232,8	234,9	220,8	236,1	227,2	293,5
M.cycle	28,6	23,3	21,2	21,3	20,7	22,0	21,4	23,5	10,6
Bus+coach	32,3	44,9	52,8	50,0	44,5	49,2	46,5	38,8	59,7
LDV < 3.5t	102,9	108,2	113,4	113,8	117,1	119,8	124,4	118,0	135,2
LDV > 3.5t	68,0	53,9	43,0	38,5	34,2	30,8	28,6	47,3	13,2
Other use v.	58,9	46,9	36,6	32,9	29,0	26,1	24,2	41,0	1,2
HGV 2 ax.	105,7	87,0	69,4	60,3	54,5	49,1	45,8	74,8	9,6
HGV 3 ax.	109,1	90,0	72,1	62,5	56,6	51,1	47,8	77,6	21,2
HGV 4 ax.	132,8	106,2	84,5	74,7	67,2	60,6	56,6	91,1	48,4
HGV 5+ ax.	166,7	134,8	109,3	191,1	171,8	155,5	144,8	118,1	168,3
	Allocated average costs (Euro / vehicle-km)								
Pass. car	0,030	0,029	0,029	0,030	0,030	0,028	0,030	0,029	0,036
M.cycle	0,037	0,039	0,047	0,052	0,058	0,035	0,038	0,048	0,021
Bus+coach	0,042	0,038	0,039	0,041	0,041	0,039	0,041	0,040	0,048
LDV < 3.5t	0,079	0,079	0,081	0,082	0,082	0,084	0,085	0,084	0,090
LDV > 3.5t	0,088	0,091	0,095	0,094	0,095	0,099	0,101	0,098	0,094
Other use v.	0,076	0,079	0,081	0,080	0,081	0,084	0,086	0,084	0,088
HGV 2 ax.	0,137	0,146	0,154	0,147	0,152	0,157	0,163	0,154	0,159
HGV 3 ax.	0,142	0,151	0,160	0,153	0,158	0,164	0,170	0,160	0,167
HGV 4 ax.	0,172	0,178	0,187	0,182	0,187	0,194	0,201	0,188	0,199
HGV 5+ ax.	0,216	0,226	0,242	0,233	0,239	0,249	0,257	0,244	0,253

Source: Own calculations

5.2 Performance of the Mertens value

Concerning the Mertens value, the composition of the traffic flow does not matter for the complexity of its computation. However, the number of iterations for computing the integral from zero to the total traffic volume, and the width of the differentiation interval (the “smoothing factor) could be of relevance of the results. Some selected configurations of these parameters are presented in Table 7. The results for a smoothing factor of 1 are not presented as the results obtained with this parameter are not convincing.

Table 7: Performance of the Mertens procedure

Vehicle class	Mertens-Value							
Iterations:	10000	10000	50000	50000	100000	100000	500000	500000
Smoothing:	3	10	3	10	3	10	3	10
	Average allocated annual costs (Euro / vehicle-km)							
Pass. car	0,0315	0,0353	0,0356	0,0355	0,0361	0,0356	0,0354	0,0353
M. cycle	0,0181	0,0214	0,0216	0,0213	0,0214	0,0213	0,0212	0,0212
Bus + coach	0,0459	0,0471	0,0476	0,0475	0,0476	0,0475	0,0475	0,0474
LDV < 3.5t	0,0886	0,0893	0,0897	0,0897	0,0898	0,0897	0,0894	0,0893
LDV > 3.5t	0,0929	0,0937	0,0940	0,0940	0,0941	0,0941	0,0937	0,0937
Other use v.	0,0869	0,0869	0,0876	0,0875	0,0878	0,0876	0,0872	0,0872
HGV 2 ax.	0,1560	0,1558	0,1591	0,1565	0,1591	0,1566	0,1584	0,1559
HGV 3 ax.	0,1642	0,1630	0,1673	0,1638	0,1673	0,1639	0,1666	0,1631
HGV 4 ax.	0,1977	0,1945	0,1988	0,1943	0,1992	0,1944	0,1981	0,1936
HGV 5+ ax.	0,2483	0,2477	0,2526	0,2472	0,2529	0,2473	0,2517	0,2462

Source: Own calculations

The numbers reveal, that the value of the smoothing factor is more relevant for the results obtained than the complexity of the procedure, which is expressed in the number of iterations. This general statement holds not true for 10000 iterations, which indicates that the least level of detail of the Mertens algorithm is somewhere between 10000 and 50000 items, However, Table 7 also reveals, that the algorithm is much more stable than the extended Shapley procedure as the variations of average costs ranges only within several percent.

5.3 The influence of network properties on the Mertens value

Table 8 presents some results obtained for nine different scenarios on the network and transport volume input data. The following variables are considered in the scenarios:

Age of assets: By default this parameter is set to 50% of the assets life expectancy (LExp) as reported in Table 3. In addition, the network scenarios consider a set of very new assets (age=0) and a set of written-down assets (age=life expectancy). Due to the long life spans of many road construction elements we observe difference in average costs of a factor three to four across all vehicle classes.

HGV-Share: In the default case the share of HGVs is approximately 9% of total traffic volume. Here, we consider higher values of 15%, 20% and 30%. It is interesting to observe, that the model outputs hardly react on these changes. Thus, the tariffs calculated can be considered stable against different traffic situations.

The reaction of the model for changing shares of fixed costs is tested using the frost sensitivity class. This directly determines the thickness of the frost preservation course. As the frost-save underground is determined by the sum of the frost preservation course and the main course, this construction element is not totally independent from traffic loads.

Respectively, the model allocates most of the additional costs to vehicles with heavy axles.

Table 8: Average costs for various network configurations

Veh..class	Mertens-Value								
Variation in:	Age of assets			HGV-Share			Frost sensitivity class		
Parameter:	0	LExp/2	LExp	15%	20%	30%	1	2	3
	Average allocated annual costs (Euro / vehicle-km)								
Pass. car	0,0499	0,0356	0,0216	0,0344	0,0357	0,0359	0,0360	0,0369	0,0356
M. cycle	0,0295	0,0213	0,0133	0,0208	0,0211	0,0211	0,0215	0,0221	0,0217
Bus + coach	0,0671	0,0475	0,0286	0,0461	0,0479	0,0481	0,0480	0,0492	0,0479
LDV < 3.5t	0,1425	0,0897	0,0491	0,0867	0,0908	0,0916	0,0905	0,0928	0,0897
LDV > 3.5t	0,1503	0,0948	0,0513	0,0902	0,0936	0,0938	0,0948	0,0971	0,0941
Other use v.	0,1413	0,0876	0,0489	0,0847	0,0883	0,0890	0,0884	0,0907	0,0929
HGV 2 ax.	0,2821	0,1566	0,0778	0,1445	0,1437	0,1395	0,1533	0,1566	0,1576
HGV 3 ax.	0,2970	0,1639	0,0819	0,1523	0,1486	0,1435	0,1606	0,1633	0,1639
HGV 4 ax.	0,3536	0,1944	0,0965	0,1815	0,1765	0,1707	0,1898	0,1930	0,1944
HGV 5+ ax.	0,4494	0,2474	0,1206	0,2283	0,2270	0,2205	0,2416	0,2464	0,2473

LEsp = Life Expectancy
Source: Own calculations

6 Conclusions

The cost model used in this paper is not totally finalised yet, as some parts or input values need further improvement. Thus, the absolute results presented in Section 5 must not be considered as recommended levels of road user charges. The values are only presented to demonstrate the structure of the solution concepts.

With this paper we were able to show, that the application of cost allocation concepts of game theory is possible even for problems with very many players and the existence of cost functions, which are not given as mathematical expressions and which do not fulfil the requirements of the space of functions pNA. By some refinements of the original formulae of the Shapley value and the Aumann-Shapley-solution for non-atomic games it was possible to reduce the usually excessive computation times of these concepts to an acceptable level.

The advantage of game-theoretic solutions is, that we first agree on a number of elementary fairness criteria, which should be fulfilled. Based on these criteria we can select a mathematical solution concept. Further, we have used standard engineering manuals to determine the characteristic function of the game. This means, that the level of arbitrariness, which is absolutely a point of criticism of the traditional cost allocation procedures, which are usually applied on road track cost accounting, can be substantially reduced.

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